NCETM
NATIONAL CENTRE FOR EXCELLENCE IN THE TEACHING OF MATHEMATICS

Multiplicative Reasoning

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

Question 3

How confident are you that you understand and can support children to quickly derive multiplication facts?

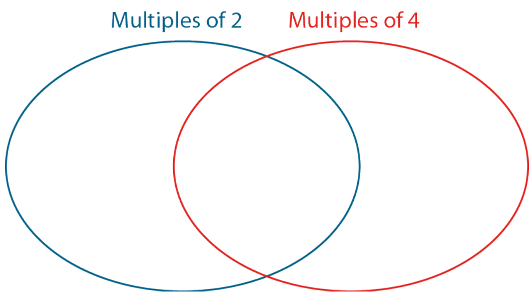
1 ☑ 2 ☐ 3 ☐ 4 ☐

How would you respond …?

a. How could the Venn diagram be used to explore the relationship between the two and four times tables?

'Place these numbers in the diagram. In the overlapping section, you should place the number that are multiples of both two and four. Numbers that are neither multiples of two nor four, should be placed outside the circles.'

0	**2**	**4**	**23**	**14**	**31**	**20**	**16**	**40**



b. How can children use their understanding of adjacent products to derive multiplication facts?

c. How can children use the relationship between the six and twelve times tables to solve this problem?
Roses are sold in bunches of 12; this is called 'a dozen' roses. If a shop has seven bunches, how many roses are there altogether?

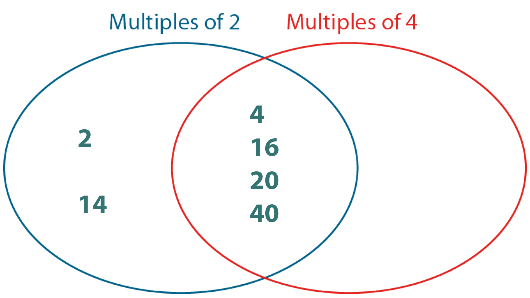
Responses

Note your responses to the questions here before you engage with the rest of this section:

Did you notice that…?

a. By sorting the numbers into a Venn diagram, children will be able to see that all multiples of four are also multiples of two. Ask questions to draw children's attention to the patterns and connections.

- *'Which section does not have any numbers in it? Why?'*
- *'What do you notice about the numbers in the section where the two sets overlap?'*
- *'What do you notice about the numbers that don't go inside the circles?'*

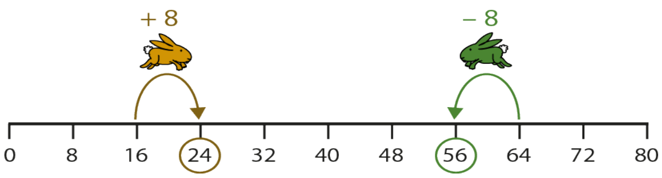


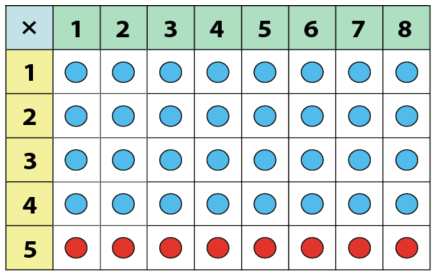
This builds understanding of the doubling and halving relationship between these times tables.

b. Children should have time to explore patterns in times tables. For example, in the eight times table, children will be able to see the following:

- The products are all even numbers.
- Products in the eight times table are also products of the two and four times tables.
- Working in sequence down the list, the product increases by eight each time.

Focus on the fact that adjacent multiples of eight have a difference of eight. This knowledge can then be used to find the next or previous multiple of eight from any given multiple, as shown on the number line and array.





c. Children can do 7 × 6 and then double to find 7 × 12.

Relationships between times tables

In this section, the relationship between the two, four, eight and three, six, nine times tables will be explored, to support children in being able to derive multiplication facts.

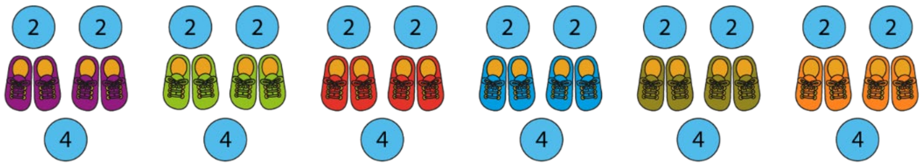
Although this section is focused on the relationships between times tables, it is worth noting that in order for children to become fluent, as well as using the connections being discussed, they must have a secure understanding of how the times tables are constructed. As with all of the times tables, regular practice will be needed to develop fluency, both when reciting the times tables, for example, *'One four is four, two fours are eight…'* and with isolated multiplication facts, for example, *'I know that seven fours are 28'*.

×	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6		12	15			24		30		
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12		24	30			48		60		
7	0	7	14		28	35			56		70		
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18		36	45			72		90		
10	0	10	20	30	40	50	60	70	80	90	10	11	12
11	0	11	22		44	55			88		11		
12	0	12	24		48	60			96		12		

Two, four and eight times tables

Children will be familiar with the two, five and ten times tables before developing the four and eight times tables. They should have also had experience with doubling and halving.

It is important that children understand that when learning the four and eight times tables, they do not have 26 new facts to learn: some facts are already known through commutativity and others are related facts from doubling. Children will have already explored the doubling relationship when learning the five and ten times tables.



2	2	2	2	2	2	2	2	2	2	2	2
4		4		4		4		4		4	

Discuss the relationship between the number of twos and the number of fours, in terms of groups of twos or groups of fours, using a pictorial representation, before using a manipulative, such as counters for the groups.

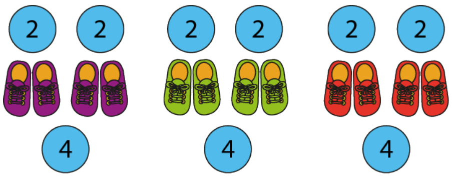
From this, children should deduce that for every one group of four, there are two groups of two.

The language of doubling and halving can then be used to compare two equations with the same product, for example $6 \times 2 = 12$ and $3 \times 4 = 12$.

If we show these calculations in a paired way …

$$\begin{array}{ccccc} 6 & \times & 2 & = & 12 \\ \downarrow & & \downarrow & & \\ 3 & \times & 4 & = & 12 \end{array}$$

you will notice the relationship between the 6 and the 3 (halves), and the 2 and the 4 (doubles).



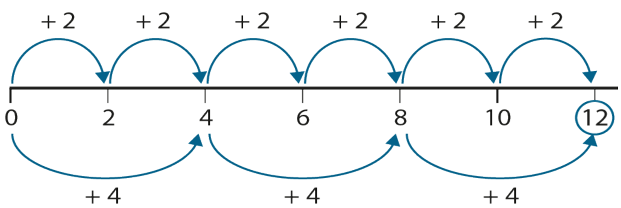
$6 \times 2 = 12$

$3 \times 4 = 12$

Again, a pictorial representation can be used, this time with a number line to demonstrate the number of jumps.

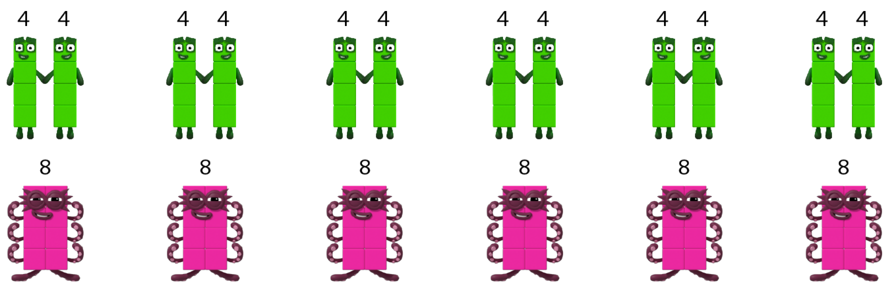
- *'<u>Three</u> times four is equal to twelve, so <u>double-three</u> times two is equal to twelve.'*
- *'<u>Six</u> times two is equal to twelve, so <u>half-of-six</u> times four is equal to twelve.'*

6 twos



3 fours

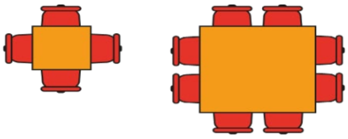
In the same way the doubling and halving relationship was developed between the two and four times tables, it can be explored between the four and eight times tables.



4	4	4	4	4	4	4	4	4	4	4	4
8		8		8		8		8		8	

Contextual problems can be used to deepen children's understanding.

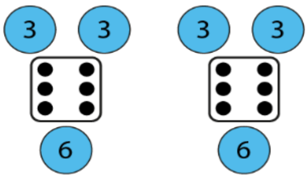
A class of children can sit around tables in eights or in fours. If they sit around tables in fours, they need six tables. How many tables will they need if they sit in eights?



Children will need to develop the understanding that a multiple of eight is also a multiple of four and two, but not all multiples of two and four are multiples of eight.

Three, six and nine times tables

The doubling and halving relationship between the three and six times tables should be explored in the same manner as for the two and four and four and eight times tables.



$4 \times 3 = 12$

$2 \times 6 = 12$

Alongside exploring the relationship between tables, children also need to understand the structure within tables, identifying that adjacent products have a difference of six. This can then be used to find the next or previous multiple of 6.

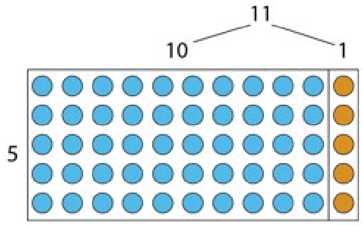
	$\times 6$		
0	0		
1	6		
2	12	$\downarrow +6$	$3 \times 6 = 2 \times 6 + 6$
3			
4	24		
5	30		
6	36		
7		$\uparrow -6$	$6 \times 6 = 7 \times 6 - 6$
8	48		
9	54		
10	60		
11	66		
12	72		

Up until this point, the focus has been on the doubling/halving relationship between times tables.

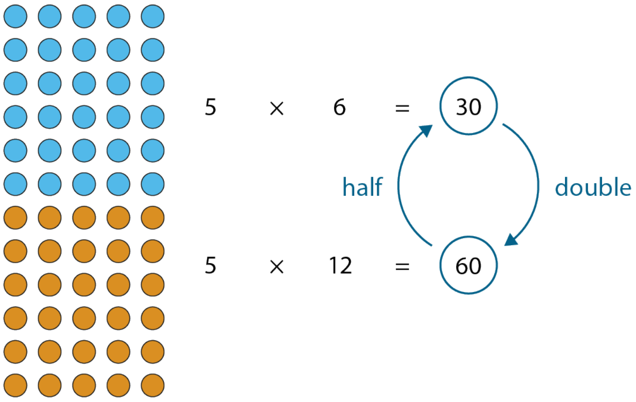
However, the relationship between the three and nine times tables is one of tripling. As with the previous examples, it is important children understand that for every one group of nine, there are three groups of three.

Distributive law, which is explored in Question 4, can be applied to enable children to find multiplication facts for the eleven and twelve times tables.

However, children could draw upon their understanding of the 'ten and a bit relationship' when finding the eleven times table. For example, $5 \times 11 = 5 \times 10 + 5$:



When deriving twelve times table facts, facts from the six times table can be applied.



'Six is half of twelve, so five sixes is half of five-twelves.'

'Twelve is double six, so five twelves is double five-sixes.'

Common errors in this area may include:

- children doubling instead of tripling from the three to the nine times table.

What to look for

Can a child:

- derive related multiplication facts from adjacent products?
- use the relationships between times tables to find related facts?

Links to supporting materials:

NCETM Primary Professional Development materials, Spine 2: Multiplication and Division:

- Topic 2.1: Counting, unitising and coins
- Topic 2.7: Times tables: 2, 4 and 8, and the relationship between them

Notes:

Key learning from support material and self-study:

What I will focus on developing in my classroom practice: