Mastery Professional Development

Multiplication and Division

2.9 Times tables: 7 and patterns within/across times tables

Teacher guide | Year 3

Teaching point 1:

Counting in multiples of seven can be represented by the seven times table. Adjacent multiples of seven have a difference of seven. Facts from the seven times table can be used to solve multiplication and division problems with different structures.

Teaching point 2:

When both factors are odd numbers, the product is an odd number; when one factor is an odd number and the other is an even number, the product is an even number; when both factors are even numbers, the product is an even number.

Teaching point 3:

When both factors have the same value, the product is called a square number; square numbers can be represented by objects arranged in square arrays.

Teaching point 4:

Divisibility rules can be used to find out whether a given number is divisible (to give a whole number) by particular divisors.

Overview of learning

In this segment children will:

- skip count in sevens and build up the seven times table
- use seven times table facts to solve contextual and abstract multiplication problems, contextual quotitive and partitive division problems, and abstract division problems
- review and explore patterns and rules across the two to ten times tables, including:
  - predicting whether a product will be odd or even depending on the factors
  - investigating square numbers and representing them using arrays, multiplication equations and through the use of the 'squared' symbol (n^2)
  - reviewing and applying the divisibility rules already learnt in segments:
    - *2.6 Structures: quotitive and partitive division* (steps *4:11–4:13;* for divisors two, five and ten)
    - *2.7 Times tables: 2, 4 and 8, and the relationship between them*
    - *2.8 Times tables: 3, 6 and 9, and the relationship between them.*

Teaching point 1 follows a similar progression to that used when learning the other times tables. By now, children should have a good grasp of how to link skip counting, grouping and multiplication equations to build up times tables. Teachers are encouraged to continue building up the class multiplication chart (first introduced in segment *2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1*):

×	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110		
12	0	12	24	36	48	60	72	84	96	108	120		

Key: 'new' facts
previously learnt facts
relevant previously learnt facts (commutativity)

The chart should be used to help children see that there are only three 'new' facts to be learnt (for 7×7, 11×7 and 12×7). However, in order for children to become fluent with the seven times table, as well as using the connections with previously learnt tables, regular practice will be needed both in reciting the times table (for example, '*One seven is seven, two sevens are fourteen…*') and with isolated multiplication facts (for example, '*I know that eight times seven is equal to fifty-six*').

As in segments *2.7* and *2.8*, since children have already been introduced to division (segment *2.6*) and calculation of quotients using multiplication facts, division is embedded in the times table practice steps of this segment. Teachers should ensure that contextual division practice encompasses both the quotitive and partitive structures of division. Similarly, children have already been introduced to the 'one equation, two interpretations' concept of commutativity (segment *2.5 Commutativity (part 2), doubling and halving*) where, for example, 4×7 can represent either four groups of seven, or seven groups of four. As such, practice also includes application of seven times table facts to solve problems about seven equal groups (distinct from problems about groups of seven).

Once children have learnt the seven times table, they will have covered all times tables from the two times table through to the ten times table. The rest of the segment provides an opportunity to 'take stock' and review patterns and rules revealed by the multiplication facts. This includes, in *Teaching point 2*, identifying the 'odd/even multiplication rules':

- **odd factor × odd factor = odd product**
- **even factor × odd factor = even product**
  and
  odd factor × even factor = even product
- **even factor × even factor = even product**

Once these rules are understood and memorised, teachers should encourage children to use them as a means of sense-checking their answers to multiplication questions throughout upcoming segments and practice.

Teaching point 4 uses familiar contexts and arrays to introduce the idea of square numbers. Children are encouraged to notice for themselves that when both factors are the same, a multiplication fact can be represented by a square array (giving rise to the term 'square number'). Children are then introduced to the 'squared' symbol, and practise linking pictorial representations (contextual or abstract arrays), multiplication equations (e.g. $7 \times 7 = 49$), and equations using the squared symbol (e.g. $7^2 = 49$). Teachers should be aware that a common misconception is for children to think that the squared symbol means '$\times 2$'; however, there is no need to specifically draw attention to this unless the misconception arises and needs to be addressed. It is recommended that teachers do not begin their exploration of squared numbers with the example of 2×2, or 2^2, since in this specific case, multiplying the like-factors gives the same result as adding the like-factors ($2 \times 2 = 2 + 2$), which could lead to confusion for some children.

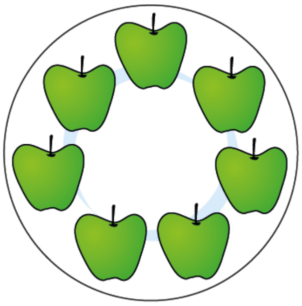
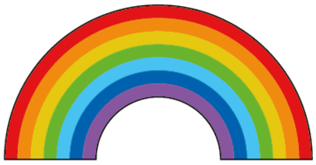
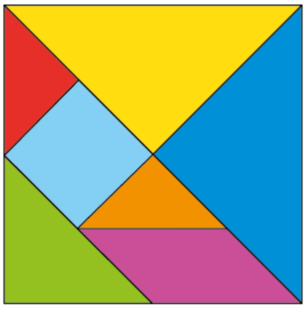
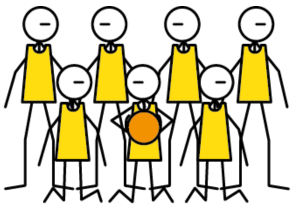
Teaching point 4 simply brings together the set of divisibility rules learnt so far (for divisibility by two, three, four, five, six, eight, nine and ten). Note that the divisibility rule for seven is too complex to be useful for children, so this is not included. The known divisibility rules are reviewed and applied to both abstract and contextual problems.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Counting in multiples of seven can be represented by the seven times table. Adjacent multiples of seven have a difference of seven. Facts from the seven times table can be used to solve multiplication and division problems with different structures.

Steps in learning

	Guidance	**Representations**
1:1	This teaching point follows a similar progression for the seven times table as that for the other times tables (see, for example, segment *2.8 Times tables: 3, 6 and 9, and the relationship between them, Teaching point 1*). Begin by exploring some groups of seven; for example, seven apples, seven colours in a rainbow, seven pieces in a tangram, seven players in a netball team, seven sides on the 50 p and 20 p coins, and seven days in the week. Ask children: • *'What's the same?'* • *'What's different?'* By now, children should recognise that items do not need to be identical to form a group; netball players can be taller or shorter, tangram pieces can be different sizes or shapes, and so on.	• *'What's the same?'* • *'What's different?'*       Mon | Tue | Wed | Thu | Fri | Sat | Sun

1:2

Taking one of the previous examples, start to explore repeated groups of seven, linking enumerating objects in groups of seven with skip counting in sevens, and writing the associated multiplication equations. For example, ask children how many players there would be if two netball teams were playing a match, as exemplified opposite. Use seven-value counters to support the idea of unitising in sevens, and use a number line with the multiples of seven highlighted for skip-counting support.

Work through several examples. In each case, ask children to describe what each number in the equation represents:

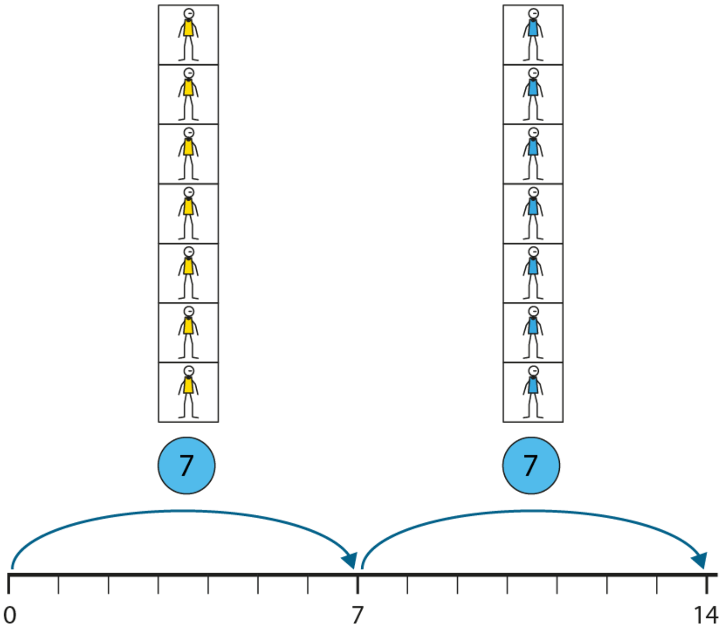
- *'What does the "2" represent?'*
  'The "2" represents the number of teams.'
- *'What does the "7" represent?'*
  'The "7" represents the number of players in each team.'
- *'What does the "14" represent?'*
  'The "14" represents how many players there are altogether.'

Remember, when describing a multiplication equation such as *$2 \times 7 = 14$*, use the language '*two times seven is equal to fourteen.*' Avoid saying '*times by*' or '*multiplied by*'. For more on this, see segment *2.2 Structures: multiplication representing equal groups, Overview of learning.*

Also continue to use the language of factors and products to describe the multiplication equation:

- ***'___ is a factor.'***
- ***'___ is a factor.'***
- ***'The product of ___ and ___ is ___.'***
- ***'___ is the product of ___ and ___.'***

'How many players are there? Count in groups of seven.'



- *'Seven, fourteen. There are fourteen players.'*
- *'There are two groups of seven; there are fourteen altogether.'*
- *'There are seven, two times; there are fourteen altogether.'*

$2 \times 7 = 14$ $\quad\quad$ $7 \times 2 = 14$

- *'Two is a factor.'*
- *'Seven is a factor.'*
- *'The product of two and seven is fourteen.'*
- *'Fourteen is the product of two and seven.'*

Children should, by now, be very familiar with the generalisation: ***'When zero is a factor, the product is zero.'***

Skip count backwards in sevens with a number line to illustrate that this is also the case for 0×7 and 7×0.

1:3

Practise skip counting, forwards and backwards in sevens between 0 and 84, regularly outside the main maths lesson, so that children begin to develop fluency with this counting sequence before moving onto the next step. Use familiar representations such as a number line and the Gattegno chart.

Number line:

0 7 14 21 28 35 42 49 56 63 70 77 84

Gattegno chart:

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1:4

Now, using a familiar context, bring together the learning from steps *1:2–1:3*, working systematically to construct the seven times table, beginning with zero sevens. You could use the idea of a netball tournament, adding teams of seven one at a time.

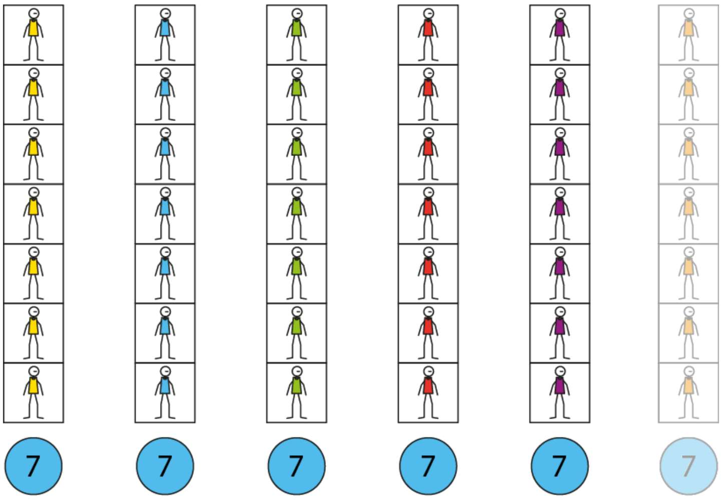
As with the other times tables, use a ratio chart to record the number of groups and the product. As you complete the ratio chart, also write the multiplication equations; write pairs of equations for each times-table fact, using the following form of language:

- $6 \times 7 = 42$
  'Six groups of seven is equal to forty-two.'
  'Six times seven is equal to forty-two.'
- $7 \times 6 = 42$
  'Seven, six times is equal to forty-two.'
  'Seven times six is equal to forty-two.'

At each stage:

- encourage children to describe what each equation represents, for example:
  - *'There are six groups of seven players.'*
  - *'There are forty-two players altogether.'*
  - *'The product of six and seven is forty-two'.*
- Then add another team, and work with children to complete the next column of the table, using their knowledge of what comes next in the counting sequence when skip counting in sevens.

Building up the seven times table:



$0 \times 7 = 0$	$7 \times 0 = 0$
$1 \times 7 = 7$	$7 \times 1 = 7$
$2 \times 7 = 14$	$7 \times 2 = 14$
$3 \times 7 = 21$	$7 \times 3 = 21$
$4 \times 7 = 28$	$7 \times 4 = 28$
$5 \times 7 = 35$	$7 \times 5 = 35$
$\mathbf{6 \times 7 = 42}$	$\mathbf{7 \times 6 = 42}$

Number of netball teams	Total number of players
0	0
1	7
2	14
3	21
4	28
5	35
6	**42**

<table>
<tr>
<td>**1:5**</td>
<td>Once the ratio chart and full set of equations are complete, ask children questions, encouraging them to use the chart/equations for support, for example:
• *'If there are nine teams, how many players are there altogether?'*
• *'How many teams are there if there are twenty-one players?'*
• *'If the product is thirty-five, what are the factors?'*
• *'Why are eight times seven and seven times eight both equal to fifty-six?'*</td>
<td>Complete ratio chart and seven times table:
<table>
<tr><th>Number of netball teams</th><th>Total number of players</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>7</td></tr>
<tr><td>2</td><td>14</td></tr>
<tr><td>3</td><td>21</td></tr>
<tr><td>4</td><td>28</td></tr>
<tr><td>5</td><td>35</td></tr>
<tr><td>6</td><td>42</td></tr>
<tr><td>7</td><td>49</td></tr>
<tr><td>8</td><td>56</td></tr>
<tr><td>9</td><td>63</td></tr>
<tr><td>10</td><td>70</td></tr>
<tr><td>11</td><td>77</td></tr>
<tr><td>12</td><td>84</td></tr>
</table>
<table>
<tr><td>$0 \times 7 = 0$
$1 \times 7 = 7$
$2 \times 7 = 14$
$3 \times 7 = 21$
$4 \times 7 = 28$
$5 \times 7 = 35$
$6 \times 7 = 42$
$7 \times 7 = 49$
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$12 \times 7 = 84$</td>
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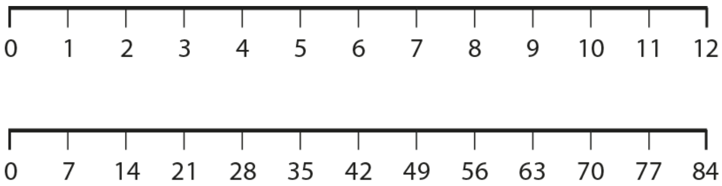
$7 \times 6 = 42$
$7 \times 7 = 49$
$7 \times 8 = 56$
$7 \times 9 = 63$
$7 \times 10 = 70$
$7 \times 11 = 77$
$7 \times 12 = 84$</td></tr>
</table></td>
</tr>
</table>

1:6	Now practise chanting the seven times table, with the written times table for support, using a variety of representations, including: • stacked number lines (as shown opposite) • the Gattegno chart • concrete representations • pictorial representations. Use the following language: • *'One group of seven is equal to seven.' 'Two groups of seven is equal to fourteen…'* • *'One times seven is equal to seven.' 'Two times seven is equal to fourteen…'* then shortening to *'One seven is seven, two sevens are fourteen…'* and • *'Seven, one time is equal to seven...' 'Seven, two times is equal to fourteen...'* • *'Seven times one is equal to seven...' 'Seven times two is equal to fourteen...'* Regular practice should be undertaken, including outside the main maths lesson, until children are fluent.	

1:7

At this point, provide practice, including:

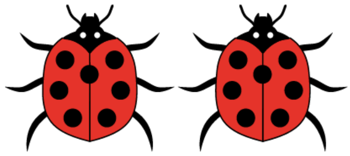
- completing/writing multiplication equations for contextual examples
- drawing/making contextual representations to match multiplication equations
- missing-number sequences and problems
- true-false style questions
- word problems, including measures contexts, for example:
  - *'What is the product of "7" and "9"?'*
  - *'Felicity goes to Australia for five weeks. How many days is this?'*
  - *'Ahmed has ten heptagons. How many sides are there altogether?'*

  Children should write a multiplication equation for each problem, rather than simply writing the product.

For word problems, ensure that some examples give seven as the second piece of information, while others give it first. However, for now, all practice should be in the context of groups of seven. The seven times table will be applied to seven equal groups in step *1:10*.

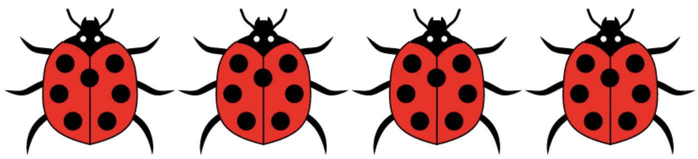
Completing multiplication equations:

'For each picture, complete the equations to show how many spots there are altogether.'



$2 \times 7 = \square$ $7 \times 2 = \square$

$\square \times 7 = \square$ $7 \times \square = \square$



$\square \times \square = \square$ $\square \times \square = \square$

Representing multiplication facts:

- *'Draw some seven-spotted ladybirds to represent:'*

  $7 \times 7 = 49$
- *'Draw an array to represent:'*

  $5 \times 7 = 35$

At this stage, children can recite the seven times table up to the number they need to find the answers, or use the multiplication chart for reference. Plenty of practice will be needed over an extended period until children are fluent in the isolated multiplication facts (for example, just knowing that six sevens are forty-two, rather than having to recite the times table up to six sevens).

Missing number sequences/problems:

'Fill in the missing numbers.'

0	7	14	21	28								

84	77	70										

$7 \times \boxed{1} = \boxed{}$

$7 \times \boxed{3} = \boxed{}$

$7 \times \boxed{5} = \boxed{}$

$7 \times \boxed{7} = \boxed{}$

$7 \times \boxed{9} = \boxed{}$

$7 \times \boxed{11} = \boxed{}$

$\boxed{0} \times 7 = \boxed{}$

$\boxed{2} \times 7 = \boxed{}$

$\boxed{4} \times 7 = \boxed{}$

$\boxed{6} \times 7 = \boxed{}$

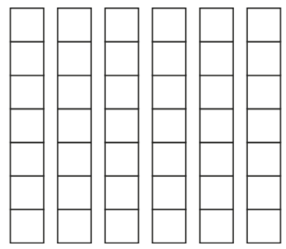
$\boxed{8} \times 7 = \boxed{}$

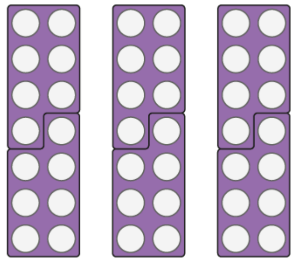
$\boxed{10} \times 7 = \boxed{}$

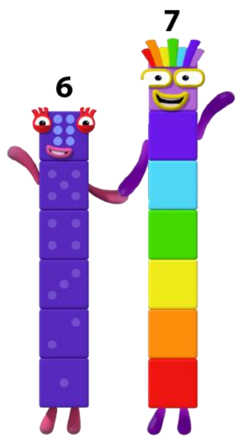
$\boxed{12} \times 7 = \boxed{}$

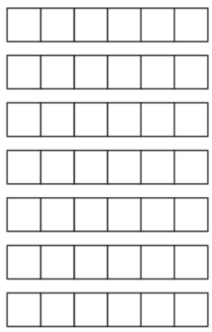
Dòng nǎo jīn:

'Which of these pictures could represent groups of seven? Write two multiplication equations for each picture that represents groups of seven.'









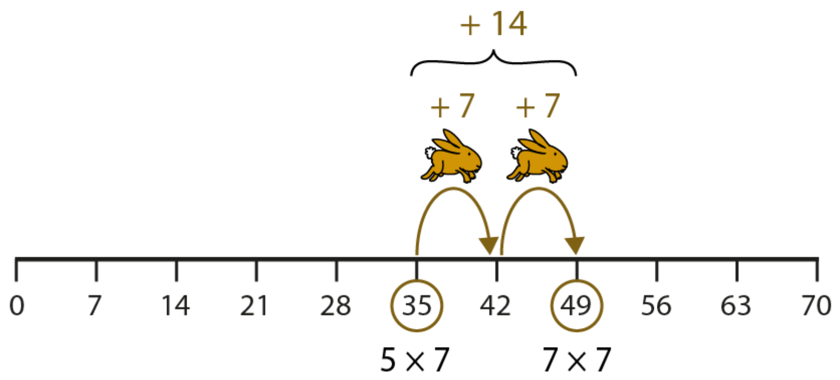
1:8	By this stage, children should be very familiar with the fact that within a times table, adjacent multiples have a constant difference. By examining the seven times table chart and the ratio chart (presented in step *1:5*), children should quickly be able to spot that adjacent multiples of seven have a difference of seven; they should also be able to see that the products alternate between odd and even. Draw attention, again, to the fact that there are only really three new facts in the seven times table to learn, discussing why we already know the given facts/where we know them from (through applying the commutative law). You can write out the seven times table, filling in the known facts, and highlighting the new/missing facts. Then ask children how we could work out the missing facts from the known facts. Initially, use the 'adjacent multiples rule', using similar representations to those used in previous segments (ratio chart, mixed-operation equations, number line and arrays) for support: • $7 \times 7 = 6 \times 7 + 7$ • $11 \times 7 = 10 \times 7 + 7$ • $12 \times 7 = 11 \times 7 + 7$ Also explore how 7×7 can be calculated through $8 \times 7 - 7$. It is not likely that children would choose this strategy in preference to $6 \times 7 + 7$. However, it is good for children to think more deeply about the structure of multiplication. Although the distributive law of multiplication is not formally considered (beyond adjacent multiples) until segment *2.10 Connecting multiplication and division, and the distributive law*, you can explore, through use of the number line or arrays, other ways that these new facts can be related to known facts: • If children are more easily able to remember the 5×7 fact compared to the 6×7 fact, they can derive 7×7 from five sevens plus two more sevens. • Similarly, children can link 12×7 to ten sevens, plus two more sevens. For now, use the representations as suggested rather than relying on writing equations of the form $7 \times 7 = 5 \times 7 + 2 \times 7$.

Three new facts:

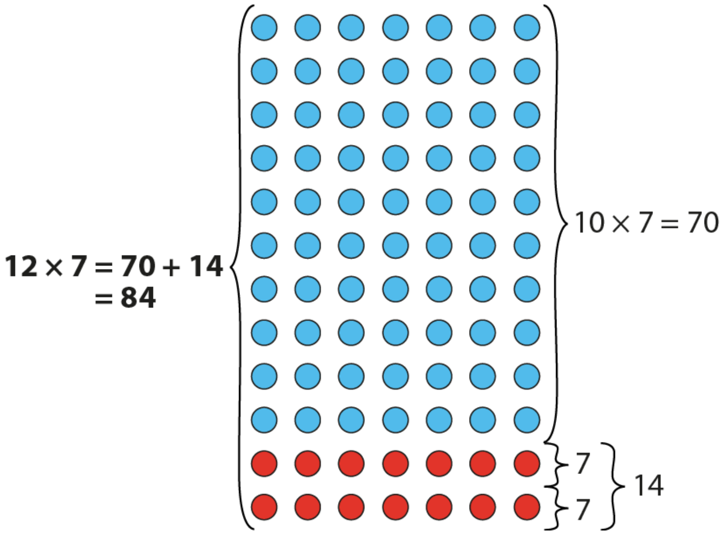
$0 \times 7 = 0$	$7 \times 0 = 0$
$1 \times 7 = 7$	$7 \times 1 = 7$
$2 \times 7 = 14$	$7 \times 2 = 14$
$3 \times 7 = 21$	$7 \times 3 = 21$
$4 \times 7 = 28$	$7 \times 4 = 28$
$5 \times 7 = 35$	$7 \times 5 = 35$
$6 \times 7 = 42$	$7 \times 6 = 42$
$7 \times 7 = ?$	**$7 \times 7 = ?$**
$8 \times 7 = 56$	$7 \times 8 = 56$
$9 \times 7 = 63$	$7 \times 9 = 63$
$10 \times 7 = 70$	$7 \times 10 = 70$
$11 \times 7 = ?$	**$7 \times 11 = ?$**
$12 \times 7 = ?$	**$7 \times 12 = ?$**

	× 7		
0	0		
1	7		
2	14		
3	21		
4	28		
5	35		
6	42	↓ + 7	$7 \times 7 = 6 \times 7 + 7$
7	**49**		
8	56		
9	63		
10	70	↓ + 7	$11 \times 7 = 10 \times 7 + 7$
11	**77**	↓ + 7	$12 \times 7 = 11 \times 7 + 7$
12	**84**		

Number line showing *7 × 7* derived from *5 × 7* fact:



Array showing 12×7 derived from 10×7 and 2×7 facts:



1:9

Provide children with varied practice based on their knowledge that adjacent multiples of seven have a difference of seven, including:

- mixed-operation missing number/symbol problems
- contextual problems, for example:
  - *'There are eight netball teams in a tournament, each containing seven players.'*
    - *'How many players are there?'*
    - *'How many players would there be if another team joined the tournament?'*
  - *'I have five bunches of flowers, each containing seven flowers.'*
    - *'How many flowers are there?'*
    - *'If I give away one bunch of flowers, how many flowers would I have left?'*

Missing number/symbol problems:

- *'Fill in the missing numbers.'*

$10 \times 7 = 9 \times 7 + \boxed{}$ $\qquad$ $6 \times 7 = \boxed{} \times 7 + 7$

$10 \times 7 - \boxed{} = 9 \times 7$ $\qquad$ $6 \times 7 - 7 = \boxed{} \times 7$

- *'Fill in the missing symbols (<, > or =).'*

$9 \times 7 \bigcirc 8 \times 7$

$9 \times \bigcirc 8 \times 7 + 7$

$9 \times \bigcirc 9 \times 7 + 7$

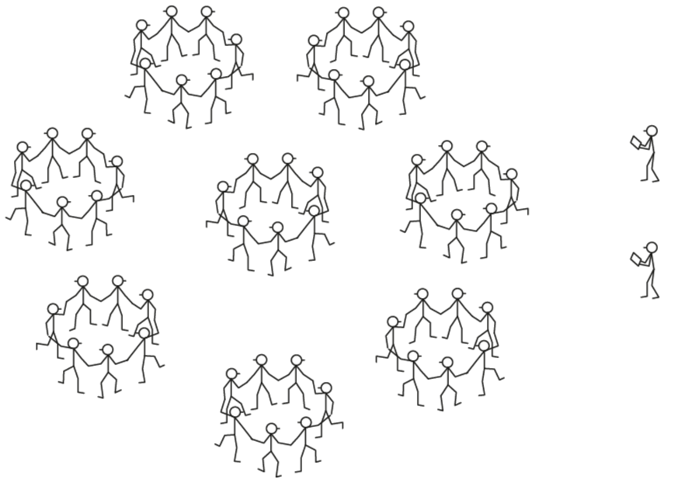
$9 \times 7 \bigcirc 10 \times 7 - 7$

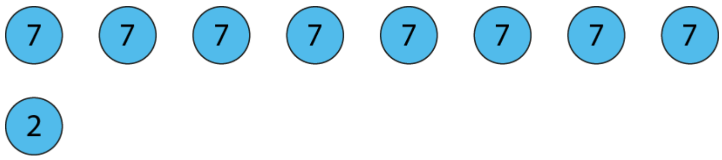
		Dòng nǎo jīn: *'Fill in the missing numbers.'* $20 \times 7 = 140$ so $21 \times 7 = \square$ $7 \times 18 = 126$ so $7 \times 19 = \square$ $15 \times 7 = 105$ so $14 \times 7 = \square$ $7 \times 17 = 119$ so $7 \times 16 = \square$ $124 \times 7 = \square$ $125 \times 7 = 875$ $126 \times 7 = \square$

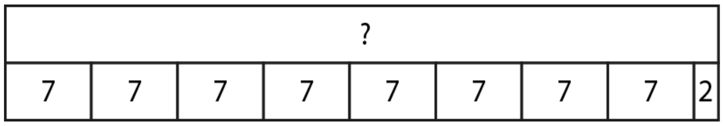
<table>
<tr>
<td>1:10</td>
<td>Mixed-operation problems, such as the example opposite, have been embedded throughout multiplication practice within the times tables teaching points across Spine 2. However, you could now take the opportunity to examine the structure of such problems more closely and discuss appropriate representations.

Present the example problem, and represent it with counters, as shown opposite. Ask children to explain why there are eight seven-value counters and one two-value counter, encouraging them to describe what the counters represent, using full sentences. Similarly, show the bar model representation and ask children to explain what each part represents. Discuss the representations, asking children which one they prefer and which one they want to record in their books; then, ask them to identify the corresponding calculation, representing it with an equation, as shown opposite, and linking it to the counter and bar model representations.

To promote and assess depth of understanding, use a dòng nǎo jīn problem in which you ask children to write or tell their own story to go with a mixed operation equation such as 6 × 5 – 2.</td>
<td>'Seven children are needed in each of the eight dance sections in the school assembly. Two children are also needed as narrators. How many children are needed altogether?'







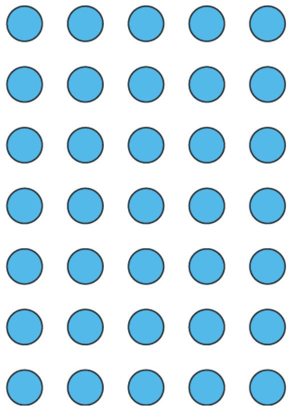
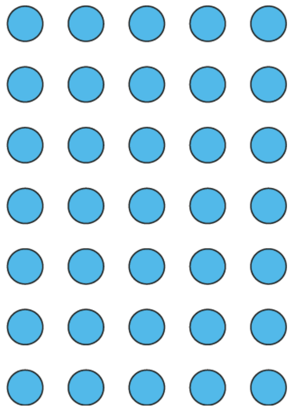
8 times 7 plus 2 narrators $= 8 \times 7 + 2$
$= 56 + 2$
$= 58$</td>
</tr>
</table>

1:11

To complete this teaching point, provide general practice that includes extending seven times table facts to:

- multiplication problems about seven equal groups (as opposed to those about groups of seven), as shown opposite and here: *'On each day of the seven days of the week, Teresa cycles three kilometres. How far does Teresa cycle altogether in one week?'*
- non-contextual division problems (writing equations; linking multiplication and division equations; missing-number problems); use intelligent practice as shown opposite
- contextual division problems, for example:
  - *'There are seven players on a netball team. If there are forty-two players in a netball tournament, how many teams are there?'* (quotitive division)
  - *'My ribbon is thirty-five centimetres long. If I split it into seven equal lengths, how long will each part be?'* (partitive division)
- multi-step contextual problems, for example:
  - *'The month of July is made up of four full seven-day weeks and three more days. How many days are there in July?'*
  - *'Charlie plans a camping trip to last three full seven-day weeks. On the last week it is raining, so Charlie ends his trip three days early. How long is Charlie's camping trip?'*

Multiplication problems about seven equal groups:

'Circle the groups of seven and complete the sentence.'  There are ___ groups of seven.	*'Circle the seven equal groups and complete the sentence.'*  There are seven groups of ___.

$7 \times \square = 35$

$\square \times 7 = 35$

Non-contextual division problems:

- *'Fill in the missing numbers.'*

$4 \times 7 = \square$ $\quad$ $9 \times 7 = \square$

$7 \times \square = 28$ $\quad$ $7 \times \square = 63$

$28 \div 7 = \square$ $\quad$ $63 \div 7 = \square$

- *'True or false?'*

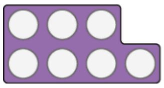
$56 \div 7 - 7 = 1 \times 7$

Teaching point 2:

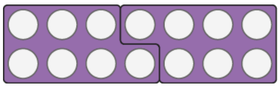
When both factors are odd numbers, the product is an odd number; when one factor is an odd number and the other is an even number, the product is an even number; when both factors are even numbers, the product is an even number.

Steps in learning

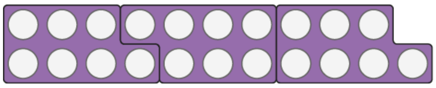
	Guidance	Representations
2:1	Now that children have covered all times tables up to and including ten, take the opportunity to explore some of the patterns revealed by these multiplication facts. This teaching point investigates whether a product will be odd or even depending on the factors. By now, children should have a solid grasp of the meaning of the terms 'odd' and 'even', based on previous work in *Spine 1: Number, Addition and Subtraction*: • Segment *1.4* introduced the terms 'odd' and 'even', and explored partitioning of odd and even numbers. • Segment *1.7* explored the difference of two between consecutive odd/even numbers. • Segment *1.10* covered the fact that we can determine whether a number is odd or even by looking only at the ones digit. Begin by briefly reviewing odd and even numbers, using true/false questions to check children's understanding, for example, *'Forty-seven is an even number because the tens digit is even – true or false?'*	Base ten number boards:



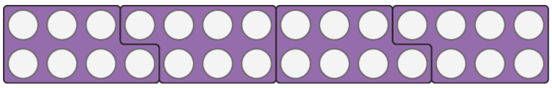
1	×	7	=	7	7	×	1	=	7
odd		odd		odd	odd		odd		odd



2	×	7	=	14	7	×	2	=	14
even		odd		even	odd		even		even



3	×	7	=	21	7	×	3	=	21
odd		odd		odd	odd		odd		odd



4	×	7	=	28	7	×	4	=	28
even		odd		even	odd		even		even

	Then begin to explore odd and even numbers within the context of patterns in the times tables. Begin with one of the factors being odd. Since children have just covered the seven times table, this is perhaps a good one to take as an exemplar, since it is fresh in children's minds, and the exploration will increase their exposure to this newly learnt times table; however, it may be appropriate for some classes to work in the lower times tables. Use base-ten number boards (or tens frames, filled 'twos-wise' as exemplified in step *2:2*) to represent consecutive multiples of seven. Begin with one seven, presenting the base-ten number board alongside the multiplication equations, as shown on the previous page. For each multiplication fact, ask children to identify each number in the equation as odd or even, using the shapes of the base-ten number boards for support.	

2:2

Children will probably have started to spot patterns during step *2:1*. Now, repeat the process using a different 'odd' times table (for example, the nine times table), both to further reveal the patterns and to demonstrate that the same patterns exist in another times table.

Ask children what patterns they can see, working towards the following generalised statements and equations:

- ***odd factor × odd factor = odd product***
  'If both factors are odd, the product is odd.'
- ***even factor × odd factor = even product***
  and
  odd factor × even factor = even product
  'If one factor is odd and the other factor is even, the product is even.'

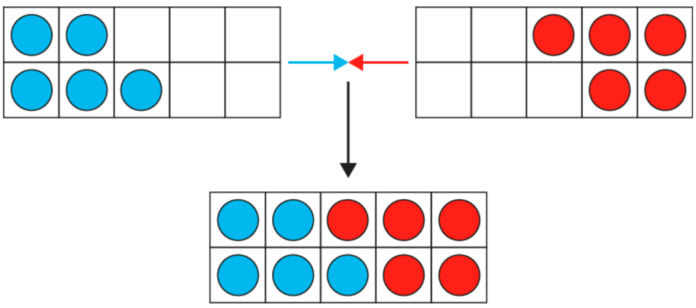
For easier memorisation, you can use the shortened sentences:

- *'Odd times odd is odd.'*
- *'Even times odd is even.'*
  and
  'Odd times even is even.'

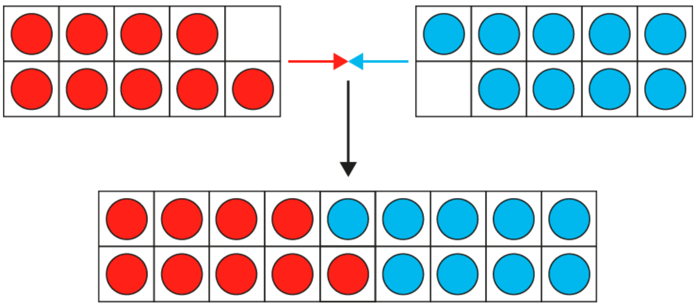
Also, make the following generalisation to aid memorisation: ***'If one of the factors is even, the product is even.'***

Note that it is implied that the other factor is a whole number since this is the context in which children are currently working.

Tens frames filled twos-wise – examples:



2	×	5	=	10	5	×	2	=	10
even		odd		even	odd		even		even



2	×	9	=	18	9	×	2	=	18
even		odd		even	odd		even		even

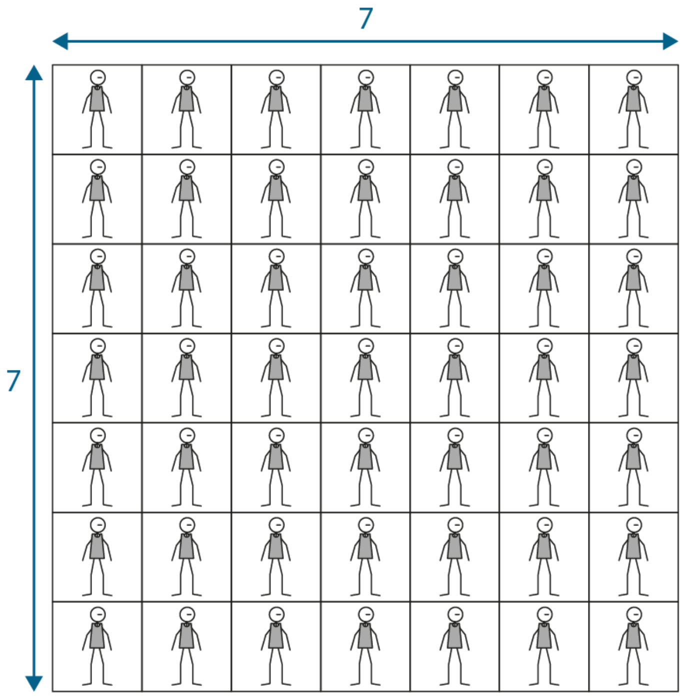
2:3	Now, repeat with an even times table (for example, the six times table) to reveal the final pattern (even × even = even) alongside the odd × even pattern. Similarly to step *2:2*, use the following generalised statement and equation: ***even* factor × *even* factor = *even* product** ***'If both factors are even, the product is even.'*** You can shorten to *'even times even is even'* to aid memorisation. Looking at the chosen even time times table, ask children why none of the products are odd. Based on their learning in steps *2:1–2:2*, they should be able to conclude that: • If one (or both) of the factors is even, the product is even. *or* Both of the factors need to be odd for the product to be odd. • Since this is an even times table, one of the factors is always even, so the product will always be even. It is useful to carry out a similar enquiry (with the base-ten number boards, or just by examination of the multiplication charts) across all other times tables.	

<table>
<tr>
<td>2:4</td>
<td>To practise and review understanding, present a series of calculations such as those shown opposite, asking children to predict whether the products will be odd or even. All numbers have been chosen in the expectation that most children would not be able to do the calculations in their heads at this stage. The focus isn't on knowing/calculating the actual product, but on predicting whether the product will be an odd or an even number. For each calculation, once children have made their prediction, reveal the product to see if they were right.</td>
<td>'For each equation, predict whether the product will be odd or even. Tick the correct column.'
<table>
<tr><th></th><th>product is odd</th><th>product is even</th></tr>
<tr><td>21×7</td><td></td><td></td></tr>
<tr><td>7×32</td><td></td><td></td></tr>
<tr><td>46×7</td><td></td><td></td></tr>
<tr><td>8×68</td><td></td><td></td></tr>
</table>
</td>
</tr>
</table>

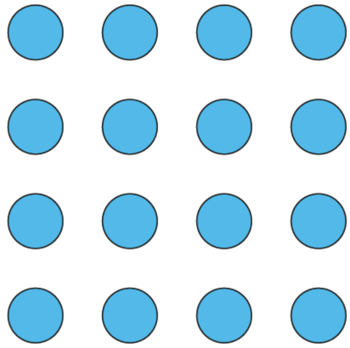
Teaching point 3:

When both factors have the same value, the product is called a square number; square numbers can be represented by objects arranged in square arrays.

Steps in learning

	Guidance	Representations
3:1	This teaching point introduces the definition of a square number, along with the corresponding mathematical notation (the squared symbol, ^{2}). Note that zero is omitted from the discussion of square numbers, since it is not possible to make and visualise a *0 × 0* array. Since children have recently been working with the seven times table, begin by looking at a seven-by-seven array. Use a familiar context, such as seven netball teams each with seven players. If necessary, review the term 'array'. Present the players as an array and ask children: • *'How many players are there altogether?'* (49) • *'What do you notice about the shape of the array?'* (It is square; all sides are the same length.) • *'How can we represent this with a multiplication equation?'* (7 × 7 = 49) • *'What do you notice about the factors in the equation?'* (They are both the same.) Explain that we call '49' a square number.	Example 1: *'There are seven netball teams, each with seven players.'*  7 × 7 = 49 • *'Both of the factors are the same, so the product is a square number.'* • *'This can be represented by a square-shaped array.'*

<table>
<tr>
<td></td>
<td>Repeat for other examples. You could use a contextual example, multilink cubes, or counters arranged in an array. For children familiar with the Numberblocks characters, you could examine the four and nine square characters.

Work towards the following generalisations:
• ‘When both factors have the same value, the product is called a square number.’
• ‘Square numbers can be represented by square-shaped arrays.’</td>
<td>Example 2:

$4 \times 4 = 16$
• ‘Both of the factors are the same, so the product is a square number.’
• ‘This can be represented by a square-shaped array.’

Examples 3 and 4:
<table>
<tr><td>
</td><td>
</td></tr>
<tr><td>$2 \times 2 = 4$</td><td>$3 \times 3 = 9$</td></tr>
</table>
</td>
</tr>
<tr>
<td>3:2</td>
<td>Once children are secure with the definition of a square number, introduce the superscript notation. Take the familiar netball-team example from step 3:1, and demonstrate that we can use the squared symbol to represent 7×7. Model and explain that the superscript ^{2} is articulated as ‘squared’ and that when it comes after a number it means that number is ‘multiplied by itself’.</td>
<td></td>
</tr>
</table>

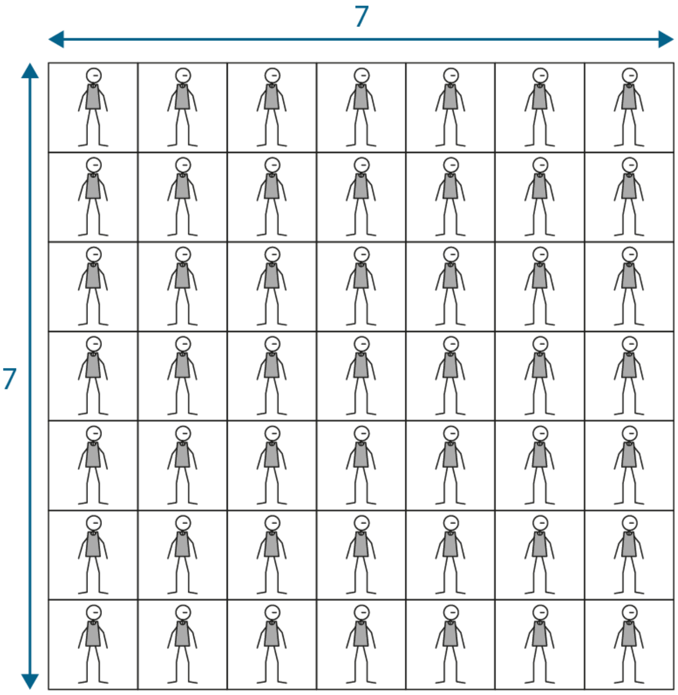
Work through several familiar examples, with children practising writing the equation with the squared symbol themselves on squared paper, as exemplified opposite. Use the following stem sentences to connect the squared symbol to the multiplication equation:

- ***'We can write this as ___ times ___ is equal to ___.'***
- ***'Both factors are the same, so we can also write this as ___ squared is equal to ___.'***

Be aware that a common misconception is for children to think that the squared symbol means *'× 2'*; there is no need to draw children's attention to this, but if the misconception arises it will need to be addressed (the set of missing-symbol practice problems in step *3:4* can help with this). The examples of 7^2 and 3^2 were used to introduce square numbers to help avoid this misconception arising; starting with 2×2 could cause unnecessary difficulty for children. Further confusion can arise due to the fact that $2 \times 2 = 2 + 2$; while, working through examples, include a 2×2 array and take time to explain that two squared has this unique property *but none of the other squared numbers follow this pattern.*

Introducing the squared symbol:

'There are seven netball teams, each with seven players.'



- *'We can write this as seven times seven is equal to forty-nine.'*
  $7 \times 7 = 49$
- *'Both factors are the same, so we can also write this as seven squared is equal to forty-nine.'*
  $7^2 = 49$

Writing the squared symbol using squared paper:

	7^2	=	4	9	

3:3 Since children will be very familiar with the multiplication chart by now, take a moment to highlight the square numbers on the class chart. It is worth noting that zero and one are both square numbers. In these cases it is either not obvious (for one) or possible (for zero) to represent them using a square array. However, they should be recognised as being square numbers because they correspond to multiplication of two equal factors (0×0 and 1×1). Note that children can already predict that 11×11 and 12×12 will both have products that are square numbers, even though they have not yet learnt these facts.

×	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110		
12	0	12	24	36	48	60	72	84	96	108	120		

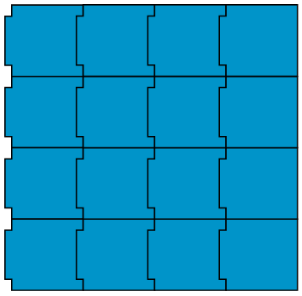
3:4

To complete this teaching point, provide children with practice, including:

- working practically to create each of the squared numbers (1^2 through to 10^2), using multilink cubes, writing both forms of the equation for each square number
- missing-number/expression/symbol problems such as those exemplified opposite.

Throughout, ensure that children write the squared symbol clearly (smaller than the numeral being squared and at the top right).

Working practically – *4 × 4* example:



$4 \times 4 = 16$

$4^2 = 16$

Missing-number problems:

'Fill in the missing parts of the equations.'

1×1	=	1^2
	=	2^2
3×3	=	3^2
4×4	=	
5×5	=	
	=	6^2
	=	7^2
8×8	=	
	=	9^2
	=	10^2

Missing-symbol problems:

'Fill in the missing symbols (<, >, or =).'

4^2 ◯ $4 + 4$

4^2 ◯ 4×2

4^2 ◯ 4×4

4^2 ◯ $4 + 2$

Teaching point 4: Divisibility rules can be used to find out whether a given number is divisible (to give a whole number) by particular divisors.	
Steps in learning	
4:1	This teaching point reviews the divisibility rules already learnt (for divisors up to and including ten). Note that the divisibility rule for division by seven is too complex to be useful for children, so this should not be included. Briefly remind children what is meant by the term 'divisible by', using a simple example such as *'Is twelve divisible by two?'* Note that, since children are working within the context of integers, the statement 'can be divided by' is used to imply 'gives a whole number when it is divided by' (or 'gives no remainder when it is divided by'). Children have already experienced the fact that being able to identify whether a dividend is divisible exactly by a particular divisor (without having to do a full calculation) is a useful skill that can be used to solve problems. Remind children of this, using a simple example such as *'Can sixty-seven children be split evenly into two teams?'*
4:2	Now, working in families of times tables, build up a list of the divisibility rules, as shown below. For each rule: • see if children can remember it, then display the rule • work through one or two abstract examples to remind children how the rule is applied, ensuring they understand the distinction between the terms 'digit' and 'number'. Once this process is complete, you could additionally display the divisibility rules in numerical order. For more detail on the divisibility rules, refer to the following segments: • For divisibility by two, five and ten, see *2.6 Structures: quotitive and partitive division* (steps *4:11–4:13*). • For divisibility by four and eight, see *2.7 Times tables: 2, 4 and 8, and the relationship between them*. • For divisibility by three, six and nine, see *2.8 Times tables: 3, 6 and 9, and the relationship between them*.

Divisibility rules in 'families'	
2	*'A number is divisible by two if the ones digit is even.'*
4	*'If halving a number gives an even value, then the number is divisible by four.'* and *'For numbers with more than two digits: if the final two digits are divisible by four then the number is divisible by four.'*
8	*'If halving a number twice gives an even value, the number is divisible by eight.'*
5	*'A number is divisible by five if the ones digit is five or zero.'*
10	*'A number is divisible by ten if the ones digit is zero.'*
3	*'For a number to be divisible by three, the sum of the digits of the number must be divisible by three.'*
6	*'For a number to be divisible by six, the number must be divisible by <u>both</u> two <u>and</u> three.'*
9	*'For a number to be divisible by nine, the sum of the digits of the number must be divisible by nine.'*

Divisibility rules in numerical order	
2	*'A number is divisible by two if the ones digit is even.'*
3	*'For a number to be divisible by three, the sum of the digits of the number must be divisible by three.'*
4	*'If halving a number gives an even value, then the number divisible by four.'* and *'For numbers with more than two digits: if the final two digits are divisible by four then the number is divisible by four.'*
5	*'A number is divisible by five if the ones digit is five or zero.'*
6	*'For a number to be divisible by six, the number must be divisible by <u>both</u> two <u>and</u> three.'*
8	*'If halving a number twice gives an even value, the number is divisible by eight.'*
9	*'For a number to be divisible by nine, the sum of the digits of the number must be divisible by nine.'*
10	*'A number is divisible by ten if the ones digit is zero.'*

4:3

Children have already practised applying the divisibility rules individually, to both abstract examples and contextual problems. Now provide more general problems, keeping to dividends with three digits or fewer.

When setting practice problems, bear in mind the following:

- Ensure that you include one-, two- and three-digit dividends in the problems, as well as some prime numbers (so that children find that some numbers meet none of the divisibility criteria).
- Include a variety of contextual problems, some of which have one 'answer' (i.e. the number given meets the divisibility criterion for only one divisor), and others that have more than one 'answer' (i.e. the number given meets the divisibility criteria for more than one divisor).
- It is useful to ask a few questions based around the multiplication facts that children sometimes find most challenging to memorise (e.g. 6×7, 7×7 and 7×8).
- Include both quotitive and partitive division contexts.
- Leave the divisibility rules on display as children work through the problems.

As children solve problems in which they must identify divisibility for more than one divisor (for example, '*Is "365" divisible by two, three, four, five, six, eight, nine and/or ten?*'), they may begin to see patterns within the families; for example, they may notice that if a number isn't divisible by two, then it won't be divisible by either four or eight, and so carrying out the tests for those divisors is unnecessary. As a class, you could make a flow chart from the divisibility rules.

Example abstract problems:

'Put a tick in the correct boxes to show which numbers are divisible by 2, 3, 4, 5, 6, 8, 9 and 10.'

	Number is divisible by…							
Number to test	**2**	**3**	**4**	**5**	**6**	**8**	**9**	**10**
5								
7								
72								
76								
180								
360								
365								
563								
960								

Contextual examples:

- *'At the Sunnyside nursery, plants need to be displayed in rows of either five or ten. Tick each quantity of plants that could be displayed in whole rows.'*

		Can be displayed in lines of 10	**Can be displayed in lines of 5**
Number of plants	450		
	975		
	273		
	465		
	890		
	501		

	Example true/false style word problems: • *'True or false? Fifty-six is divisible by five because one of the digits is five.'* • *'True or false? One hundred and three is divisible by ten because the tens digit is zero.'* • *'Sometimes, always or never true? A number divisible by five is also divisible by ten.'*	• *'For each example below, circle the numbers that are possible.'* • A cake factory needs to put an equal number of cakes in every box. If the factory makes 492 cakes, how many could go in each box? 3 5 6 (quotitive division) • Staff at a factory must sit down to eat at the same time. If there are 372 staff, and an equal number of people need to sit at each table, how many tables could there be? 2 3 4 5 6 8 (partitive division) • 415 children voted in the school-council election. Each candidate got the same number of votes. How many candidates could there have been? 5 10 2 (partitive division) • Rachel is packing 240 apples into bags. Each bag must contain the same number of apples. How many apples could there be in each bag? 2 4 5 10 (quotitive division) • Daffodils are sold in bunches of 7. How many daffodils should the pickers collect to make full bunches? 47 48 49 (partitive division)