MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

6 Geometry

6.2 Perimeter, area and volume

Guidance document | Key Stage 3

Making connections

The NCETM has identified a set of six 'mathematical themes' within Key Stage 3 mathematics that bring together a group of 'core concepts'.

The sixth of these themes is *Geometry*, which covers the following interconnected core concepts:

6.1 Geometrical properties

6.2 **Perimeter, area and volume**

6.3 Transforming shapes

6.4 Constructions

This guidance document breaks down core concept *6.2 Perimeter, area and volume* into three statements of knowledge, skills and understanding:

6.2.1 Understand the concept of perimeter and use it in a range of problem-solving situations

6.2.2 Understand the concept of area and use it in a range of problem-solving situations

6.2.3 Understand the concept of volume and use it in a range of problem-solving situations

Then, for each of these statements of knowledge, skills and understanding we offer a set of key ideas to help guide teacher planning.

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Overview

At Key Stage 2, students will have had the opportunity to measure the perimeter of simple 2D shapes, find the area by counting squares and estimate volume by counting blocks. They should have calculated the area of rectangles, triangles and parallelograms, and the volume of cubes and cuboids using formulae. They should also have had opportunities to develop their conceptual understanding by relating the area of rectangles to parallelograms and triangles.

The extent to which students have explored these concepts may vary. There is a danger that the study of this element of the curriculum could be reduced to the mere memorising of formulae and the execution of learnt procedures. It is important that students have a secure and deep understanding of perimeter, area and volume before further developing these ideas in Key Stage 3. Students should fully understand the concepts involved, appreciate how the various formulae are derived and connected, and reason mathematically to solve a wide range of problems, including those in new and unfamiliar situations.

At Key Stage 3, when calculating perimeters, students will use the properties of parallelograms, isosceles triangles and trapezia, as well as non-standard shapes, and reason mathematically to deduce missing information. They will learn about the perimeter (circumference) of circles and that the ratio between circumference and diameter is the same for all circles. When calculating areas and volumes, this will include students using their knowledge of area of circles and the surface area and volume of prisms (including cylinders).

Prior learning

Before beginning to teach *Perimeter, area and volume* at Key Stage 3, students should already have a secure understanding of the following from previous study:

Key stage	Learning outcome
Upper Key Stage 2	• Find the area of rectilinear shapes by counting squares • Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres • Calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes • Estimate volume [for example, using 1 cm^3 blocks to build cuboids (including cubes)] and capacity [for example, using water] • Recognise that shapes with the same areas can have different perimeters and vice versa • Recognise when it is possible to use formulae for area and volume of shapes • Calculate the area of parallelograms and triangles • Calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3]

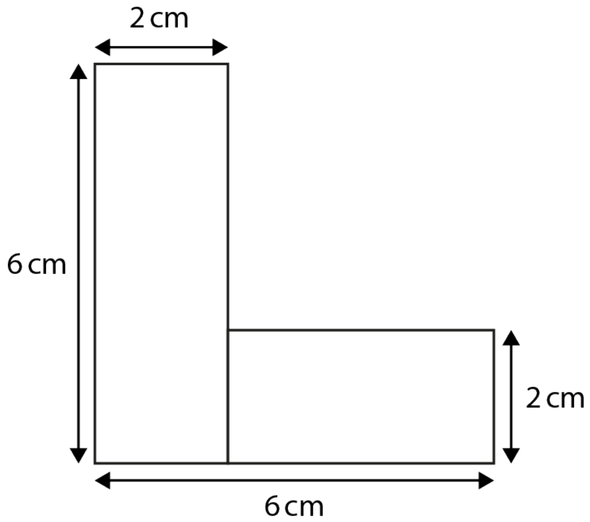
You may find it useful to speak to your partner schools to see how the above has been covered and the language used.

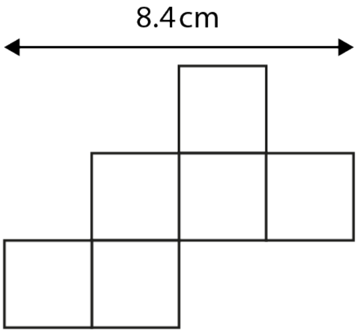
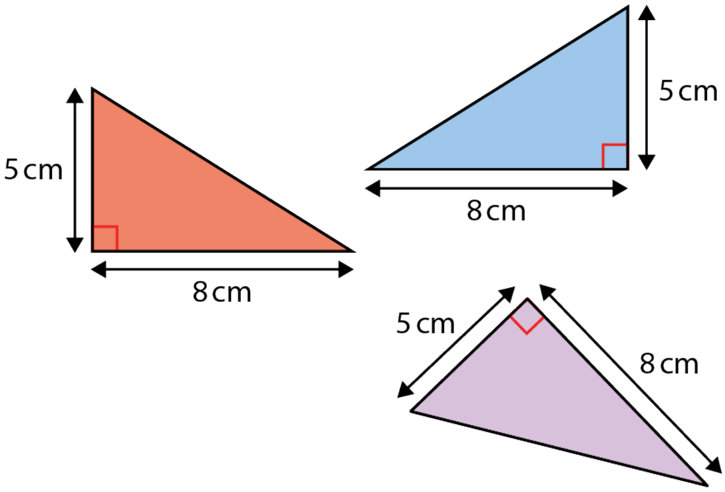
You can find further details regarding prior learning in the following segments of the NCETM primary mastery professional development materials[1]:

- Year 4: 2.16 Multiplicative contexts: area and perimeter 1
- Year 6: 2.30 Multiplicative contexts: area and perimeter 2

Checking prior learning

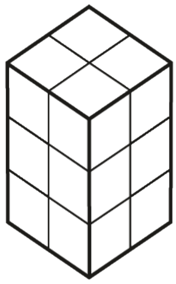
The following activities from the NCETM primary assessment materials[2] and the Standards & Testing Agency's past mathematics papers[3] offer useful ideas for assessment, which you can use in your classes to check whether prior learning is secure:

Reference	Activity
Year 4 page 22	The shape below is made from two rectangles. Identify the perimeter of each of the two rectangles. How many 1 cm squares would fit into the smaller rectangle? How many more squares fit into the larger rectangle? 

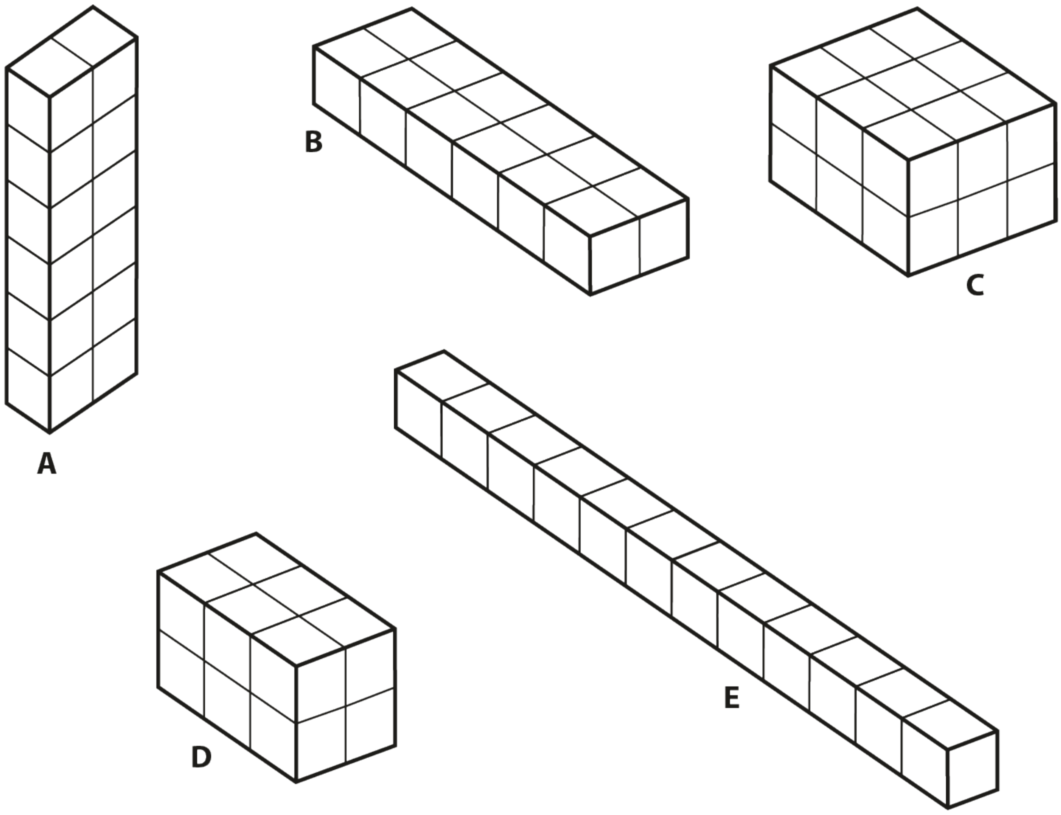
Year 6 page 31	Here is a tiled floor pattern. It is made from squares. Work out the perimeter of the design. Give your answer in metres. 
Year 6 page 32	Which of these right-angled triangles have an area of 20 cm^2? 
Year 6 page 32	Think about these rectangles: • a 4 cm by 6 cm rectangle • a 12 cm by 2 cm rectangle • a 3 cm by 8 cm rectangle. What's the same? What's different?
Year 6 page 32	The diameter of a golf ball is 4 cm. I want to make a box which will hold six golf balls. What size could my box be? Is there more than one answer?

2016 Key Stage 2
Mathematics
Paper 3: reasoning
Question 10

Emma makes a cuboid using 12 cubes.



Write the letter of the cuboid that has a **different** volume from Emma's cuboid.



Source: *Standards & Testing Agency*
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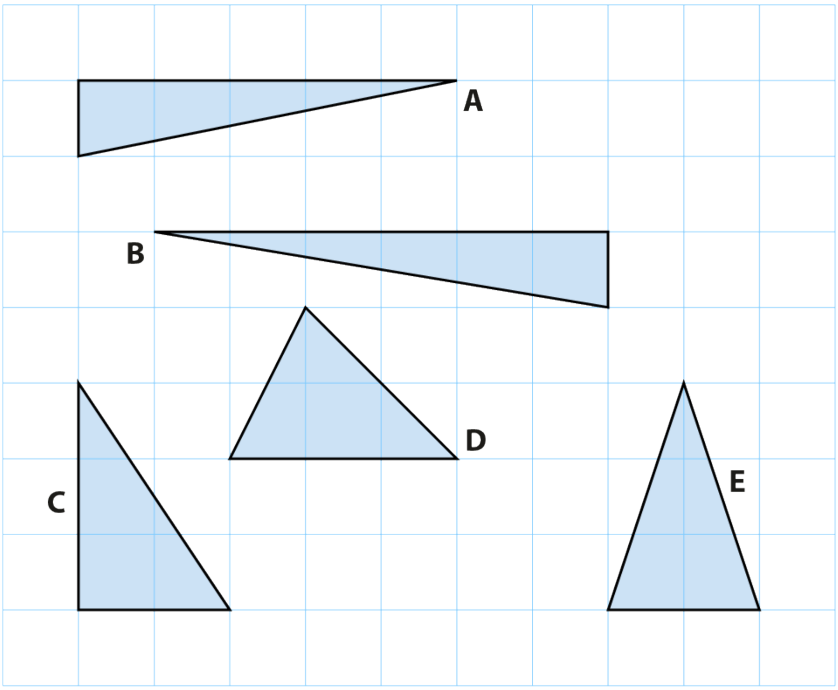
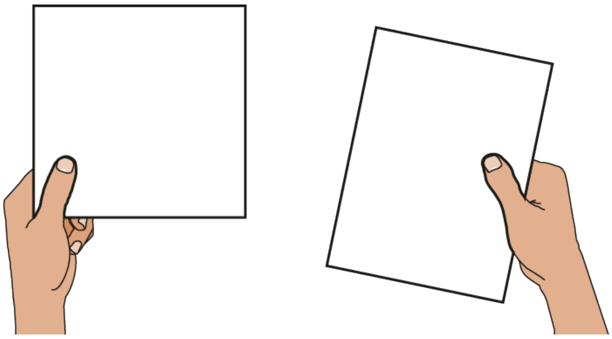
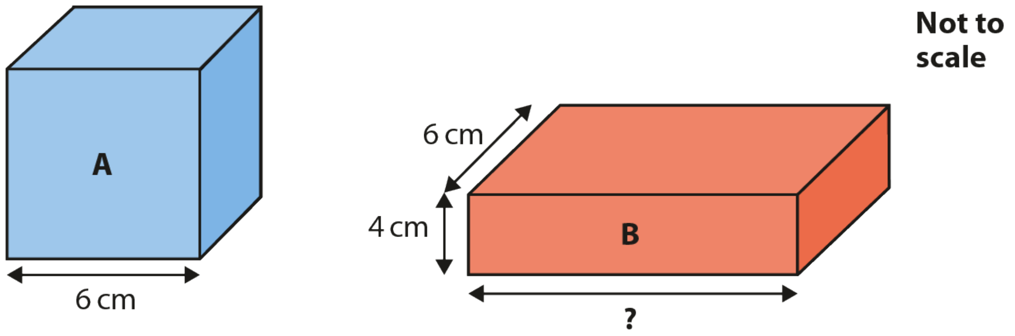
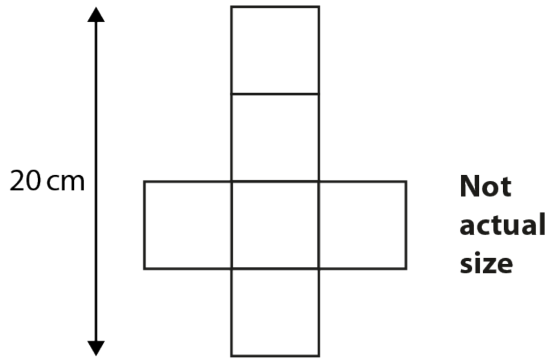
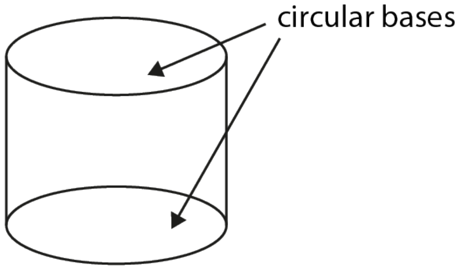
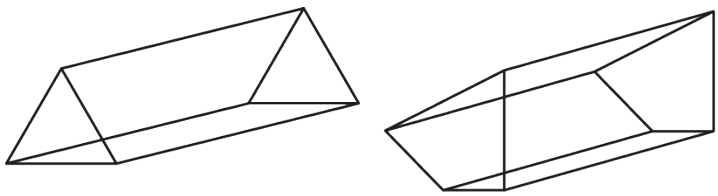
2016 Key Stage 2 Mathematics Paper 3: reasoning Question 17	Here are five triangles on a square grid.  Four of the triangles have the same area. Which triangle has a **different** area? Source: *Standards & Testing Agency* Public sector information licensed under the Open Government Licence v3.0
2017 Key Stage 2 Mathematics Paper 3: reasoning Question 20	 A square tile measures 20 cm by 20 cm. A rectangular tile is 3 cm **longer** and 2 cm **narrower** than the square tile. What is the **difference in area** between the two tiles? Show your method. Source: *Standards & Testing Agency* Public sector information licensed under the Open Government Licence v3.0

2017 Key Stage 2 Mathematics Paper 3: reasoning Question 24	Cube A and cuboid B have the same volume.  Calculate the missing length on cuboid B. Show your method. Source: *Standards & Testing Agency* Public sector information licensed under the Open Government Licence v3.0
2018 Key Stage 2 Mathematics Paper 2: reasoning Question 22	This is the net of a cube.  What is the **volume** of the cube? Source: *Standards & Testing Agency* Public sector information licensed under the Open Government Licence v3.0

Key vocabulary

Term	Definition
cylinder	A 3D object whose uniform cross-section is a circle. A right cylinder can be defined as having circular bases with a curved surface joining them, this surface formed by line segments joining corresponding points on the circles. The centre of one base lies over the centre of the second.  Right cylinder
pi (π)	The ratio of the circumference of a circle to the length of its diameter is a constant called pi (symbol: π). Pi is an irrational number and so cannot be written as a finite decimal or as a fraction. One common approximation for π is $\frac{22}{7}$. 3.14159265 is a more accurate approximation, to 8 decimal places.
prism	A solid bounded by two congruent polygons that are parallel (the bases) and parallelograms (lateral faces) formed by joining the corresponding vertices of the polygons. Prisms are named according to the base, e.g. triangular prism, quadrangular prism, pentagonal prism, etc. Examples:  If the lateral faces are rectangular and perpendicular to the bases, the prism is a right prism.
rectilinear	Bounded by straight lines. A closed rectilinear shape is also a polygon. A rectilinear shape can be divided into rectangles and triangles for the purpose of calculating its area.
surface area	The surface area of a 3D figure is a measure of the area covered by all of the surfaces of the figure.
trapezium	A quadrilateral with exactly one pair of sides parallel.

Collaborative planning

Below we break down each of the three statements within *Perimeter, area and volume* into a set of key ideas to support more detailed discussion and planning within your department. You may choose to break them down differently depending on the needs of your students and timetabling; however, we hope that our suggestions help you and your colleagues to focus your teaching on the key points and avoid conflating too many ideas.

Please note: We make no suggestion that each key idea represents a lesson. Rather, the 'fine-grained' distinctions we offer are intended to help you think about the learning journey irrespective of the number of lessons taught. Not all key ideas are equal in length and the amount of classroom time required for them to be mastered will vary, but each is a noteworthy contribution to the statement of knowledge, skills and understanding with which it is associated.

The following letters draw attention to particular features:

D Suggested opportunities for **deepening** students' understanding through encouraging mathematical thinking.

L Examples of shared use of **language** that can help students to understand the structure of the mathematics. For example, sentences that all students might say together and be encouraged to use individually in their talk and their thinking to support their understanding (for example, *'The smaller the denominator, the bigger the fraction.'*).

R Suggestions for use of **representations** that support students in developing conceptual understanding as well as procedural fluency.

V Examples of the use of **variation** to draw students' attention to the important points and help them to see the mathematical structures and relationships.

PD Suggestions of questions and prompts that you can use to support a **professional development** session.

For selected key ideas, marked with an asterisk (*), we exemplify the common difficulties and misconceptions that students might have and include elements of what teaching for mastery may look like. We provide examples of possible student tasks and teaching approaches, together with suggestions and prompts to support professional development and collaborative planning. You can find these at the end of the set of key ideas.

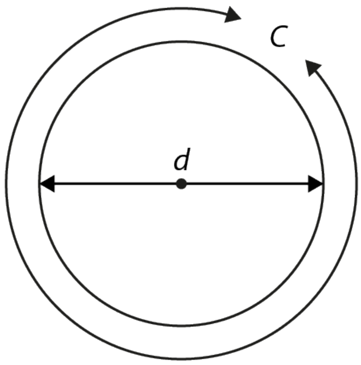
Key ideas

6.2.1 Understand the concept of perimeter and use it in a range of problem-solving situations

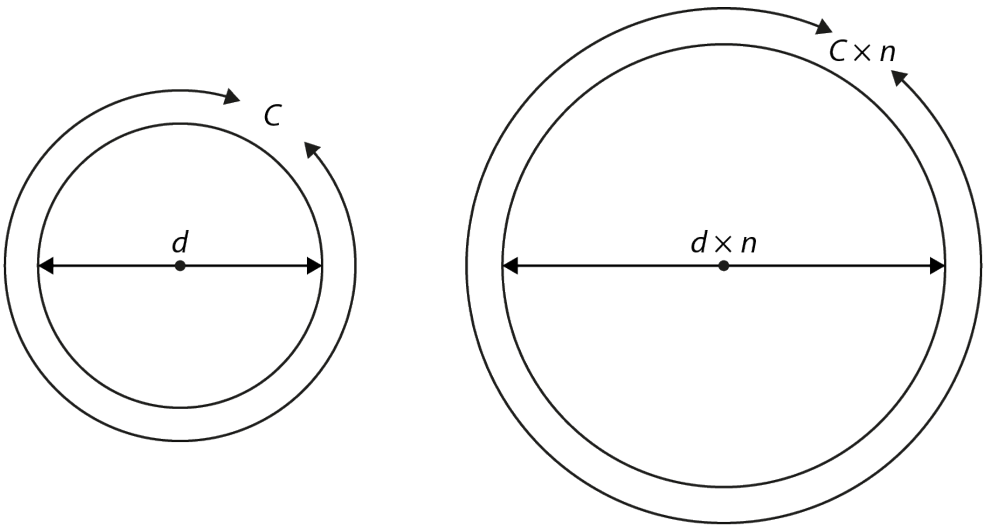
Students should be exposed to a range of problems involving the perimeter of rectilinear shapes and circles. These problems should require students to choose which lengths to include, which lengths not to include and which lengths must be found by reasoning. Students should also work with problems where the perimeter is stated and the side lengths need to be found.

Where the formula for finding the perimeter of a rectangle is used (i.e. $P = 2(l + \mathrm{w})$ or $P = 2l + 2w$), students should appreciate the reasoning behind the formula and know that it cannot be used for finding the perimeter of other shapes. Students should also understand that perimeter is a one-dimensional measure and be able to distinguish it from area, which is two-dimensional (two ideas that are often confused).

When circles and the ratio π are introduced, a key awareness is that, no matter how large or small the circle, the ratio between its circumference and its diameter is always the same. This is the classic multiplicative relationship *within* every circle, which is encapsulated by the formula $C = \pi d$ or $\pi = \frac{C}{d}$.



Students should also be aware of the corresponding multiplicative relationship between any two circles – i.e. if one circle has a diameter n times the length of another, then its circumference will be n times the circumference of the other.



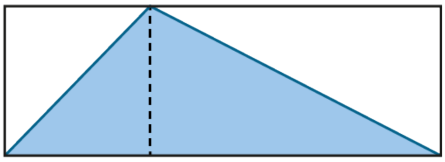
6.2.1.1 Use the properties of a range of polygons to deduce their perimeters

6.2.1.2 Recognise that there is a constant multiplicative relationship (π) between the diameter and circumference of a circle

6.2.1.3 Use the relationship $C = \pi d$ to calculate unknown lengths in contexts involving the circumference of circles

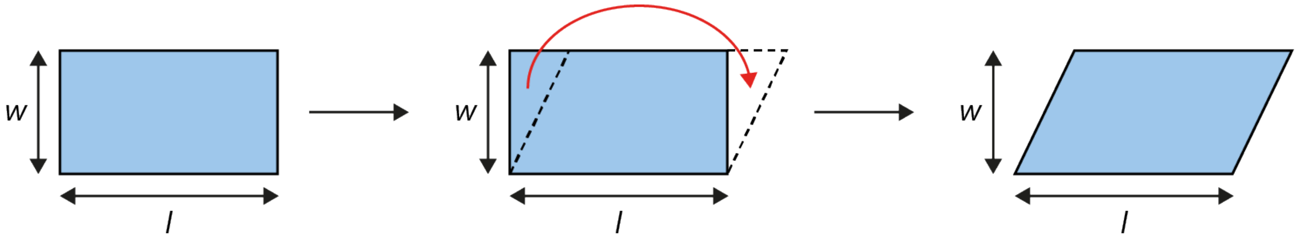
6.2.2 Understand the concept of area and use it in a range of problem-solving situations

Students will be familiar with the area of a triangle from Key Stage 2 and be able to calculate it, using the formula Area = $\frac{1}{2}$ × base × height. They should understand how a triangle can be placed inside a rectangle and, by partitioning the triangle as shown:



that each part of the triangle is half of the smaller rectangle in which it sits and, therefore, the whole triangle is half of the large rectangle.

If the expectations of the national curriculum Key Stage 2 programme of study have been fully met, this idea will have been generalised still further and students will be aware that any parallelogram has the same area as a rectangle with the same base and **perpendicular** height.



Furthermore, they will be also aware that any triangle has an area that is half the area of a parallelogram with the same base and **perpendicular** height.

These are important ideas because they support students' developing awareness that all such formulae arise as a result of reasoning about the geometry of the shape and are not arbitrary collections of symbols to be memorised without meaning.

At Key Stage 3, such reasoning will be applied to other shapes. Students should be encouraged to explore how they might find areas in different ways and to see how these ways can all be generalised to a formula. For example, students should fully understand how the formulae for the area of a trapezium = $\frac{1}{2}(a+b)h$ and the area of a circle = πr^2 are derived from other known facts.

Additionally, the concept of surface area will provide an ideal opportunity for students to make connections between two and three dimensions, and apply and consolidate their understanding of the area and properties of 3D shapes from Key Stage 2.

6.2.2.1* Derive and use the formula for the area of a trapezium

6.2.2.2 Understand that the areas of composite shapes can be found in different ways

6.2.2.3* Understand the derivation of, and use the formula for, the area of a circle

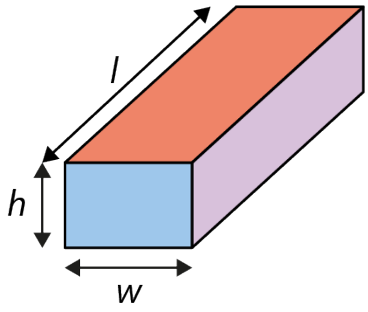
6.2.2.4 Solve area problems of composite shapes involving whole and/or part circles, including finding the radius or diameter given the area

6.2.2.5* Understand the concept of surface area and find the surface area of 3D shapes in an efficient way

6.2.3 Understand the concept of volume and use it in a range of problem-solving situations

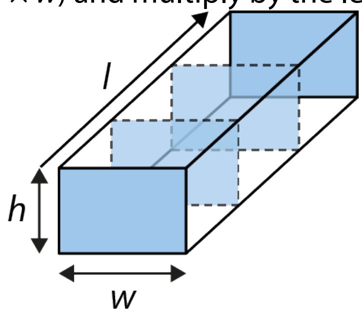
Students will be familiar with finding the volume of cubes and cuboids from Key Stage 2 and will have used the formula Volume = width × height × length (or similar) to calculate volumes. At Key Stage 3, these ideas are developed to include the volume of prisms more generally.

For example, when considering a cuboid, such as this:

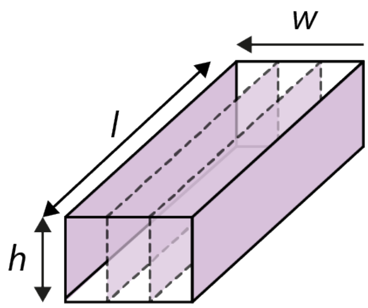


there are various ways of calculating the volume:

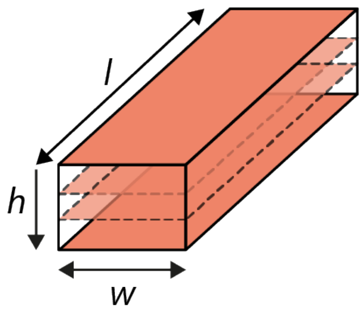
- Find the area of the blue face ($h \times w$) and multiply by the length (l).



- Find the area of the purple face ($h \times l$) and multiply by the width (w).



- Find the area of the red face ($w \times l$) and multiply by the height (h).



Through this sort of analysis, students will realise that the volume of a cuboid is actually the area of one of the faces multiplied by the other dimension. This can then be generalised in Key Stage 3 to other prisms and to the formula Volume of a prism = area of cross-section × length.

Students will use and apply their knowledge of the area of 2D shapes to calculate the cross-sectional area of a variety of prisms.

Although a cylinder is not strictly a prism (a prism has a polygonal uniform cross-section), it is important for students to appreciate that it has the same structure as a prism (with the uniform cross-section being a circle) and its volume can be calculated in a similar way. Thereby, students will see the formula $V = \pi r^2 h$ as an example of a general geometrical property of cylinders that has meaning, and not just a collection of symbols to be memorised.

6.2.3.1 Be aware that all prisms have two congruent polygonal parallel faces (bases) with parallelogram faces joining the corresponding vertices of the bases

6.2.3.2 Use the constant cross-sectional area property of prisms and cylinders to determine their volume

Exemplified key ideas

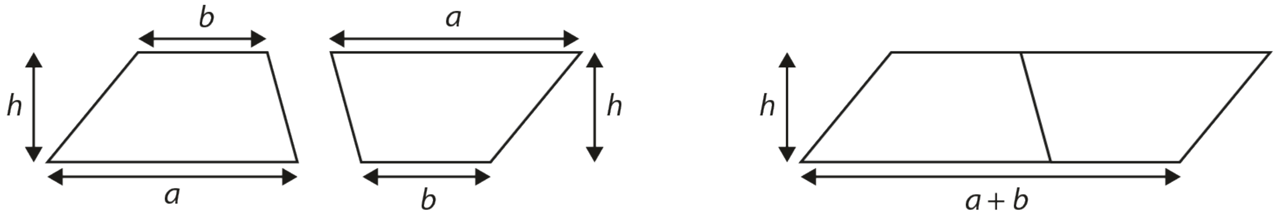
6.2.2.1 Derive and use the formula for the area of a trapezium

Common difficulties and misconceptions

When calculating the area of a trapezium, students may think that the only method is to divide the trapezium into rectangles and triangles, based on their experiences of finding the area of rectilinear shapes in Key Stage 2. They may simply memorise a formula, and then substitute numbers into that formula, without appreciating the importance of the pair of parallel sides and the significance of the $\frac{1}{2}$ in the formula Area of a trapezium = $\frac{1}{2}(a+b)h$.

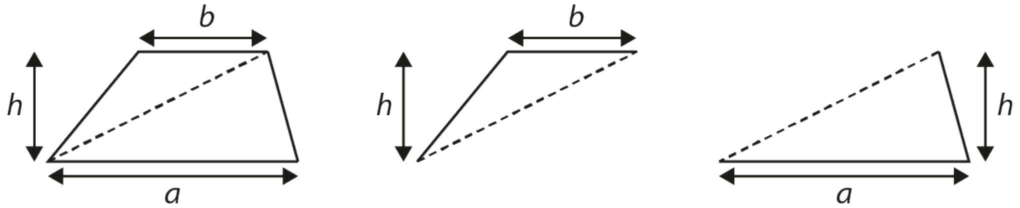
R There are many ways to explore the formula for the area of a trapezium using representations and making connections with the area of a parallelogram, triangle and rectangle.

Use two congruent trapezia to construct a parallelogram

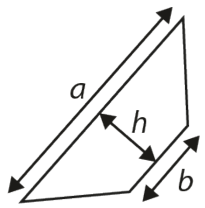
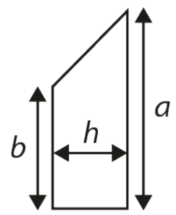
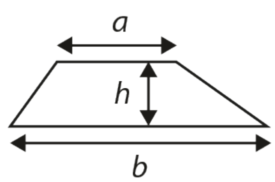
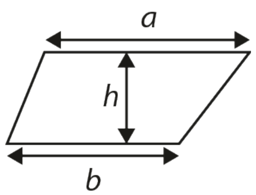


This is a useful representation to support students' understanding of the significance of the $\frac{1}{2}$ and the 'sum of the parallel sides' parts of the formula for the area of a trapezium. Students should be able to see that, since the area of the parallelogram = $(a + b)h$, the area of the trapezium must be half, or $\frac{1}{2}(a+b)h$.

Partitioning a trapezium into two triangles



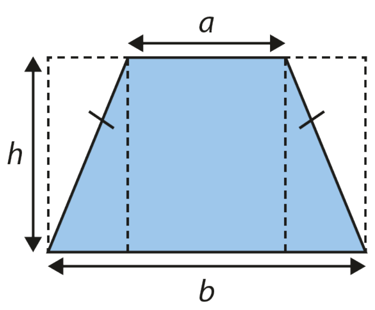
This derivation helps to support students' understanding of the significance of the 'perpendicular height' part of the formula as they realise that the area of a trapezium is the sum of the area of two triangles, or $\frac{1}{2}b\times h+\frac{1}{2}a\times h=\frac{1}{2}(a+b)h$.

What students need to understand	Guidance, discussion points and prompts
Understand that the formula to calculate the area of a trapezium can be applied to any trapezium – isosceles, right-angled or scalene. *Example 1:* *Find the area of the trapezia below in terms of a, b and h.* *a)*  *b)*  *c)*  *d)* 	**V** The choice of what and what not to vary can draw students' attention to the key ideas. In *Example 1*, the trapezia have been carefully chosen to encourage students to notice: • there is always one pair of parallel sides • *a* is not always the longer parallel side • the perpendicular height is not always vertical (in the diagram) • of the parallel sides, the longer side does not have to extend beyond the shorter side in both directions.

The formula for the area of a trapezium can be derived by examining the shape's geometrical structure.

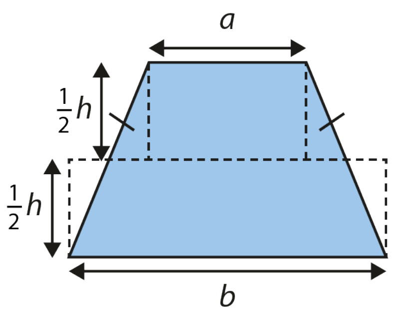
Example 2:

Ayesha:



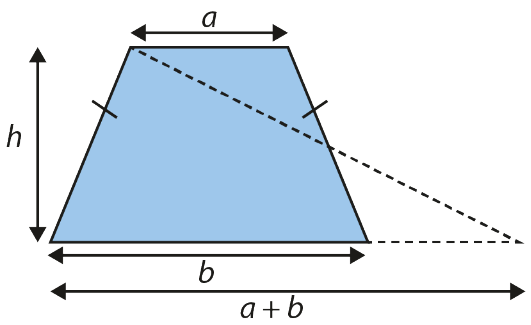
$b \times h - \frac{1}{2} \times (b - a) \times h$

Bronwyn:



$(a \times \frac{1}{2}h) + (b \times \frac{1}{2}h)$

David:



$\frac{1}{2}(a + b) \times h$

a) *Explain how each person has derived their formula.*

b) *Justify that all of these formulae are equivalent to* $A = \frac{1}{2}(a + b)h$.

V *Example 2* presents the derivation of the formula for the area of a trapezium in a variety of different ways. This will help students to see the mathematical structures and relationships, and to make sense of the important elements of the formula.

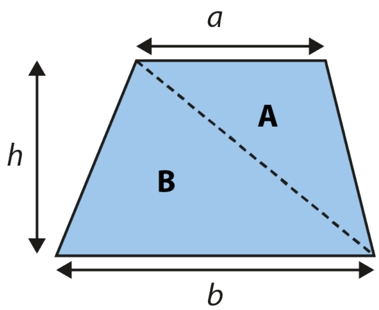
R The different representations provide opportunities for students to experiment, investigate, reason and prove the formula.

D Extra challenge can be offered by asking students to think about how these methods may need to be modified if the trapezium is not isosceles.

PD What other opportunities might there be in your geometry curriculum (or more widely) where students are presented with multiple proofs and are encouraged to justify and convince?

Example 3:

Josh is deriving the formula for the area of a trapezium by thinking of the shape as split into two triangles:



He comes up with the following formula:

$$\left(\frac{1}{2}\times a\times h\right)+\left(\frac{1}{2}\times b\times h\right)$$

Rearrange the statements below to show what Josh's reasoning might have been.

Divide the trapezium into two triangles, A and B
Area of trapezium $= \frac{1}{2}h(a+b)$
Area of trapezium $= \frac{1}{2}(ah+bh)$
The area of triangle B $= \frac{1}{2}\times b\times h$
Area of trapezium $= \left(\frac{1}{2}\times a\times h\right)+\left(\frac{1}{2}\times b\times h\right)$
The area of the trapezium = area of triangle A + area of triangle B
The area of triangle A $= \frac{1}{2}\times a\times h$

D At Key Stage 3, it is important for students to develop the ability to construct a deductive argument or proof. One strategy to support this is to present students with proofs for them to discuss. In *Example 3*, the strategy of offering a set of statements to put in the correct order provides additional support for this deeper level of thinking.

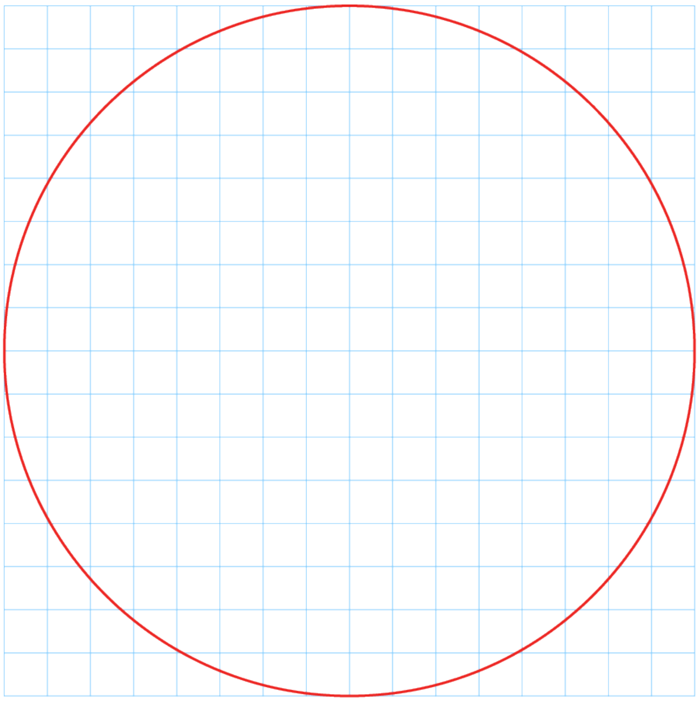
6.2.2.3 Understand the derivation of, and use the formula for, the area of a circle

Common difficulties and misconceptions

If the concept of the area of a circle is introduced at the same time as the circumference of a circle, then some students may confuse the definitions and/or formulae. This can be avoided by introducing the circumference of a circle when thinking about perimeter alongside other shapes, and the area of a circle when considering area in general.

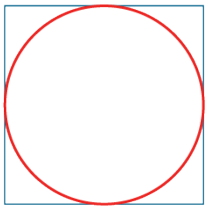
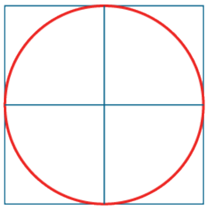
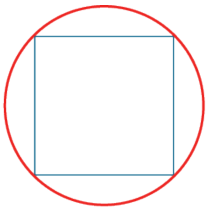
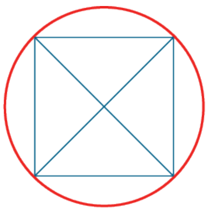
Students may have difficulty finding the area of a shape with no straight sides, since the areas of other shapes considered previously (such as parallelograms, triangles and trapezia) were all derived from the area of a rectangle and so involved multiplying two dimensions together.

Asking students questions, such as, *'How can we find the area, measured in cm^2 (**square** centimetres) of something that is circular?'* and presenting a diagram, such as the one below, could help students appreciate this apparent conundrum and begin to think about the problem of finding the area of a circle:



Example 1, below, will help students to get a feel for the value of $\frac{A}{r^2}$ and will set up the surprising but intriguing idea that it might just be the same as the value of $\frac{C}{d}$!

The area of a sector is typically encountered at Key Stage 4, but there is an opportunity at Key Stage 3 for students to make reasonable conjectures about how to find the area of sectors, using their knowledge of angles, fractions and the area of circles.

What students need to understand	Guidance, discussion points and prompts
Understand the approximate relationship between the area of a circle and the square of its radius. *Example 1:* a) *Close your eyes. Imagine a circle with a radius of r sitting inside a square. Grow the circle until it is the biggest it can be but still fitting inside the square.* *Now open your eyes and draw what you have imagined.* *Share your diagram with your desk partner and try to agree between you:* (i) *What is the area of the square?* (ii) *What can you say about the area of the circle?* b) *Now imagine a square inside the circle; grow it until it's as large as possible whilst remaining inside the circle.* *Share your diagram with your desk partner and try to agree between you:* (i) *What is the area of the square?* (ii) *What can you say about the area of the circle?* c) *What would happen if you started with a larger circle? Or a smaller circle?*	**R** If any representation is to effectively support understanding, students need to internalise it and use it as a mental tool to think and reason with. A visualisation exercise, such as the one offered in *Example 1*, encourages students to begin with their own mental image and, through discussion with desk partners, the teacher and the whole class, use this to get a sense of the area of a circle as some proportion of r^2. Such an exercise may result in a diagram like this:  which, through teacher-led discussion (to highlight how the radius of the circle relates to the area of the square), might evolve into:  The second part of the exercise leads to this diagram:  Similarly, adding the radii makes it clear that the area of the circle is greater than $2r^2$: 

Discussion of the diagrams that students come up with is likely to result in the conclusion:

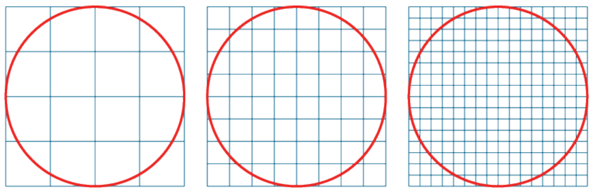
$$2r^2 < \text{Area of circle} < 4r^2$$

And that an average of these, $3r^2$, would be a good approximation for the area of a circle.

D In part c), it is important for students to realise that whatever size the circle is, it will always be the same proportion of the square and, therefore, there exists a constant multiplier that can be used with r^2 to find the area of a circle.

PD Representations can support students in seeing the mathematical structure in a situation. What are the pros and cons of asking students to construct their own mental image of a situation first, before offering them an image yourself? In what other areas of the curriculum might such a strategy be useful?

D The diagrams above could be further refined and discussed using images like these:

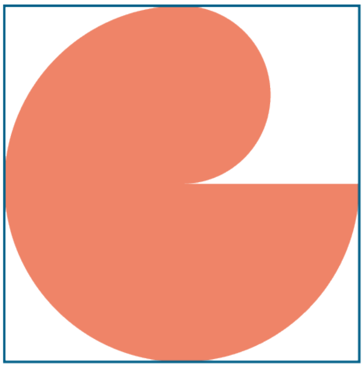


Students could be asked to look for those smaller squares that are more than half-captured by the circle and so estimate the fraction of the large square captured in order to find an approximation for the area of the circle.

Analysing the first diagram results in an approximation of $3r^2$. The second one gives an approximation of $3\frac{1}{4}r^2$.

Example 2:

This shape is cut from a square piece of card measuring 12 cm by 12 cm. Approximately what area of card remains?



Students could be asked to tackle *Example 2* by using one of the approximations obtained from the previous example, i.e.

$$A \approx 3r^2 \text{ or } A \approx 3\tfrac{1}{4}r^2$$

Alternatively, students could be encouraged to accept the fact that they are not sure what this value is and, therefore, use

$$A = kr^2 \text{ (where } k \text{ is some constant).}$$

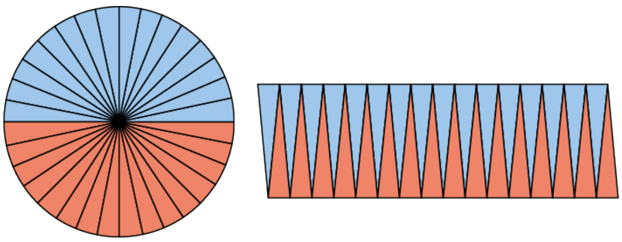
Note that this would be good preparation for Key Stage 4, where it is often more appropriate to leave answers as a multiple of π.

PD Discuss with colleagues whether and when it is appropriate to use an approximation for π, what approximations you use and when, and whether it is useful to introduce the idea of using the symbol π in calculations and answers.

Appreciate that the formula for the area of a circle can be derived from prior knowledge.

Example 3:

Look at the images below.



a) *What do you see?*

b) *What do you know about the area of the circle on the left and the area of the shape on the right?*

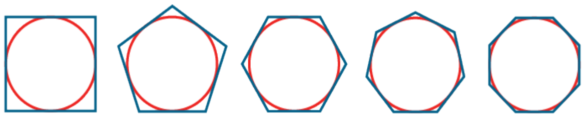
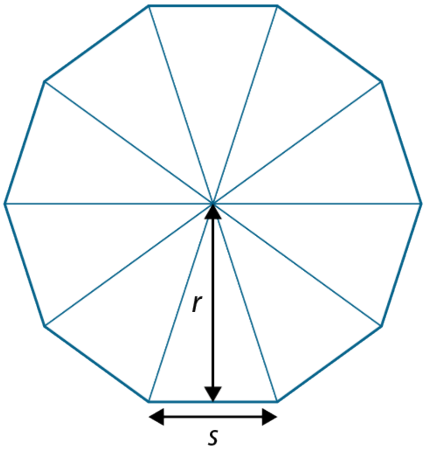
c) *How might you find a good approximation of the area of the shape on the right?*

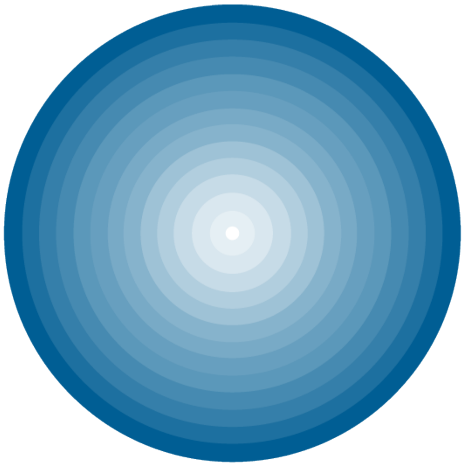
The key points to draw out in any whole-class discussion about *Example 3* are:

- that the shape on the right is a pretty good approximation to a rectangle (and that this transformation cleverly turns the problem of finding areas of circles into one of finding areas of rectangles)
- that the length of the rectangle which approximates to this shape is half the circumference of the circle
- that the height of the rectangle which approximates to this shape is the radius of the circle.

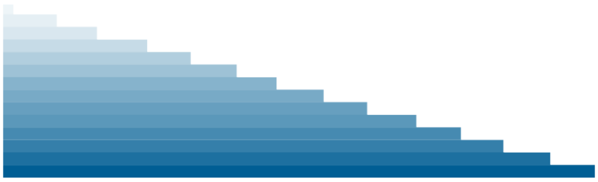
PD Static images like this are useful; however, one of the key ideas here is thinking of the limit as the size of each sector becomes smaller, the number of sectors increases, and the right-hand diagram approaches a rectangle. How might the use of dynamic geometry software, such as GeoGebra, support the development of this idea?

R Presenting students with multiple ways of deriving the formula for the area of a circle can lead to a deeper understanding. For example, students could be encouraged to

	think about how the area of a polygon approaches the area of a circle as the number of sides increases:  reasoning that the area of any polygon is the area of one triangle $\times$ the number of sides. $= \frac{1}{2} \times s \times r \times$ (number of sides) $= \frac{1}{2} \times r \times (s \times$ number of sides) $= \frac{1}{2} \times r \times$ perimeter of polygon  And, therefore, as the number of sides increases, the polygon approaches a circle, the perimeter approaches $C = 2\pi r$ and the area approaches πr^2. Alternatively, students can think of the circle as made up of an infinite number of line segments representing the circumference of ever-decreasing circles inside the original circle, like this:



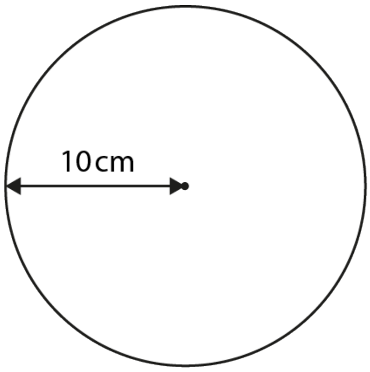
and imagine all of these line segments rearranged to form an approximation of a triangle, like this:



which has height equal to the radius of the circle and length equal to its circumference.

Calculate the area of a circle and appreciate the effect of taking different values of π.

Example 4:



a) What is the area of the circle:

(i) as a multiple of π?

(ii) if π is taken as approximately 3?

(iii) if π is taken as approximately 3.1?

(iv) if π is taken as approximately $\frac{22}{7}$?

(v) if the π button on your calculator is used?

b) Which of the answers above is most accurate?

V Building a sequence of questions with different required degrees of accuracy can raise some important issues regarding students' understanding of accuracy and to what extent rounding the final answer is appropriate.

In *Example 4*, students may say the answer with the most decimal places is the most accurate, whereas the value 100π is not only the most accurate but also the most convenient. This example should reinforce the idea that any particular number used for π will be an approximation and the only precise way of signifying π is to use its symbol.

Example 5:

a) A circle has an area of 25π cm^2. What is the length of its radius?

b) A circle has an area of 100 cm^2. What is the length of its radius?

V *Example 5* requires students to work with the formula for the area of a circle in reverse.

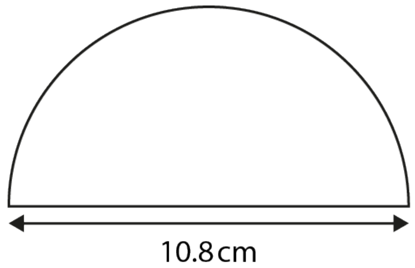
Part a) is in terms of π and a square number, thus the underlying structure is clearer.

Part b) is slightly more complicated, owing to the nature of the numbers involved.

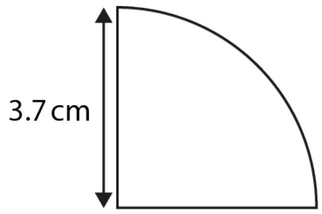
Solve problems involving calculating the area of a circle in context.

Example 6:

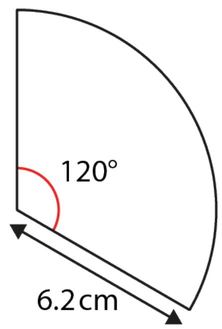
a) What is the area of this semi-circle?



b) What is the area of this quadrant?



c) What is the area of this sector?

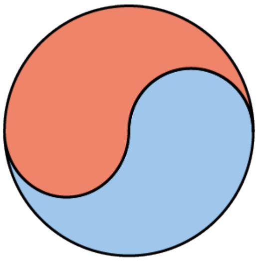
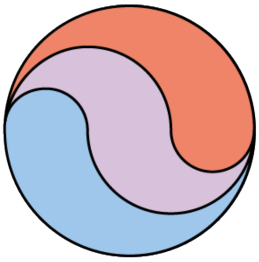


D *Example 6* encourages students to make connections with their knowledge of fractions. It lays the foundations for a more generalised approach to finding the area of sectors at Key Stage 4.

Example 7:

A takeaway sells two sizes of pizzas: 10" and 12". The Hawaiian 10" pizza costs £5.40. The 12" costs £7.60. Which pizza is better value for money?

PD *Example 7* provides students with the opportunity to apply their knowledge of the area formula. How might students interpret their calculations given the context? What additional awarenesses about the area of a circle might arise from discussion of such problems?

Example 8: *a) Using a pair of compasses, can you construct a diagram like this?*  *If the radius of the circle is 10 cm, what is the area of the red and blue shapes?* *b) Construct this diagram:*  *If the radius of the circle is 10 cm, what is the area of the red, purple and blue shapes?*	**R** Getting students to construct the diagram in part a) first is a good way to support them to see how each of the red and blue shapes is formed and therefore find their area. **D** You can challenge students to generalise this result by considering diagrams such as the one in part b). Students may be surprised by the fact that the areas are equal. Note that this problem is an ideal context in which to encourage students to use π in their calculations rather than a numerical approximation.
Example 9: *Are all circles similar figures? Explain your answer.*	**D** *Example 9* encourages students to make connections with their knowledge of proportional reasoning and scale factors.

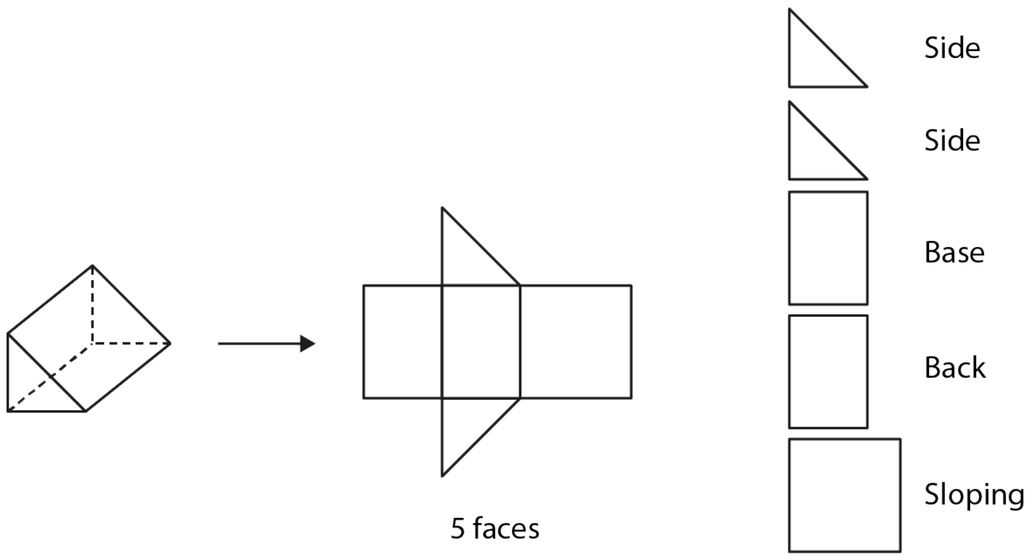
6.2.2.5 Understand the concept of surface area and find the surface area of 3D shapes in an efficient way

Common difficulties and misconceptions

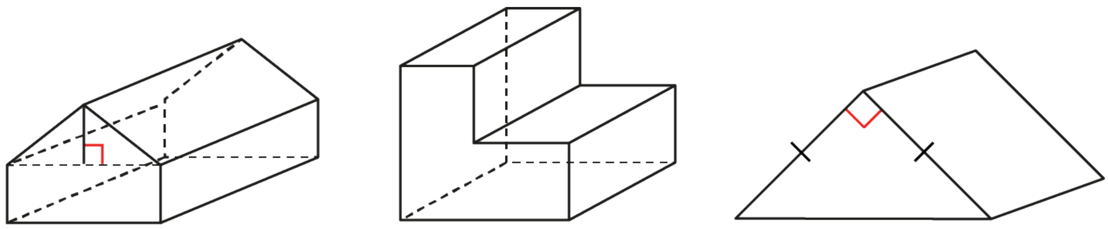
Students may have difficulty visualising how many surfaces certain prisms have. They can be supported in this by being given the opportunity to handle 3D models of prisms and to describe how many faces there are and what shape they are.

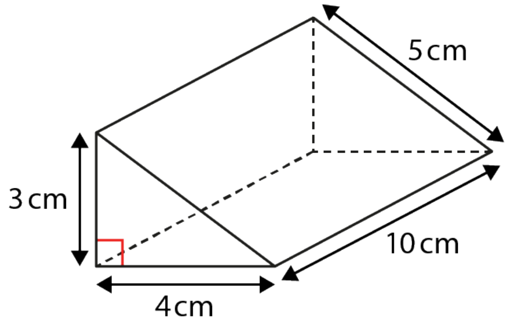
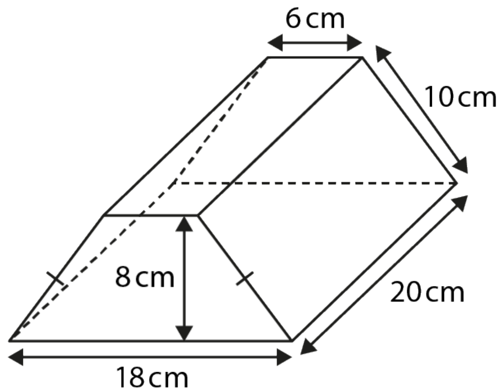
Unthinking memorisation and use of formulae (e.g. $SA = 2(xy + xz + yz)$ or $2xy + 2xz + 2yz$ for the surface area of a cuboid, where x, y and z are its dimensions) should be avoided, as this can lead to over-generalising that the surface area of all prisms can be calculated in this way (e.g. as the sum of the area of *six* rectangular faces). It is important for students to appreciate the structure of all surface area calculations, so that they correctly generalise that to find the surface area of a prism, they need to identify all of the surfaces (two of which will be the ends and, therefore, have the same area) and a number of rectangular faces depending on the nature of the polygonal ends.

R Asking students to sketch 2D representations of a prism will help them to identify the number of faces, the shapes of the faces and their dimensions. This should consolidate the idea that the surface area of a prism is the sum of the areas of all its faces. If such sketching of the net is difficult, identifying and sketching the faces independently, possibly by using a 3D model as support, will help students make this connection. This is particularly helpful when students are trying to identify the 'sloping' rectangular faces of, for example, triangular prisms.



Presenting students with a range of prisms and asking them to calculate the surface area should help students to realise the key idea that the surface area of a prism is equal to the sum of the areas of all its faces.



What students need to understand	Guidance, discussion points and prompts
Understand that prisms can have a different number of faces and, therefore, the surface area can be the sum of a different number of areas. *Example 1:* *For each prism:* *a) How many faces does the prism have?* *b) Describe and sketch the shape of each face, including dimensions.* *(i)*  *(ii)*  	**V** The choice of what and what not to vary can draw students' attention to the key ideas. In *Example 1*, the prisms have been chosen to support students in noticing that prisms can have a different number of faces and different shaped faces. The prisms have also been carefully chosen as the dimensions of each face are easy to deduce. **PD** Students often struggle when working with 2D representations of 3D objects. What prisms are there readily 'to hand' in your classroom or department? What familiar objects could you easily present students with to help them to access this work?

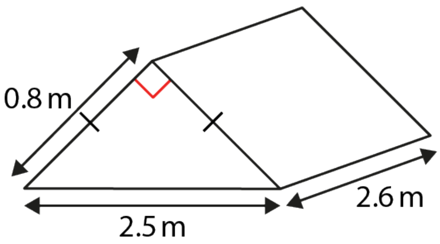
Example 2:

For each prism:

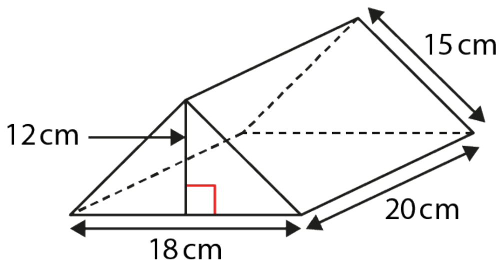
a) How many faces does the prism have?

b) Describe and sketch the shape of each face, including dimensions.

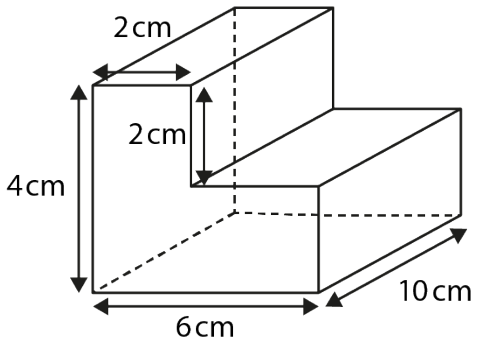
(i)



(ii)



(iii)

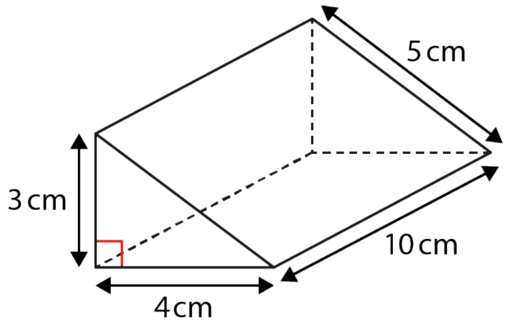


V In *Example 2*, the prisms have been carefully chosen to encourage students to not only notice the number of faces, but also that sometimes the dimensions may have to be worked out from the given information, especially when the shape of a face is not a common one, such as the L-shape in prism (iii).

PD What examples of prisms do students currently experience? Are the prisms that they see (from their textbooks or in the examples you draw on the board) always in a 'standard' orientation (i.e. with a horizontal base)? To what extent would presenting them in different orientations help students to identify the shapes of the different faces?

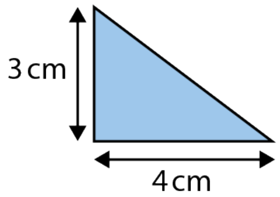
Understand that the surface area of a prism is the sum of the area of **all** the faces and not just the visible ones.

Example 3:



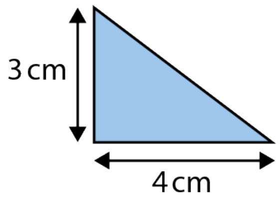
Kagendo is calculating the total surface area of the prism:

Front:



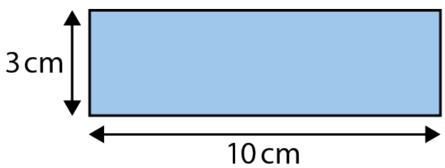
Area $= \frac{1}{2} \times 3 \times 4 = 6\ cm^2$

Back:



Area $= \frac{1}{2} \times 3 \times 4 = 6\ cm^2$

Side:



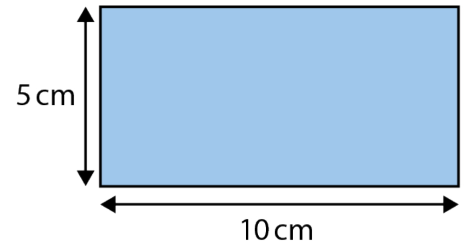
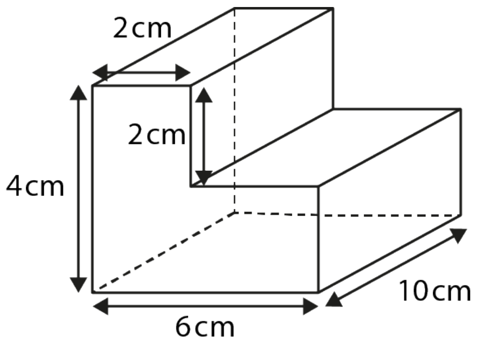
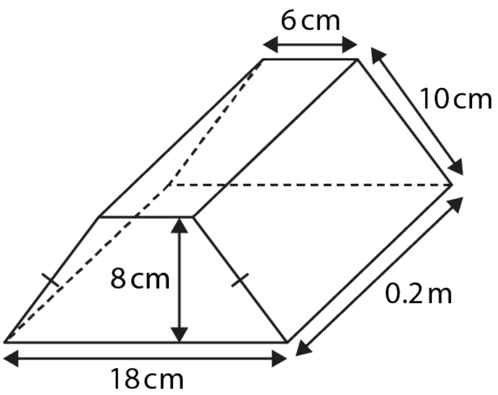
Area $= 3 \times 10 = 30\ cm^2$

V An aspect of variation is to offer examples of 'what it's not' (as well as 'what it is') to help students clarify the idea in their minds.

Example 3 is designed to encourage students to notice that not all the faces have been used in the calculation of the surface area of the prism.

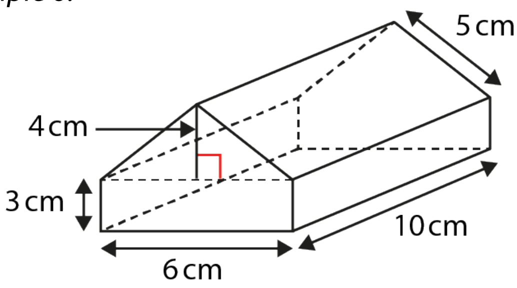
L Consider encouraging students to verbalise the formula using correct terminology, i.e. *'The surface area of a prism is equal to the sum of the areas of all its faces.'*

PD Can you construct other 'what it's not' examples that might highlight the importance of identifying all the faces of a prism?

Sloping side:  *Area* $= 5 \times 10 = 50\ cm^2$ *Surface area* $= 6 + 6 + 30 + 50 = 92\ cm^2$ *Do you agree with Kagendo? Explain your answer.*	
Understand how to use the correct dimensions to calculate the area of 2D shapes. *Example 4:* *Calculate the surface area of this prism.* 	**V** As noted above, the choice of what and what not to vary can draw students' attention to the key ideas. In *Example 4*, the prism has been chosen because the L-shaped face may cause difficulties for some students. They may think that the area of each face can be calculated using a standard formula, such as $A = l \times w$ or $A = \frac{1}{2} \times b \times h$. Being flexible is an important aspect of fluency.
Example 5: *Calculate the surface area of this prism.* 	**V** In *Example 5*, students need to notice that the unit of one dimension (0.2 m) differs from the units of the other dimensions. Students should realise that all dimensions must be in the same unit before they can calculate the area of a face.

There is no standard formula for calculating the surface area of a prism.

Example 6:



John thinks the surface area of the prism is:

Surface area $= 6 \times 3 \times 4 \times 10 = 720$ cm^2

Do you agree with John? Explain your answer.

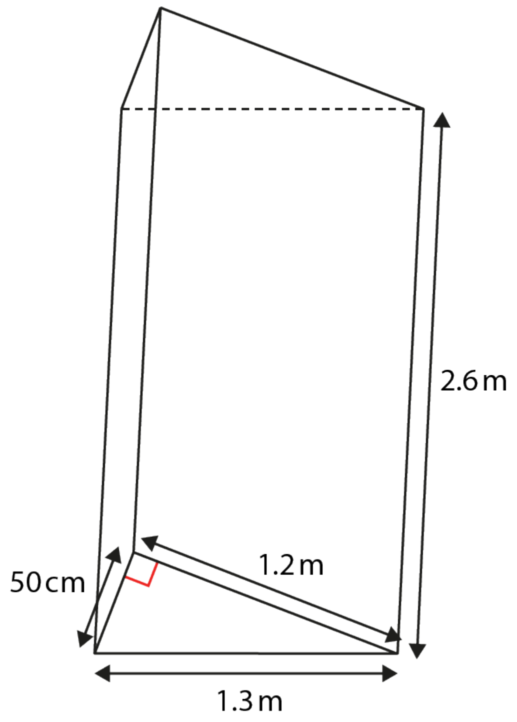
V *Example 6* is designed to encourage students to notice that there is not a single formula for calculating the surface area of any prism. Each prism must be considered on its own merits.

PD Can you construct other 'what it's not' examples that might highlight the importance of identifying the number, and type, of faces?

The units of surface area are squared.

Example 7:

Zarina works out the total surface area of this prism:



Area of front face $= \frac{1}{2} \times 50 \times 1.2 = 30$

Area of rear face $= \frac{1}{2} \times 50 \times 1.2 = 30$

Area of base $= 1.3 \times 2.6 = 3.38$

Area of sloping face $= 1.2 \times 2.6 = 3.12$

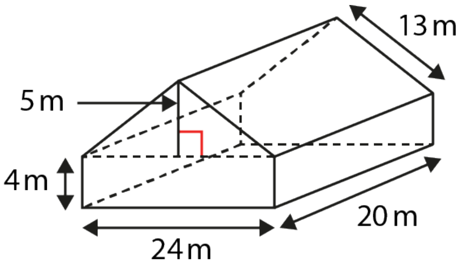
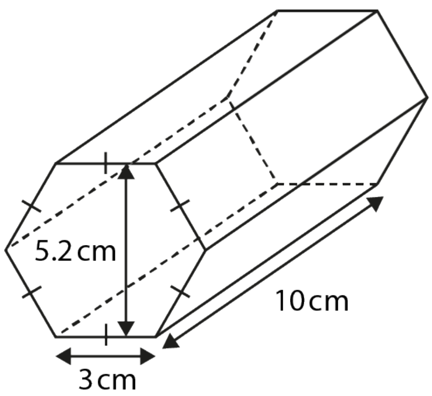
Therefore, total surface area of prism $= 11.3$ m^3

There are three mistakes in Zarina's working out. Can you find them?

Getting students to discuss and explain why certain statements are wrong is a strategy to encourage reasoning (a fundamental aim of the national curriculum) and explicitly address common mistakes or misconceptions.

The three mistakes in *Example 7* are:

- not changing all measurements to the same unit
- not finding the surface area of every surface
- using the incorrect unit (m^3) for the total surface area.

Solve familiar and unfamiliar problems, including real-life applications. *Example 8:* *Roberto is painting his barn. He wants to paint all of the walls and the roof.*  *One tin of paint covers 15 m^2.* *How many tins of paint are needed?*	Problems provide opportunities for students to intelligently practise their understanding of a concept (rather than mechanical repetition), to focus on relationships – not just the procedure – and make connections.
Example 9: *A cuboid is created from 24 linking cubes.* *What could its surface area be?*	**D** Students can begin to link volume and surface area, and realise that for constant volume, a range of surface areas is possible, just as a constant area of a rectangle gives rise to a range of possible perimeters.
Example 10: *Explain (with reasons) how you would calculate the surface area of a hexagonal prism.*  	**D** The subtle shift involved in asking students to explain **how** they would calculate something, rather than produce the calculation, often encourages reasoning and the construction of a convincing argument, which all contribute to deeper thinking.

Solve problems where there is more than one answer and there are elements of experimentation, investigation, checking, reasoning, proof, etc. *Example 11:* *Sketch a prism with a surface area of 24 cm^2.* *And another, and another.*	Students who have demonstrated a secure understanding of calculating the area of triangles should be encouraged to go deeper by solving more complex problems, such as exploring all possibilities, creating their own examples and testing conjectures. **R** If students are struggling, you could encourage them to think about 2D representations of prisms. **D** For problems such as *Example 11*, encourage students to find non-integer values and 'unusual' solutions (i.e. solutions that no one else in the class will find!).

Weblinks

1 NCETM primary mastery professional development materials https://www.ncetm.org.uk/resources/50639

2 NCETM primary assessment materials https://www.ncetm.org.uk/resources/46689

3 Standards & Testing Agency past mathematics papers https://www.gov.uk/government/collections/national-curriculum-assessments-practice-materials#key-stage-2-past-papers