The NCETM Podcast Episode 80

Early maths concepts: doubles

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My name's Rebecca Fisher [RF], and today we're going to do things a little bit differently on the podcast. Normally it would be my job to ask all the questions and interview someone about a specific topic, but today I'm here with my colleague, Sue Evans [SE], to have a discussion about an area of early maths that we think is really interesting, and hopefully you will too. We're going to be talking about the foundations of doubles, how it starts in the Early Years and builds through KS1 and beyond. Just for a bit of background, prior to my role here as a Communications Manager at the NCETM, I was a Reception class teacher for many years, I was involved in the work of my local Maths Hub, leading some of the programmes there, and I was a maths lead as well. Sue, I'll let you introduce yourself a little bit now.

SE: Hi, thanks Rebecca. I'm Sue Evans. I'm an Assistant Director with the Primary Team here at the NCETM. Prior to my role at the NCETM, I was a primary school teacher and maths lead, and I also worked with my local Maths Hub. I've got a particular interest in early maths and working with Reception and KS1 teachers.

RF: Excellent, thanks Sue. Let's get started then – why have we chosen this topic to discuss today? What's so important about doubles, Sue?

SE: Well, as you say, Rebecca, I think doubles are really interesting. We're going to be thinking about doubles as addition facts in this podcast. They're part of a larger set of facts that we want children to become fluent in, but I think that doubles are really good value facts. Just to give you an example, if I know that 7 plus 7 is 14, then actually I can use that to derive some other quite tricky facts. If I know 7 plus 7 is 14, I can work out that 7 plus 8, a near double, must be one more than that and therefore 15, or 7 plus 6 is one less. And of course, as with any additive facts, if we know that say 7 plus 7 is 14, we also know that 14 minus 7 must be 7. We describe that as an additive relationship. There's a relationship between those three facts. Although we're going to talk about doubles facts, we don't want to just think of this as something separate.

It's all about developing children's number sense. We're going to look at the foundations of this in Reception, how we can build really firm foundations to develop children's number sense so that they really understand what doubles are and how they're made, and then they can use those doubles for all sorts of maths. And of course, it's not just addition facts. We can use it to find products in the two times table. If we can double a number, then the two times table becomes easy, and we can also double and double again for the four times table. So, as I say, really worthwhile learning these facts.

RF: Yes, I would agree. I think it's all about the relationships, like you say, that you can learn about through understanding doubles. They are really good value facts to know, and it helps with fluency as well.

SE: I'd also say, Rebecca, something that's important to think about is it's not just knowing the facts. As a maths lead, I really wanted children to know 7 plus 7 is 14, but it doesn't stop there. We want all of our children to understand **why** double 7 is 14. There's nothing in that number word 14 that gives us a clue that there's a relationship with 7. I want children to understand why, and I want them to be able to tell me, 'Well, it can't be 15 because that's an odd number'. You know, why does doubling any whole number give an even number? It's that deep understanding, isn't it?

Let's think then, Rebecca, about what this looks like in the really early days in Reception. What sort of things did you do with children early on?

RF: Well, first of all, I think that we need to recognise, as teachers, that building that prior knowledge ready for understanding the facts as they move on is not something that we start in the summer term, just to tick that box. All that number work that we're doing in the autumn term is supporting future learning.

In my Reception class, I'd be making sure children had lots of experiences where they're recognising small quantities, subitising, especially on fingers, because fingers are the best kind of representation.

SE: Absolutely. They're always there, aren't they? They're so useful.

RF: Exactly! And gestures as well – getting children familiar with the gesture of 'altogether'. For those of you listening, I'm making a wide circle with my hand to show that we're talking about all the objects or quantities as a whole. Also, recognising that numbers are made of other numbers, because really a double is just a special way that two parts can make a whole. They need to understand composition of number and we'd spend so much time on this as children move through Reception.

We want children to develop a habit of noticing things and being mathematically observant. I would be asking children to tell me what they notice at different points during a game, or even specifically sharing an image on the interactive whiteboard of something like a scenario or a scene, for children just to think about what they notice. At first those conversations would be non-mathematical, thinking about colour or what type of objects they can see. But as time goes on – and I'd be modelling that mathematical thinking – noticing when two sets are the same, or when there's an equality or inequality between two groups of quantities. That also works for games as well, for example, when playing a dice game, thinking about when two children are at the same point within the game, or they roll the same number or not the same. All those comparison and composition conversations are key.

SE: Absolutely. I think there's some really important things you've talked about there. You talked about subitising, we know that's something that we want children to develop. It's building on something that is innate as a skill, to be able to look at a quantity and start to know how many objects there are, is something that we know we can build children's skill with. For example, they'll come to school and often be able to tell you that this is two. (I've got a board here and I've got some counters, I'm just showing two). I want children to be able to look at it and say, 'It's two'. We know that's something that they are capable of doing. Then we might offer them, say, four counters and we are not saying to children, 'Oh, I've got two and another two, what's that altogether?', we're offering them four counters and saying, 'here's four counters. You know it's one, two, three, four, I know it's four. How can you see the parts inside four?'. This is where they use their subitising skills to understand how numbers are composed. They might make an

arrangement, so they've got three together and one just moved away, and they might say, 'Well, inside four I can see that there's three and one, altogether four can be made of three and one'. And of course, they'll also start to notice, especially if they're making symmetrical patterns, that four can be made of two and two. We're not saying, 'What's 2 + 2?', we're saying, 'How can four be made? Use your subitising skills to really understand the composition of those numbers'. You've already got two doubles here, really, haven't you? If you can see that two is made of one and one, that's our first double. And if you can see that four is made of two and two, then you've got another double fact. Without even talking to children that they're learning doubles – we don't need to be doing that early on – we know that we're giving them the skills that they need to be able to start to understand how these small numbers are composed and can be composed with two equal parts.

RF: Yeah, that's exactly right. Also, with you showing that there, children being able to physically move the manipulatives, that's really powerful. Splitting them up, bringing them back together in lots of different ways, they need that experience of doing that from very early on.

SE: Let's think about the representations that you would use with children, when we are starting to want to introduce these numbers and to see these doubles facts. What have you found to be helpful, Rebecca? In recent years, what have you been using?

RF: Well, again, the thing that comes to mind is finger patterns. After all, as you said before, they are a resource that we've just got with us all the time. Children need to be able to manipulate their own fingers because that's a really useful tool. Creating double patterns for themselves, being able to show (I'm showing now on my fingers people who are listening) a two and a two. Also in Reception, we do a lot of matching them up. So being able to put the fingers on both hands together and I would often say, 'Make sure they've got a partner, and we can see if it's a double or not'. If each finger on both hands has got a partner, then we know that it's a double because it's the same – talking about the 'sameness'. And then as children get familiar with using their own fingers to represent those two parts, they can do that with a partner, which then frees up another hand to be able to gesture the circle around the two hands showing how many 'altogether'. If I had a partner and they put their hand to match up, the other person could then say, 'Altogether it makes four… two and two. Oh, there must be a double because each finger's got a partner'.

SE: I love the idea of doing it with a partner, actually, Rebecca, because I think it's easier when you're doing it yourself, isn't it? But when you're doing it with a partner, you've actually got to start really noticing how many fingers the other person is holding up. You are sort of shifting your attention away from what you are doing and looking at what somebody else is doing, and then matching it. It's a little bit of a step on, isn't it? Deepening that understanding and that skill. So yeah, finger patterns, and I'm going to come back to finger patterns when we talk about learning doubles beyond double five. But absolutely finger patterns are key. What other things might you use?

RF: There's quite a few actually that I can think of. One of them being dice patterns and dice representations. We would use dice a lot in Reception to play games, even just quite simple games, they don't have complicated. Just practising rolling dice, lots of repetition of rolling, noticing the spots and thinking about doubles on a dice is such a powerful tool. And they'll be able to play that with adults at home as well and older siblings, so that's really useful.

Just a few more I can think of representation wise, we've got Numicon, which is a great tool to use that shows the evenness of doubles. When two Numicon are the same, then we can see that it's a double and the colour within that as well is powerful.

Then tens frames, which links to that, particularly if you use it cleverly with colour as well to highlight the evenness of doubles. Thinking about the way I would do it, we would use ten frames in a vertical and a landscape way. But I feel that with it shown in landscape, with the two rows of five, it links to the rekenrek later on, and it reflects that pattern that they'll see as they move through school. I think often we would do doubles on there and have them above each other, but obviously children do need to be familiar with using the tens frame in all orientations really.

Then *Numberblocks* as well. We would often use some of the *Numberblocks* episodes alongside connecting cubes and then activities would spawn from that. Some of those episodes are quite good for composition and comparison, and obviously some of them are about doubles, where often a mirror is used as well. Sometimes we would use connecting cubes in a mirror to see the mirror image, and then see if a partner could make the same to create two connecting cube towers so it becomes a double. So yes, lots of quite simple ideas of representations available there.

SE: Thank you – so those are the sorts of things that you might be using in your whole class teaching, I'd like to pick up on what you said about using those dice patterns, Rebecca. Rolling two dice and spotting when it's the same children and knowing that's a double is really key. But I'd also like to stress, we might use a different sort of tens frame.

This is called a Hungarian number frame – I'm holding a picture here and we'll put it in the show notes – it's a tens frame that is made in the arrangement of two dice patterns of five. If I clear my frame, all I've got is an empty frame with ten round spaces, where I'm going to put counters. As I say, it's a five and another five together. We call it a Hungarian number frame or a double dice pattern with the children. I can use this and I could perhaps make three, I could make it in a die arrangement for three, but I could make it in all sorts of ways with three in going around a corner. But if I just make it like that and say, well, if we've got three and we have another three. Children often like symmetry, so I'm going to make that as a symmetrical pattern. Now I've got three on each side, in a diagonal pattern on my Hungarian number frame. Children will be able to start to say, 'I can see three and I can see another three. I can make that on my fingers and show that it is a double'. Now how are we going to see that it is six? Well, there's different ways, but we want to avoid always having to count. If we're going to use our subitising skills and build up this idea of how numbers are composed being the basis of learning our number facts, well, we could arrange it in a way that children can more easily see six. We might arrange it like that, or we could of course just fill up one side of our frame with five, and then we've got one more – I'm going to describe this – we've got six shown as five on one side and one on the other. And again, we can link that to our number patterns. We know that double three must be six because I can rearrange it. I haven't added any counters, I haven't taken any off. I've just rearranged it and I can see now that it's six because it's made of five and one. We call that the five-and-a-bit structure, and that's going to be important later on. Children knowing that these numbers six, seven, eight, and nine are made of five-and-a-bit.

But that's just a simple idea that you could be doing with all of your class. Now let's think then, Rebecca, those are some of the things that we might talk with children about as a class. Of course, we don't want to just leave it there, we want children to really embed their learning through continuous provision and small-group activities. What sort of things would you do with your children to really embed these doubles?

RF: Yeah, of course. I found games were always really popular with children and dice really linked well to playing games. And you'll know if the children in your class enjoy competition or not, so it might be that you create a game where there's an element of competition, or it might be that it's just a game where there's a lot of repetition, and a lot of pausing to notice and talk.

Things that can be played independently or in small groups or pairs are great. For example, things like having two giant dice outside, so two children in a pair taking turns to roll the giant dice, and then subitising to see what it lands on, predicting then whether they think their partner is going to get the double of that and have the same number when they roll the dice. Think whether they're going to have more spots on their die or fewer than their partner, and then the other person having a turn, subitising, and then thinking, 'Oh, was it a double?'. It's better when there are two visible dice so they can compare the quantities side-by-side. Playing something like this sounds really simple, and it just sounds a little bit like, 'Oh, are they going to enjoy that?', but often children enjoy those more simple activities that they can understand. There aren't any complicated rules. As an adult, sometimes when we plan activities, we are thinking, 'How can we make it more challenging? How can we do this?', but it's all about noticing the doubles, noticing the equivalence, and also reasoning. If it's not a double, then how do you know it's not a double?

As a teacher I would often, in activities with children, offer examples of a pattern that's not a double to get the children to explain why it's not, and then see if they can change that pattern to become a double. Maybe if it was using counters, like you've just showed with the magnetic whiteboard and the magnetic counters, could one of the children come and move it? So, turning an arrangement of four and two into a three and a three to make six, but shown as a double.

But what I would say here is, often we get drawn into, especially if you google 'double activities', it comes up with butterflies or ladybirds with spots. Yes, using spots on ladybirds does sound appealing, but maybe think about that as a distraction from the mathematics. I found that using butterfly wings – painting one side with however many shapes or colours and then folding it over to create that symmetrical pattern – of course symmetry is a representation of double, but it's a distraction, because they're thinking about how they're painting the pattern and they're thinking more about the design of the butterfly wings rather than the actual mathematics. Things like using Play-Doh to make balls or punching or poking with their finger double patterns into Play-Doh, that can easily be flattened out and done. Again, lots of repetition available, whereas if you've painted butterfly wings it's more about the outcome than the actual process. We are thinking about it the other way around, where we want to think about the mathematics as the process rather than the outcome that we can put up on the wall. It's just all about keeping it simple!

SE: I really like the idea of the Play-Doh. We can link that with our finger patterns or with Numicon or whatever we're using. But I really like the idea of making a doubles pattern on your fingers, and then plunging your fingers into the Play-Doh and the teacher can say, 'Well, you had double three, what is double three altogether?', and you can still see those six dots. It's a way then of connecting finger patterns to something, another representation.

Those are the really important ways to represent these doubles, or any addition facts, in reception. We don't need children in reception writing equations for any of the addition facts. We don't need them to be writing something like 3 + 3 = 6, that can absolutely be left for Year 1.

RF: So now let's think a little bit about Key Stage 1. What are we moving on to as children leave Reception, Sue?

SE: So listeners might be thinking that the next natural step is in Year 1 to learn double six, double seven, and so on. Of course, we do want children to be learning those doubles beyond double five, but if we are going to teach children in a way that all children can access this learning, we need to think really, really carefully about the sort of progression in learning that we offer children.

You've talked about dice games, Rebecca. Lots of children will start to remember that double 6 is 12. If you've got a good memory, then that's great, you'll just remember double 6 is 12. I'd also like you, even if you can remember that double 6 is 12, I'd like you to know **why** double 6 is 12. But we want all children to be able to access these rather more difficult doubles beyond double five.

I'm going to suggest that we think about this in Year 2, and I'm going to come back to Year 1 and think about what we do in Year 1 in a moment. I talked earlier about the Hungarian number pattern showing that six can be made of five and one, and I refer to that as the five-and-a-bit structure. All we mean by the five-and-a-bit structure is six is made of five and one, seven is five and two, eight is five and three, and as I'm talking, I'm holding up my fingers. So five fingers on one hand and the bit on the other hand, all the way up to 5 + 4 = 9. If children understand how those numbers can be composed with five as a part, we can use that to build their knowledge of doubles. They don't have to learn double six and double seven as a separate fact. We can just build on those doubles that they've become fluent with in Reception.

I'm going to use a rekenrek at this point. For listeners who aren't familiar with a rekenrek, it's a sort of abacus. It's got two rows of beads, ten on the top and ten on the bottom, and each row has got five red beads and five white beads. This is a really useful tool for when we're working with that five and a bit structure. If I know that seven is made of five and two, or can be composed of five and two on my rekenrek, I can do what we call a ‘one push’ seven. I can just subitise that, so I'm drawing on those subitising skills. I know that seven is made a five and two, so I can do a ‘one push’ seven. There's my seven. And then I can make double seven by showing the same number on the bottom. It's really clearly two equal groups because the beads are the same length, aren't they? I know it must be double seven because it's seven on the top and seven on the bottom.

And the really lovely thing here is that I don't have to remember what double seven is. if I just move these beads, I'm just going to use this to move the beads a little bit along, we can then think of double seven as double five – that's easy because we know double five, we've used our finger patterns for double five – and double two – that’s easy again, we've used our finger patterns and we know that double two is four – so double seven must be a combination then of ten and four. So double seven is equal to ten and four. That's using that five-and-a-bit structure to unpack those trickier doubles beyond double five.

As I said earlier, we can show this in different ways. I've got an image of a tens frame (we'll pop this in the show notes) and we could use this representation. This is eight: I've got two tens frames together, next to each other, and I'm showing eight on the top as five and three and eight on the bottom as five and three. I've used colour too. I've put the first tens frame which is full, with red counters, and the other tens frame which has got obviously three and three on, as yellow. You can use colour there to draw attention to that five in a bit structure.

I think I really like the rekenrek because we can do those ‘one push’ numbers. We can show that we can subitise and look at seven as a group of five and two, and we can also just manipulate it and move the beads away to really draw attention to the idea that double seven is made of double five and double two.

Going back to finger patterns, Rebecca, you were talking about playing that doubles game with a friend. Of course, we can do this with our doubles past five. If I was to hold up my fingers and show seven and say, ‘I have seven’, and if you were in the same room as me, Rebecca, you could put your hands against mine and copy my pattern just as you would in Reception – we'll try and do it – Rebecca could reply, ‘I have seven’. And then we could put our hands together to

show that, ‘we know that we've got a double five, we almost don't need to think about that, but it's the ‘bit’, that's tricky’. If I can recognise that that 2 + 2 = 4, I can start to reason why 7 + 7 = 14.

That's what it might look like to unpack that in Year 2. Of course, thinking about the prerequisite facts that are going to help all children access that, we need to do a lot of work with Year 1 children on that five-and-a-bit structure. So really, really knowing and not having to think about it. When I go into Year 2, I just know that seven is made of five and two I, I've seen it a lot, I've seen it represented, I've talked about ‘seven is made of five and two, five and two make seven’. We need to really be secure with that.

And of course, the other really tricky thing any Year 1 teacher will know is that knowing what ten plus four is equal to is no mean feat, it's really tricky for children. We know that 20 + 4 = 24, 30 + 4 = 34, there's a clue isn't there in the number words how those numbers are made up, but why is 14 made of 10 and 4? I used to talk to my Year 1 children about how we ought to rename the tricky teen numbers. We ought to call them ‘tenty-four’ and ‘tenty-five’ and ‘tenty-six’, and we used to say we need to write to somebody important to get the number names changed, but of course we can't do that! But it was a way of saying to children, ‘those teen numbers are tricky’. We need to do a lot of work on helping children become fluent in knowing that 10 + 4 = 14 and 10 + 6 = 16.

But I would say that, for some children, it's going to take a bit more work, perhaps if they've not had those early experiences or enough repetition of those early experiences. If you're a maths lead in a primary school, or if you're a KS2 teacher and you know that some of your children haven't got security with those doubles, I hope that you've got some ways to start to think, ‘I need to check – are they secure with knowing what doubles are? Are they secure with being able to subitise those really small quantities? If I show them three and three, can they recognise it as three and three? Do they pair their fingers and say that it must be a double because I've got two of the same number. Are they secure within those doubles up to double five? Do they know that six, seven, eight, and nine are made of five-and-a-bit? Are they fluent with knowing what ten plus a one-digit number is?’. It's absolutely worth spending the time on unpacking some of those earliest skills that they might have missed. If you've got Year 6 children who don't know doubles, it's absolutely appropriate to go back to some of those earlier experiences and support them.

I hope that's given listeners a sense of how they might develop children's understanding of this important set of number facts. Obviously, we've not had very long to talk today and there's lots more things that we'd love to be able to share with you.

So, Rebecca, if listeners as either KS1 or Reception teachers or maths leads or KS2 teachers are interested in developing their understanding further, what do you recommend they could do?

RF: Well the first thing I'd recommend to do is, if you're not already in touch with your local Maths Hub, is to visit our website – we'll put the link in the show notes to where you can find where your local Maths Hub is based and how to get into contact with them. Then there are lots of programmes on offer with your local Maths Hub, including the Mastering Number Programme and Specialist Knowledge Work Groups as well. There's lots of support out there, lots of ways that you can develop your practice. I'd really recommend the Mastering Number Programme in particular, because it links to everything we talked about today, including developing fluency and number sense in Reception and KS1. Definitely have a look into that if you're not already involved.

So, we've come to the end of the episode Sue! It's been really interesting to delve deeper into one specific area of early maths together with you today. Thank you very much for chatting with me.

SE: Oh, it's an absolute pleasure. Thank you, Rebecca.

RF: So finally, if you've enjoyed the episode today, make sure that you are following us over on Instagram (@TheMathsPodcast) where you can keep up to date with all the latest episodes, and remember to subscribe wherever you get your podcasts. Thanks for listening!