MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.23 Multiplication strategies for larger numbers and long multiplication

Teacher guide | Year 6

Teaching point 1:

When multiplying two numbers that are multiples of 10, 100 or 1,000, multiply the number of tens, hundreds or thousands and then adjust the product using place value.

Teaching point 2:

When multiplying two numbers where one number is a multiple of 10, 100 or 1,000, use short multiplication and adjust the product using place value.

Teaching point 3:

Two two-digit numbers can be multiplied by partitioning one of the factors, calculating partial products and then adding these partial products. This method can be extended to multiplication of three-digit numbers by two-digit numbers.

Teaching point 4:

'Long multiplication' is an algorithm involving multiplication, then addition of partial products, which supports multiplication of two numbers with two or more digits.

Teaching point 5:

Multiplication where one of the factors is a composite number can be carried out by multiplying one factor and then the other factor.

Overview of learning

In this segment children will:

- use prior knowledge of multiplication by 10, 100 and 1,000, and apply this to calculations where 10, 100 or 1,000 is a factor of a number
- review prior knowledge of short multiplication and apply this to multiplying by multiples of 10, 100 and 1,000 by removing the zeros at the end of the factors, multiplying the significant digits and replacing the zeros at the end of the product
- multiply by two-digit numbers, using short multiplication to multiply by the ones and tens separately, then adding the partial products together
- develop the long multiplication algorithm to multiply up to four-digit numbers by two-digit numbers, including regrouping and how to record this
- apply prior knowledge of factorising, composite numbers and prime numbers to multiplying by two-digit numbers, investigating different ways of solving the same problem.

This segment draws together several concepts/strategies from previous segments and applies them to multiplication by a two-digit number. Before beginning on this segment, it is important that children have already mastered multiplication by 10 or 100, and short multiplication, both of which they will have first met in Year 4 (see segments *2.13 Calculation: multiplying and dividing by 10 or 100* and *2.14 Multiplication: partitioning leading to short multiplication*).

In *Teaching points 1* and *2*, children will explore efficient strategies for multiplying by multiples of 10, 100 or 1,000, drawing on their knowledge of place value. They will remove the zeros from both factors or just the multiplier, and then scale the product by a factor of 10, 100, 1,000 and so on, by replacing the zero(s).

In *Teaching point 3*, children will explore multiplication by a two-digit number by partitioning the multiplier into tens and ones, and then using their knowledge of short multiplication to find two partial products. They will see that the total of the partial products is the same as the product of the original two factors.

This leads to the long multiplication algorithm in *Teaching point 4*, which is also based on the addition of partial products. Long multiplication supports multiplication of two numbers with two or more digits, but this segment is limited to cases where the multiplier is a two-digit number and the multiplicand has two, three or four digits. In much the same way as the short multiplication algorithm was introduced in segment *2.14* with an 'expanded' layout, before progressing children to a 'compact' layout, an expanded layout is also used to introduce the long multiplication algorithm. It is suggested that children use the expanded layout until they are confident working without place-value headings. Initially, calculations are limited to those only requiring regrouping of tens into hundreds, while children are becoming confident with the algorithm. Regrouping of ones into tens and then regrouping in both partial products follows.

Finally, children will draw on their knowledge of the associative law (see segment *2.20 Multiplication with three factors and volume*) and composite and prime numbers (see segment *2.21 Factors, multiples, prime numbers and composite numbers*) to explore alternative strategies for multiplication by two-digit numbers. They will complete the segment by comparing different strategies for the same calculations, discussing why one strategy might be more efficient, or elegant, than another.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

When multiplying two numbers that are multiples of 10, 100 or 1,000, multiply the number of tens, hundreds or thousands and then adjust the product using place value.

Steps in learning

1:1

This teaching point builds on segment *2.13 Calculation: multiplying and dividing by 10 or 100, Teaching point 6* (multiplying single-digit numbers by multiples of ten). As a review of the previous learning, begin with a real-life example such as: *'There are ten sweets in one bag. If four children have three bags of sweets each, how many sweets do they have altogether?'*

First, represent this with Dienes tens rods as four groups of three tens. Ask children to explain how the Dienes represents the problem (one row of three tens represents the sweets for one child; the four rows represent the four children).

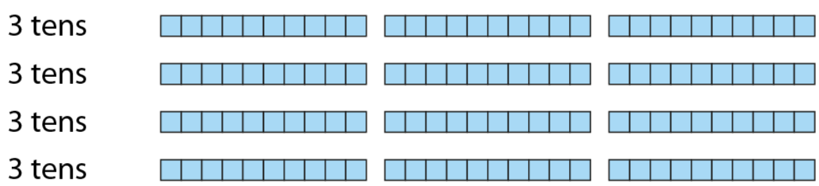
Then ask children to discuss the following scenarios, using the Dienes to demonstrate the reasoning behind each idea:

- *'Ralphie says that you can calculate it as four times three tens.'*
- *'Lily says that you can calculate it as three times four tens.'*

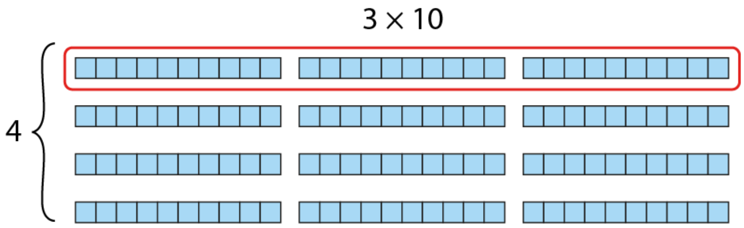
Whichever way around we do the calculation, we are making 3×4 or 4×3 ten times the size.

Revisit the generalisation from segment *2.13*, *Teaching point 6*: ***'If one factor is made ten times the size, the product will be ten times the size.'***

'There are ten sweets in one bag. If four children have three bags of sweets each, how many sweets do they have altogether?'



Ralphie:

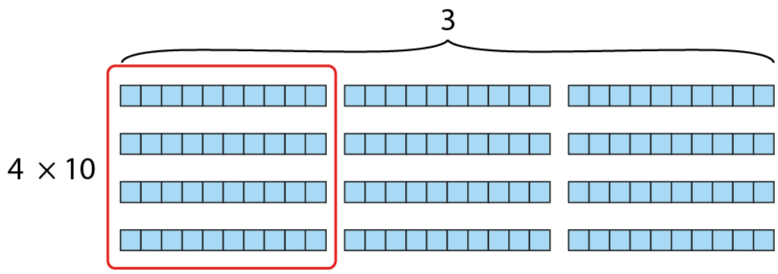


4×3 tens $= 12$ tens

$= 12 \times 10$

$= 120$

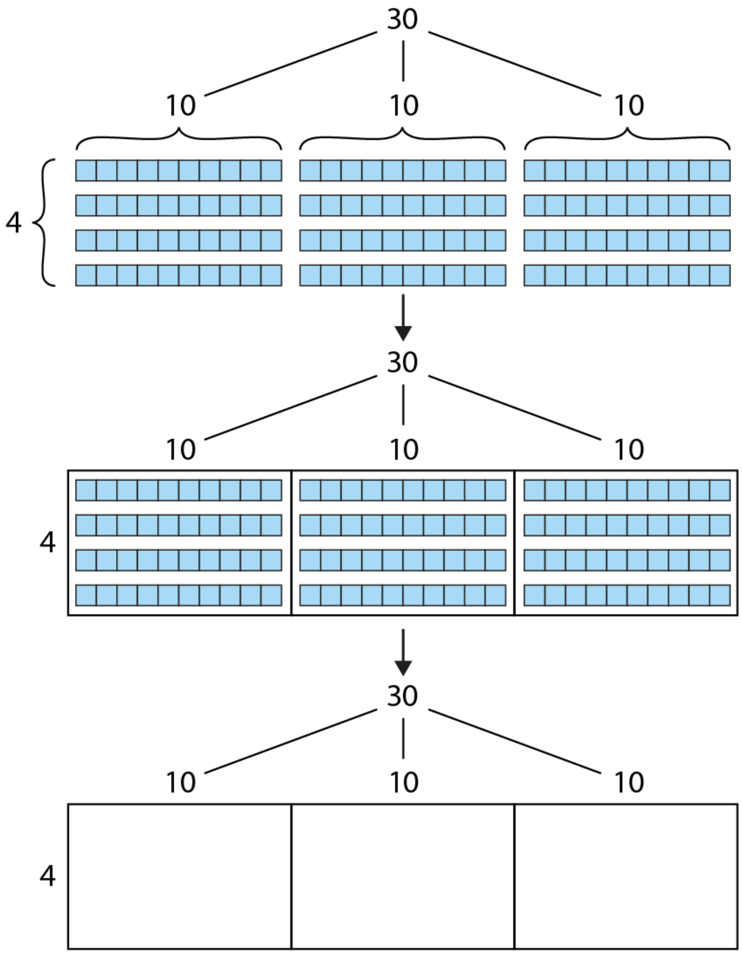
Lily:

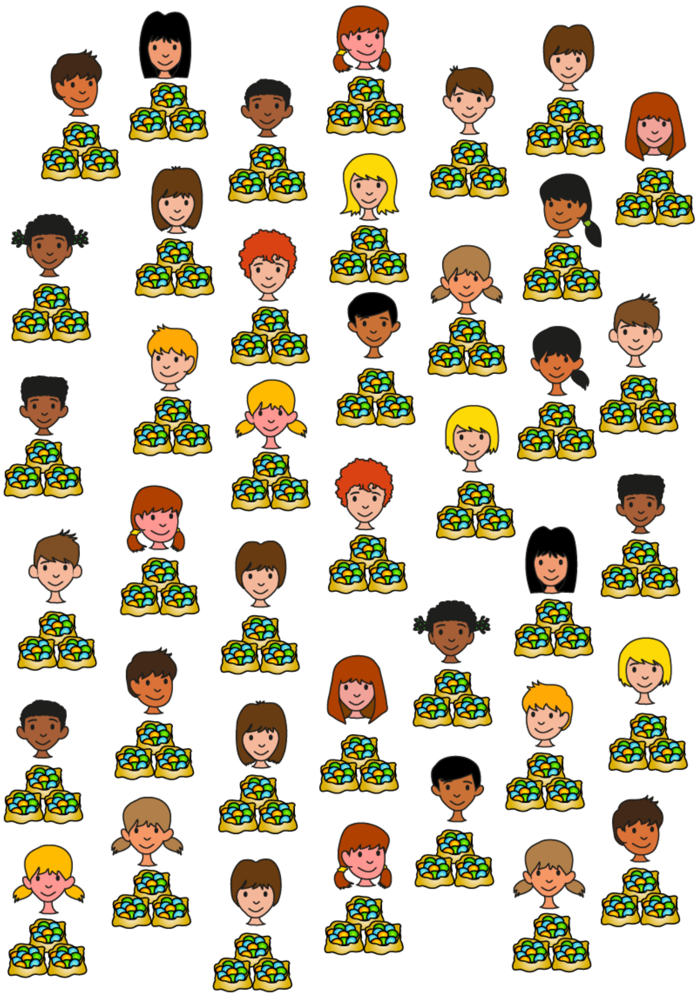


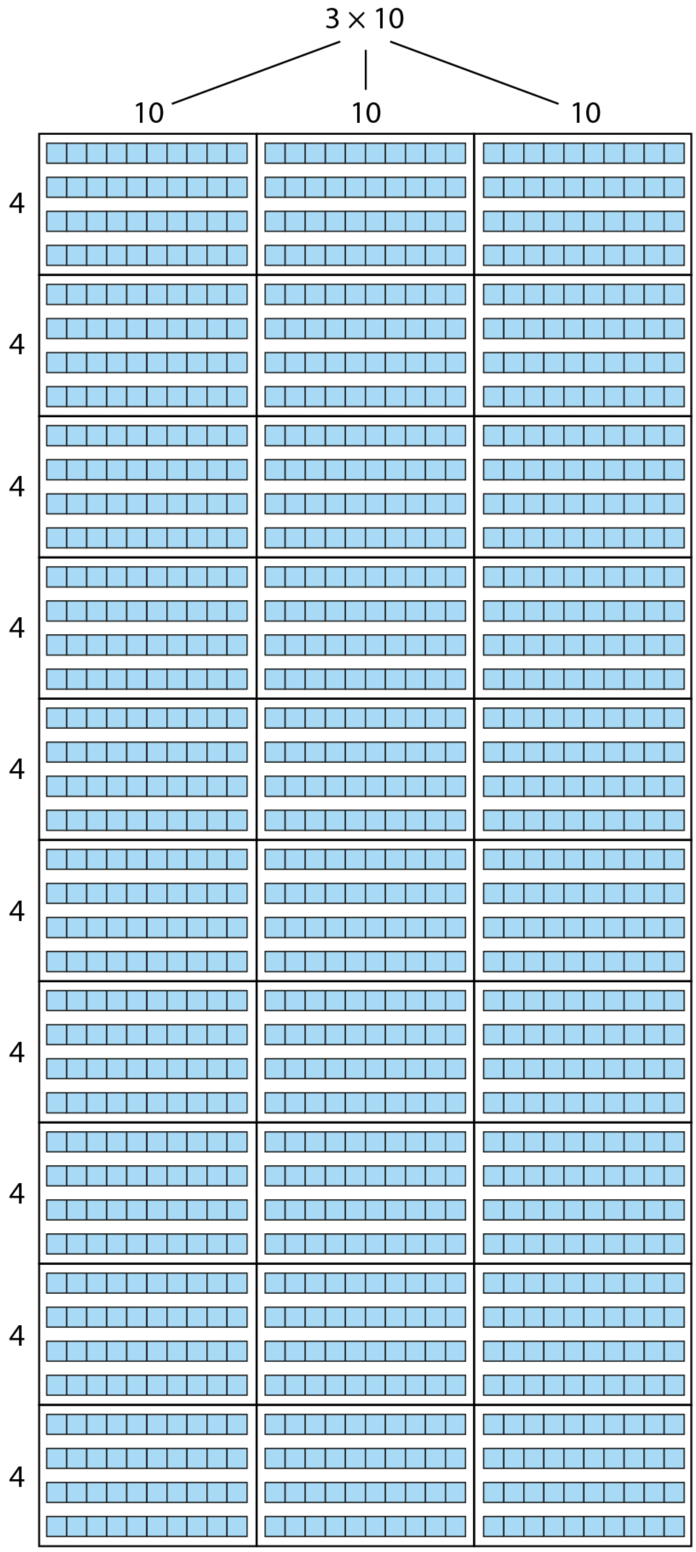
3×4 tens $= 12$ tens

$= 12 \times 10$

$= 120$

1:2	Go back to the original problem from the previous step, to now demonstrate how we can represent the calculation *4 × 30* in an array/grid. This allows us to represent the multiplication of larger factors more easily. Point out to children that the dimensions of the representation might not measure exactly the numbers it represents, but it is a representation to help us visualise what we are doing. Ask the children what the '30' represents (the number of sweets for one child) and what the '4' represents (the number of children).
	Array/grid: 

1:3	Now move on to multiplying two numbers that are *both* multiples of ten. Using the same story as in the previous steps, ask: *'What if there are forty children rather than four?'* That is, the number of children is ten times the size. We now need ten times our previous array/grid. This is a useful way to demonstrate that the product is going to be ten times the size. Again, using the generalised statement from step *1:1*, ***'If one factor is made ten times the size, the product will be ten times the size'***, display the calculations: $30 \times 4 = 120$ $30 \times 40 = 120 \times 10$ Draw attention to the fact that this calculation can be written in different ways, as shown above. You may wish to refer to place-value charts for extra support, as used in segment *2.13 Calculation: multiplying and dividing by 10 or 100*. Remind children that each time we multiply a number by ten, we move the digits one place to the left. Here, you can also make links to the commutative law and the area model for multiplication.
	'There are ten sweets in one bag. If forty children have three bags of sweets each, how many sweets do they have altogether?' 



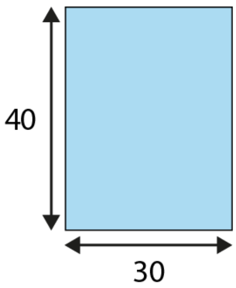
$$
\begin{aligned}
30 \times 40 &= 3 \times 10 \times 4 \times 10 \\
&= 3 \times 4 \times 10 \times 10 \\
&= 3 \times 4 \times 100 \\
&= 1{,}200
\end{aligned}
$$

Place-value chart:

1,000s	100s	10s	1s	
			4	↓ × 10
		4	0	
			3	↓ × 10
		3	0	
		1	2	↓ × 10
	1	2	0	↓ × 10
1	2	0	0	

ten times the size ten times the size ten times the size

Area model:



$30 \times 40 = 120$ $40 \times 30 = 120$

1:4

At this point, provide children with varied practice like the examples shown opposite. The final example has several correct solutions – how many can children find?

Missing-number problems:

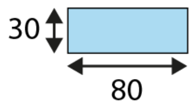
'Fill in the missing numbers.'

$6 \times 50 =$ □ $70 \times 80 =$ □

□ $= 4 \times 50$ $80 \times 50 =$ □

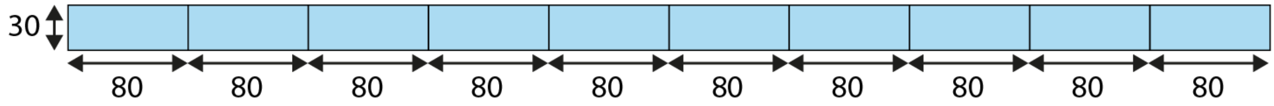
□ $= 30 \times 60$ $90 \times$ □ $= 10{,}800$

$70 \times$ □ $= 3{,}500$ ___0 $\times$ ___0 $= 3{,}600$

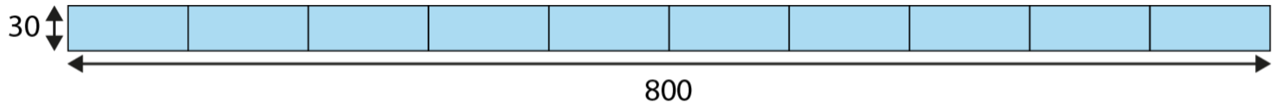
1:5	Next, develop this learning further by finding the product using multiples of 10 and 100. Begin with a similar problem to the one used earlier, for example: *'Eggs come in boxes of thirty. A supermarket orders eighty boxes of eggs in one week. How many eggs does the farmer need to supply?'* Use an area model or grid to represent the equation, as shown below. (Arrays may still be used, as in step *1:2*, but are less practical for larger numbers.) Work through the calculation and continue to make links to commutativity. An error some children might make is to write, for example: 3 tens × 8 tens = 24 tens Address this by drawing attention to the fact that: 3 × 8 tens = 24 tens Again, you could represent this using a place-value chart if necessary, as in step *1:3*. Now make one of the factors ten times the size: *'A lorry can carry eight hundred boxes of eggs. How many eggs does the farmer need to fill the lorry?'* Work through the calculation, as shown as shown below, drawing on place-value charts if necessary and continuing to link to commutativity. Then make one of the factors ten times the size again: *'In one month, a lorry delivers three hundred loads of eight hundred boxes. How many boxes is that?'* Work through the calculation as before.
	• *'Eggs come in boxes of thirty. A supermarket orders eighty boxes of eggs in one week. How many eggs does the farmer need to supply?'*  80 boxes of 30 eggs: 30 × 80 = 3 × 8 × 10 × 10 = 3 × 8 × 100 = 2,400 30 × 80 = 2,400 80 × 30 = 2,400 • *'The farmer needs to supply 2,400 eggs.'*

- *'A lorry can carry eight hundred boxes of eggs. How many eggs does the farmer need to fill the lorry?'*

10 times 80 × 30:



becomes:



800 boxes of 30 eggs:

$30 \times 800 = 10 \times 30 \times 80$

$= 10 \times 2{,}400$

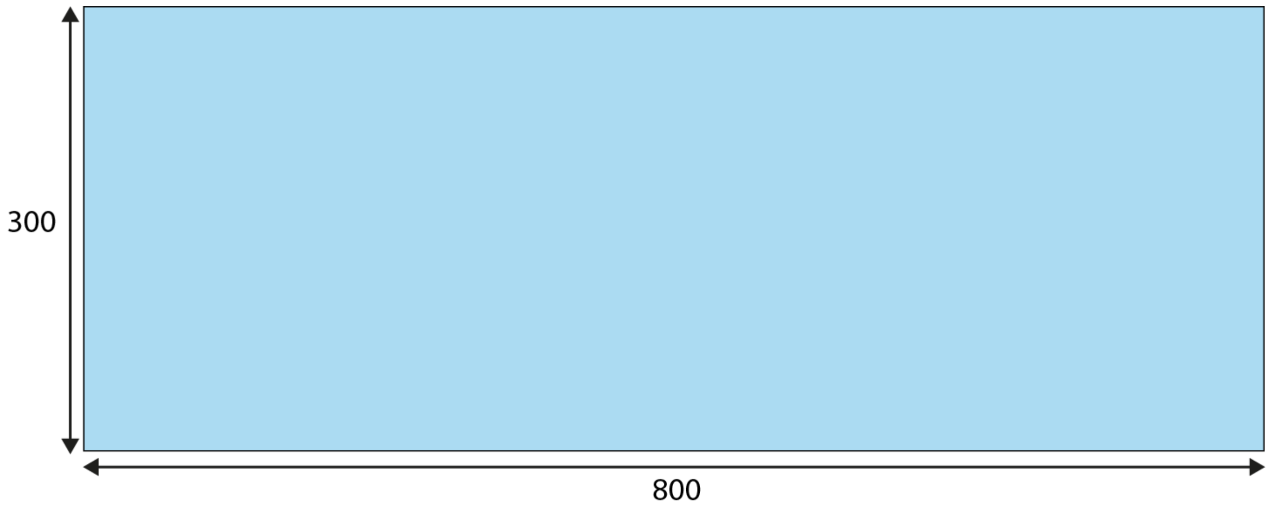
$= 24{,}000$

$30 \times 800 = 24{,}000$ $800 \times 30 = 24{,}000$

- *'The farmer needs 24,000 eggs.'*

- *'In one month, a lorry delivers three hundred loads of eight hundred boxes. How many boxes is that?'*

10 times 800 × 30:



300 loads of 800 boxes of eggs:

$300 \times 800 = 10 \times 30 \times 800$

$= 10 \times 24{,}000$

$= 240{,}000$

$300 \times 800 = 240{,}000$ $800 \times 300 = 240{,}000$

- *'The lorry delivers 240,000 boxes of eggs.'*

1:6	Once you have worked through several calculations, present the results in a table, as shown opposite. Looking at the calculations, ask children: • *'What's the same?'* • *'What's different?'* They should notice that the known fact 3×8 is same in each case and that the number of zeros in the product is the same as the number of zeros in the factors combined. Draw out that each time one of the factors becomes ten times the size, the product becomes ten times the size. Each time there is another zero at the end of one of the factors, there is another zero at the end of the product. As above, you could use place-value charts to support this. Ask the children to come up with their own example sets of calculations based on the same known fact to ensure that they have understood the concept. That is, we can find the product of the single-digit numbers and then scale the product according to the place value of the factors. Work towards the generalisation: ***'To multiply multiples of ten, one hundred or one thousand, remove the zeros, find the product of the single-digit numbers and then replace the zeros.'***	• *'What's the same?'* • *'What's different?'* $3 \times 8 = 24$ $30 \times 8 = 240$ $30 \times 80 = 2{,}400$ $30 \times 800 = 24{,}000$ $300 \times 800 = 240{,}000$ $3000 \times 800 = 2{,}400{,}000$
1:7	Once children have understood the meaning of the generalisation, consolidate it with varied practice, such as the examples shown opposite and on the next page. Note that the examples with known facts of 2×5 and 5×8 are included intentionally, as their products are also multiples of ten. Reinforce the generalised statement – remove the zeros, find the product of the single-digit numbers (which, in these cases,	Missing-number problems: *'Fill in the missing numbers.'* $70 \times 900 = \square$ $200 \times 5{,}000 = \square$ $\square = 50 \times 9{,}000$ $63{,}000 = 90 \times \square$ $\square \times 900 = 45{,}000$ $8{,}000 = 500 \times \square$

will give a product with a zero in the ones) and then replace the zeros.

Children should also use their knowledge of multiplying multiples of ten to solve problems with real-life contexts using the different structures for multiplication, such as:

- *'A recipe for twenty cupcakes uses 500 g flour, 300 g sugar and 400 g butter. If a school kitchen needs to make enough cupcakes for four hundred children, how much of each ingredient would they need in kilograms?'*
- *'A school buys fifty packs of paper, each containing 3,000 sheets. A teacher removes one sheet each for her class of thirty children. How many sheets are left?'*
- *'Faisal collects 50 p coins and Adaline collects 20 p coins. Faisal has collected eighty coins and Adaline has collected two hundred coins. Adaline says that she has collected more money. Do you agree with her? Explain your answer.'*
- *'The entrance to a school fete costs 40 p for children and 50 p for adults. If three hundred adults and four hundred children attend the fete, how much money will the school make from the entrance fees?'*
- *'Becca's sunflower is 20 mm tall. Over the summer, it grows so that it is one hundred times as tall. Ryan's sunflower is 3 cm tall. Over the summer, it grows so that is fifty times as tall. Whose sunflower is taller?'*

True/false-style problems:

'Use a tick or a cross to show whether each equation is correct or not.'

$4{,}900 = 700 \times 700$

$800 \times 5{,}000 = 40{,}000$

$70{,}000 \times 60 = 4{,}200{,}000$

$1{,}000 = 20 \times 500$

Matching:

'Match each calculation on the left with the calculation on the right that has the same product.'

$36 \times 10{,}000$	90×400
$36 \times 1{,}000$	$600 \times 6{,}000$
$36 \times 100{,}000$	$1{,}200 \times 300$

Missing-symbol problems:

'Fill in the missing symbols (<, > or =).'

120×50 ◯ 300×200

$90 \times 4{,}000$ ◯ $3{,}000 \times 1{,}200$

$400 \times 5{,}000$ ◯ $2{,}000 \times 2{,}000$

Dòng nǎo jīn:

'How many solutions can you find?'

$\square \times \square = 24{,}000$

Teaching point 2:

When multiplying two numbers where one number is a multiple of 10, 100 or 1,000, use short multiplication and adjust the product using place value.

Steps in learning

2:1

Guidance

This teaching point combines the learning from *Teaching point 1* with the short multiplication skills developed in segment *2.14 Multiplication: partitioning leading to short multiplication*. Begin with a review of short multiplication, using a series of examples from real-life contexts, such as:

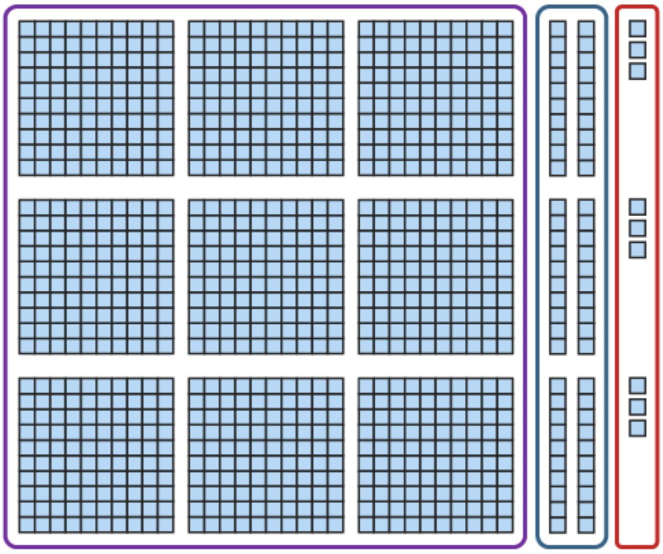
- *'Ahmed has three hundred and twenty-three books on his bookcase. His friend has a bookcase that holds three times as many. How many books does it hold?'*
  (no regrouping)
- *'If a bakery can produce eight hundred and thirty-two cupcakes in one day, how many cupcakes will it have produced after three days?'*
  (regrouping of hundreds into thousands)
- *'One side of an equilateral triangle is 424 mm. What is the perimeter of the triangle?'*
  (regrouping of ones into tens and hundreds into thousands)
- *'A cinema can seat five hundred and seventy-six people at one time. If there are four shows in one day, how many tickets are available each day?'*
  (regrouping of ones into tens, tens into hundreds and hundreds into thousands)

Initially, you may wish to work through the Dienes representation, expanded layout and compact layout, as shown opposite, to remind children of the short multiplication algorithm. However, they should be confident

Representations

'Ahmed has three hundred and twenty-three books on his bookcase. His friend has a bookcase that holds three times as many. How many books does it hold?'

Dienes representation:



Multiplication algorithm – expanded layout:

	100s	10s	1s	
	3	2	3	
×			3	
			9	3 × 3 ones = 9 ones
		6	0	3 × 2 tens = 6 tens
	9	0	0	3 × 3 hundreds = 9 hundreds
	9	6	9	

Multiplication algorithm – compact layout:

	3	2	3
×			3
	9	6	9

- *'His friend's bookcase holds nine hundred and sixty-nine books.'*

	with this from segment *2.14* and be able to work with just the compact layout of the algorithm relatively quickly. If children need extra support with regrouping, you can use place-value counters as in segment *2.14*.	
2:2	Next, introduce another real-life example, beginning with short multiplication, but then make one factor ten times the size. For example: *'A lorry driver takes deliveries from Swindon to York once a week. The journey there and back is 472 miles. How far does she travel in three weeks?'* Solve this together with the children, using short multiplication as before. Then ask: *'The driver does this journey for 30 weeks each year. How far will she have travelled in one year?'* Draw out that this is ten times as many as three weeks, so she will have travelled ten times the distance. Refer to the generalisation from step *1:1*: ***'If one factor is made ten times the size, the product will be ten times the size.'***	• *'A lorry driver takes deliveries from Swindon to York once a week. The journey there and back is 472 miles. How far does she travel in three weeks?'* $\begin{array}{r} 4\ 7\ 2 \\ \times\qquad 3 \\ \hline 1\ 4\ 1\ 6 \\ \hline {\scriptstyle 2}\quad \end{array}$ • *'The lorry driver drives 1,416 miles in three weeks.'* • *'The driver does this journey for thirty weeks each year. How far will she have travelled in one year?'* $472 \times 3 = 1{,}416$ $472 \times 30 = 14{,}160$ • *'The lorry driver will have travelled 14,160 miles in one year.'*
2:3	Introduce children to the two methods shown opposite and ask them to discuss what's the same and what's different. • Ezra has chosen to remove the zero, use short multiplication and then scale the product by a factor of ten by replacing the zero. • Ling has chosen to use short multiplication to multiply by 30 by placing a zero in the ones column to show that she is multiplying by a multiple of ten before she starts. Children should notice that the product is the same in each case. Ask *'Whose method do you think is the most efficient?'* Draw out the generalisation: ***'To multiply by a multiple of ten, use short***	• *'What's the same?'* • *'What's different?'* **Ezra's method:** 472×30 $\begin{array}{r} 4\ 7\ 2 \\ \times\qquad 3 \\ \hline 1\ 4\ 1\ 6 \\ \hline {\scriptstyle 2}\quad \end{array}$ $1{,}416 \times 10 = 14{,}160$ **Ling's method:** 472×30 $\begin{array}{r} 4\ 7\ 2 \\ \times\qquad 3\ 0 \\ \hline 1\ 4\ 1\ 6\ 0 \\ \hline {\scriptstyle 2}\qquad \end{array}$

	multiplication by a single-digit number and then multiply by ten.' Note that the focus here is on Ezra's method. However, it only applies when one of the factors is ten or a multiple of ten. Later, we will return to Ling's method for multiplication involving non-multiples of ten (see step *3:1*).	
2:4	At this point, give children opportunities for varied practise, as shown opposite and below. Include some problems with mixed operations and real-life contexts: • *'One bottle of juice contains 564 ml, which is enough for two children. A school needs to provide juice for eighty children. How many litres of juice is this?'* (This can be correctly solved by calculating *564 × 40* or *282 × 80*. Once children have solved it in one way, ask if they can solve it in another way. Which way do they prefer?) • *'Janice has worked out that she walks 432 m to school each day. There are six weeks in one half-term. How far does she walk in one half-term on her journeys to and from school?'*	Missing-number problems: *'Fill in the missing numbers.'* $30 \times 522 = \square$ $718 \times 60 = \square$ $567 \times 40 = \square$ $796 \times 30 = \square$ $907 \times 80 = \square$ $498 \times 70 = \square$ $498 + 60 \times 325 = \square$ $20 \times 4 \times 427 = \square$ $40 \times (342 - 45) = \square$

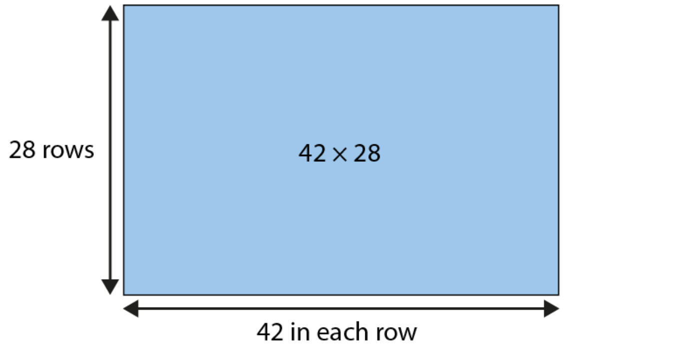
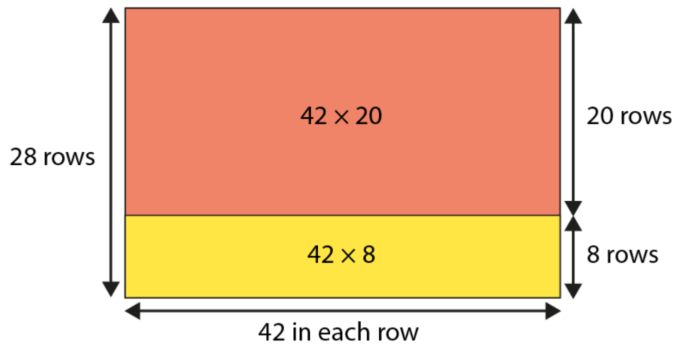
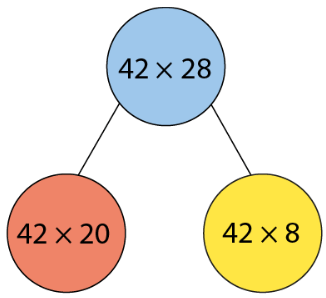
2:5	Once the children are confident with multiplying three-digit numbers by multiples of ten, go back to the lorry-driver problem from step *2:2*, but now make the number of weeks ten times as many again: *'How far would the lorry driver travel in three hundred weeks?'*
	Draw out that this is the same principle, so they can use Ezra's method again, this time scaling the product by a factor of 100.
	This leads to another generalisation: ***'To multiply by a multiple of one hundred, use short multiplication by a single-digit number and then multiply by one hundred.'***
		'A lorry driver takes deliveries from Swindon to York once a week. The journey there and back is 472 miles. How far does she travel in three hundred weeks?'
		    4 7 2
		× 3
		1 4 1 6
		    2
		$472 \times 3 = 1{,}416$
		$1{,}416 \times 100 = 141{,}600$
		• *'The lorry driver will have travelled 141,600 miles in three hundred weeks.'*

2:6	Provide children with varied practice multiplying by multiples of 10 and 100, as shown opposite and below. Again, include some problems with mixed operations and real-life contexts: • *'A model rocket is 218 mm tall. If the real rocket is three hundred times the size of the model, how tall is it in metres?'* • *'One fence panel is 246 cm long. If a farmer buys six hundred panels, will he have enough to go around a field that has a perimeter of 1480 m?'*	Missing-number problems: *'Fill in the missing numbers.'* $500 \times 623 = \square$ $\quad 495 \times 70 = \square$ $806 \times 300 = \square$ $\quad 839 \times 40 = \square$ $907 \times 80 = \square$ $\quad 498 \times 700 = \square$ $300 \times 892 \times 2 = \square$ $(86 - 16) \times 642 = \square$ $586 + 400 \times 897 = \square$
2:7	Now revisit the original calculation of *472 × 3* and ask children to discuss how they might calculate *472 × 3,000*. They should, by now, notice that this would also be the same principle as multiplying by multiples of 10 and multiples of 100, leading to a new generalisation: ***'To multiply by a multiple of one thousand, use short multiplication by a single-digit number and then multiply by one thousand.'***	$472 \times 3{,}000 = ?$ $\begin{array}{r} 4\ 7\ 2 \\ \times \quad\quad 3 \\ \hline 1\ 4\ 1\ 6 \\ \hline {}_{2}\quad \end{array}$ $472 \times 3 = 1{,}416$ $1{,}416 \times 1{,}000 = 1{,}416{,}000$
2:8	Complete this teaching point by providing children with varied practice covering all three generalisations: multiplying by multiples of 10, 100 and 1,000. As before, include some problems with mixed operations and real-life contexts: • *'An author writes a book with three hundred and ninety-eight pages. She then writes a sequel with three hundred and fifty-four pages. If 4,000 copies of each book are printed, how many pages does the publisher have to print?'* • *'There are 6,000 seats in a football stand for season-ticket holders. If a season ticket costs £462, how much money could the football club make from that one stand?'*	Missing-number problems: *'Fill in the missing numbers.'* $4{,}000 \times 948 = \square$ $\quad 800 \times 529 = \square$ $709 \times 90 = \square$ $\quad 368 \times 6{,}000 = \square$ $296 \times 7{,}000 = \square$ $\quad 40 \times 824 = \square$ $(300 + 400) \times 564 = \square$ $3{,}000 \times 648 \times 3 = \square$ $482 \times 600 + 98 = \square$

Teaching point 3:

Two two-digit numbers can be multiplied by partitioning one of the factors, calculating partial products and then adding these partial products. This method can be extended to multiplication of three-digit numbers by two-digit numbers.

Steps in learning

	Guidance	Representations
3:1	This teaching point builds on the previous two teaching points by combining multiplication of two two-digit numbers with short multiplication. Present children with a problem that involves multiplication of two two-digit numbers, for example: *'Seats in one section of a stadium are in rows of forty-two. If there are twenty-eight rows, how many seats are there in this section?'* Represent this with an area model, as used in *Teaching point 1*, and show how this can be partitioned into two parts (see opposite). You can also represent this using a part–part–whole model. Model how each part is calculated separately using short multiplication to give a partial product and then add these partial products to give the total product, answering the original problem. Ask children to repeat the process, using the generalisation: ***'To multiply two two-digit numbers, first multiply by the ones, then multiply by the tens, and then add them together.'*** Note that we have used Ling's method from step *2:3* to calculate the partial product *42 × 20*. This is the standard method for multiplication involving non-multiples of ten.	*'Seats in one section of a stadium are in rows of forty-two. If there are twenty-eight rows, how many seats are there in this section?'* Area model/grid:   Part–part–whole model: 

<table>
<tr>
<td></td>
<td></td>
<td>Short multiplication and combining partial products:

$$\begin{array}{r} 4\;2 \\ \times\quad 8 \\ \hline 3\;3\;6 \\ \hline {}_{1}\;\; \end{array} \qquad \begin{array}{r} 4\;2 \\ \times\; 2\;0 \\ \hline 8\;4\;0 \\ \hline \end{array}$$

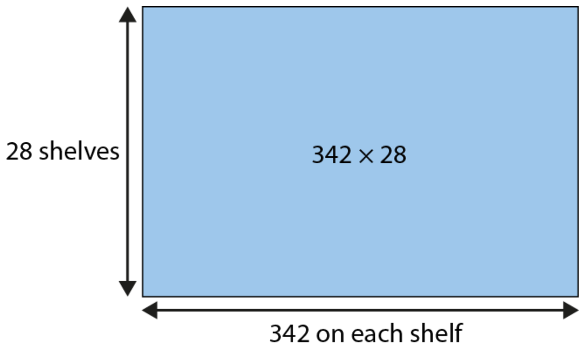
$$\begin{array}{r} 8\;4\;0 \\ +\; 3\;3\;6 \\ \hline 1\;1\;7\;6 \\ \hline \end{array}$$

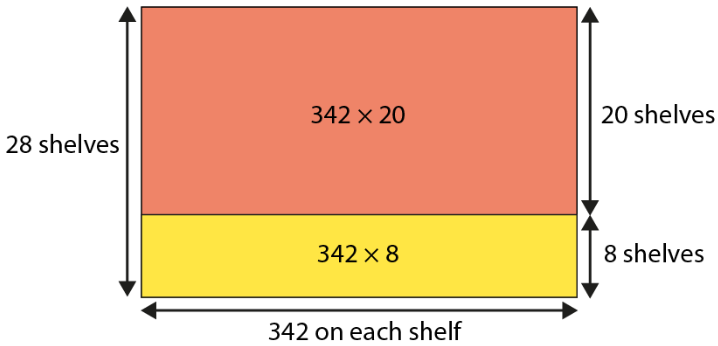
- *'There are 1,176 seats in this section of the stadium.'*</td>
</tr>
<tr>
<td>3:2</td>
<td>Now extend to multiplication of three-digit numbers by two-digit numbers using the same process.

Again, present children with a problem. For example: 'A factory can store three hundred and forty-two bottles of juice on one shelf. If it has twenty-eight full shelves in the storeroom, how many bottles of juice does it have to sell?'

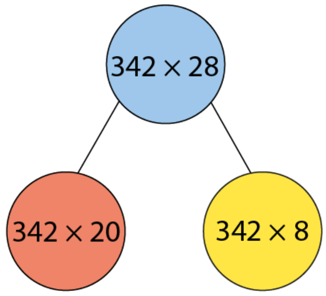
Represent this using an area model, as in step 3:1, to demonstrate the partitioning into two partial products. Point out to children that we partition the two-digit number, not the three-digit number, as this is a more efficient calculation.

Ask children to repeat the process again, using the generalisation: 'To multiply a three-digit number by a two-digit number, first multiply by the ones, then multiply by the tens, and then add them together.'</td>
<td>'A factory can store three hundred and forty-two bottles of juice on one shelf. If it has twenty-eight full shelves in the storeroom, how many bottles of juice does it have to sell?'




</td>
</tr>
</table>

Part–part–whole model:



Short multiplication and combining partial products:

$$\begin{array}{r} 342 \\ \times \quad 8 \\ \hline 2736 \\ \hline {\scriptstyle 3\,1\,} \end{array} \qquad \begin{array}{r} 342 \\ \times \quad 20 \\ \hline 6840 \\ \hline \end{array}$$

$$\begin{array}{r} 6840 \\ +\ 2736 \\ \hline 9576 \\ \hline {\scriptstyle 1\,\,\,\,\,\,} \end{array}$$

- *'The factory has 9,576 bottles of juice to sell.'*

3:3

Complete this teaching point by giving children opportunity, through varied practice, to multiply by partitioning into partial products.

To help children understand and visualise what they are doing, it is useful to ask them to draw an area model or a grid each time to show how they would partition the calculation, as in steps *3:1* and *3:2*. Children should then solve these using short multiplication of partial products and adding the results to find the total product.

Children may need to be reminded that while the calculations are sometimes written with a two-digit number first and sometimes with a three-digit number first, we always partition the two-digit number because this is more efficient.

Missing-number problems:

'Fill in in the missing numbers.'

$27 \times 29 = 27 \times 20 + 27 \times \square = \square$

$223 \times 34 = 223 \times 30 + 223 \times \square = \square$

$47 \times 73 = 47 \times \square + 47 \times \square = \square$

$43 \times 376 = 376 \times \square + 376 \times \square = \square$

$53 \times 67 = \square \times \square + \square \times \square = \square$

$534 \times 48 = \square \times \square + \square \times \square = \square$

$64 \times 809 = \square \times \square + \square \times \square = \square$

$43 \times 33 = \square$

$23 \times 782 = \square$

	Also, provide children with opportunities to solve problems in real-life contexts, including with measures and more than one step, for example:
	• *'One floor tile has an area of 508 cm^2. If Mrs Miggins buys twenty-eight tiles, what is the area they will cover?'*
	• *'To make cookies, a class of thirty-five children need 452 g of flour each.*
	    • *'How much flour does the teacher need to buy?'*
	    • *'If flour comes in bags of 1 kg, how many bags does the teacher need to buy?'*	

Teaching point 4:

'Long multiplication' is an algorithm involving multiplication, then addition of partial products, which supports multiplication of two numbers with two or more digits.

Steps in learning

	Guidance	Representations
4:1	See below	See below

4:1 Guidance

The next step is to compare the strategy from *Teaching point 3* with the long multiplication algorithm. Begin with a multiplication of two two-digit numbers that does not require any regrouping of ones into tens, only regrouping of tens into hundreds (e.g. *31 × 24*). You may wish to present this to children in a real-life context, for example: *'Martha wants to calculate how many hours there are in January.'* As in the previous teaching points, use an area model or a grid to represent what is happening in the calculation.

Show children the previous strategy for calculating, as in step *3:1*. That is, working out the partial products (here: *31 × 4* and *31 × 20*) using short multiplication and then adding the partial products to find the answer to the original problem.

Then, alongside, model the long multiplication recording of the same calculation. At this stage, use the 'compact' layout as shown (opposite). This will be worked through using the 'expanded' layout in the next step. For now, ask children:

- *'What's the same?'*
- *'What's different?'*

Draw attention to where the partial products (*124* and *620*) are in the two strategies, as well as the product of the original factors (*744*):

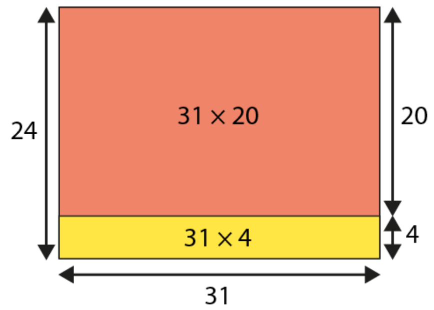
- *'Where is 31 × 4?'*
- *'Where is 31 × 20?'*
- *'Where is the product of 31 and 24?'*

4:1 Representations

'Martha wants to calculate how many hours there are in January.'

31 × 24

Area model/grid:



Short multiplication and combining partial products compared with long multiplication:

$$\begin{array}{r} 3\ 1 \\ \times\quad 4 \\ \hline 1\ 2\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 3\ 1 \\ \times\ 2\ 0 \\ \hline 6\ 2\ 0 \\ \hline \end{array} \qquad \begin{array}{r} 6\ 2\ 0 \\ +\ 1\ 2\ 4 \\ \hline 7\ 4\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3\ 1 \\ \times\ 2\ 4 \\ \hline 1\ 2\ 4 \\ 6\ 2\ 0 \\ \hline 7\ 4\ 4 \\ \hline \end{array}$$

- *'There are seven hundred and forty-four hours in January.'*

4:2

Now spend some time working through the long multiplication algorithm using an expanded layout as shown below and on the next page. Work through one step at a time in a similar way to how the short multiplication algorithm was introduced in segment *2.14 Multiplication: partitioning leading to short multiplication*. Remember that, at this stage, we are focusing on multiplications that only require *regrouping of tens into hundreds*.

Although the two two-digit factors can be written in either order, it is beneficial to get children into the practice of writing the largest factor first, as this will become necessary when one of the factors has more than two digits (see steps *4:9* and *4:11*).

Draw attention to the following key points, encouraging children to repeat the instructions at each step:

- *'First, write the largerfactor: 31.'*
- *'Then write the smaller factor below, lining up the digits: 24.'*
- *'Now multiply by the ones digit to give a partial product: to multiply thirty-one by four…'*
- *'…multiply one one by four to give four ones; write "4" in the ones column…'*
- *'…then multiply three tens by four to give twelve tens…'*
- *'…and regroup: twelve tens is equal to one hundred, and two tens; write "1" in the hundreds column and "2" in the tens column.'*
- *'Then, place a zero to show that it's ten times the size.'*
- *'Next, multiply by the tens digit to give a partial product: to multiply thirty-one by twenty…'*
- *'…multiply one one by two tens to give two tens; write "2" in the tens column…'*
- *'…then multiply three tens by two tens to give six hundreds; write "6" in the hundreds column.'*
- *'Finally, add the partial products: 124 plus 620 is equal to 744.'*

If children need additional support with regrouping, they can use place-value counters as in segment *2.14*.

Look more closely at the expanded algorithm and point out that, just like in short multiplication:

- when the factors are recorded, the digits are aligned correctly (ones with ones, etc.)
- when the partial products are recorded, the digits are aligned correctly.

Complete a few more examples (e.g. *32 × 14* and *34 × 22*) together in the same way, talking through each step.

Multiplication algorithm – expanded layout:

Step 1 – write the factors

	100s	10s	1s
		3	1
×		2	4

Step 2 – multiply the ones digit by the ones digit

	100s	10s	1s	
		3	1	
×		2	4	
			4	4 × 1 one = 4 ones

Step 3 – multiply the tens digit by the ones digit and regroup

	100s	10s	1s
		3	1
×		2	4
	1	2	4

4 × 3 tens = 12 tens

= 1 hundred + 2 tens

Step 4 – place a zero to show that it's ten times the size

	100s	10s	1s
		3	1
×		2	4
	1	2	4
			0

Step 5 – multiply the ones digit by the tens digit

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	
		2	0	2 tens × 1 one = 2 tens

Step 6 – multiply the tens digit by the tens digit

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	
	6	2	0	2 tens × 3 tens = 6 hundreds

Step 7 – add the partial products

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	31 × 4
	6	2	0	31 × 20
	7	4	4	

4:3

When children are confident using the expanded layout from step *4:2*, compare it with the compact layout they were shown in step *4:1* and ask:

- *'What's the same?'*
- *'What's different?'*

- *'What's the same?'*
- *'What's different?'*

Multiplication algorithm – expanded layout:

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	31 × 4
	6	2	0	31 × 20
	7	4	4	

Multiplication algorithm – compact layout:

		3	1
×		2	4
	1	2	4
	6	2	0
	7	4	4

4:4	Give children examples to practise. Initially, children may need to return to the expanded layout, but they should move to more independence with setting out the calculations. Choose examples carefully, such as those shown opposite, so that children do not need to regroup ones into tens at this point. This avoids the recording of regrouping while children are becoming confident with the algorithm. To check secure understanding, include some completed examples that have common errors, such as the final example opposite.	Applying the long multiplication algorithm: *'Complete the calculations.'* 32×13 $\begin{array}{rrr} & 3 & 2 \\ \times & 1 & 3 \\ \hline & 9 & 6 \\ & & 0 \\ \hline \\ \hline \end{array}$ $\begin{array}{l} 32 \times 3 \\ 32 \times 10 \end{array}$ 13×23 $\begin{array}{rrr} & 1 & 3 \\ \times & 2 & 3 \\ \hline & 3 & 9 \\ \\ \hline \\ \hline \end{array}$ $\begin{array}{l} 13 \times 3 \\ 13 \times 20 \end{array}$ 42×14 $\begin{array}{rrr} & 4 & 2 \\ \times & 1 & 4 \\ \hline 1 & 6 & 8 \\ \\ \hline \\ \hline \end{array}$ Laying out and applying the long multiplication algorithm: *'Use long multiplication to do these calculations.'* $14 \times 22 =$ $22 \times 32 =$ $22 \times 43 =$ $42 \times 22 =$ Reasoning: *'Taron has done this calculation. Do you agree with him? Explain your answer.'* $\begin{array}{rrr} & 2 & 2 \\ \times & 2 & 3 \\ \hline & 6 & 6 \\ & 4 & 4 \\ \hline 1 & 1 & 0 \\ \hline & {\scriptstyle 1} & \end{array}$
4:5	Once children are confident with the algorithm, move on to calculations where regrouping ones into tens is required when multiplying by the ones (e.g. *47 × 18*). Repeat the steps using the expanded layout, as in step *4:2*, encouraging children to describe the steps in laying out and applying the algorithm, this time drawing attention to where we record the *regrouping of the ones*:	

- *'First, write the larger factor: 47.'*
- *'Then write the smaller factor below, lining up the digits: 18.'*
- *'Now multiply by the ones digit to give a partial product: to multiply forty-seven by eight…'*
- *'…multiply seven ones by eight to give fifty-six ones…'*
- *'…<u>and regroup: fifty-six ones is equal to five tens and six ones; write "5" above the tens column and "6" in the ones column</u>…'*
- *'…then multiply four tens by eight to give thirty-two tens…'*
- *'….and add the five tens from regrouping to give thirty-seven tens…'*
- *'<u>…and regroup again: thirty-seven tens is equal to three hundreds and seven tens; write "3" in the hundreds column and "7" in the tens column.'</u>*
- *'Then, place a zero to show that it's ten times the size.'*
- *'Next, multiply by the tens digit to give a partial product: to multiply forty-seven by ten…'*
- *'…multiply seven ones by one ten to give seven tens; write "7" in the tens column…'*
- *'…then multiply four tens by one ten to give four hundreds; write "4" in the hundreds column.'*
- *'Finally, add the partial products together: 376 plus 470 is equal to 846.'*

Step 1 – write the factors

	100s	10s	1s
		4	7
×		1	8

Step 2 – multiply the ones digit by the ones digit and regroup

	100s	10s	1s
		5	
		4	7
×		1	8
			6

8 × 7 ones = 56 ones
= 5 tens + 6 ones

Step 3 – multiply the tens digit by the ones digit, add the regrouped tens and regroup again

	100s	10s	1s
		~~5~~	
		4	7
×		1	8
	3	7	6

8 × 4 tens = 32 tens
32 tens + 5 tens = 37 tens
= 3 hundreds + 7 tens

Step 4 – place a zero to show that it's ten times the size

	100s	10s	1s
		~~5~~	
		4	7
×		1	8
	3	7	6
			0

Step 5 – multiply the ones digit by the tens digit

	100s	10s	1s
		~~5~~	
		4	7
×		1	8
	3	7	6
		7	0

1 ten × 7 ones = 7 tens

Step 6 – multiply the tens digit by the tens digit

	100s	10s	1s
		~~5~~	
		4	7
×		1	8
	3	7	6
	4	7	0

1 ten × 4 tens = 4 hundreds

Step 7 – add the partial products

	100s	10s	1s	
		~~5~~		
		4	7	
×		1	8	
	3	7	6	47 × 8
	4	7	0	47 × 10
	8	4	6	
	1			

4:6

Give children the opportunity to practise setting out and completing calculations in which regrouping ones into tens is required when multiplying by the ones using the algorithm.

Choose examples, such as those shown opposite, that do not require regrouping tens into hundreds when multiplying by the tens.

Initially, children may use the expanded layout until they are confident enough to use the compact layout without place-value headings.

'Use long multiplication to do these calculations.'

23 × 18 = 46 × 18 = 82 × 18 = 62 × 48 =

4:7

Now move on to calculations where regrouping is required for multiplying both partial products (e.g. *27 × 23*). Following the same process as in steps *4:2* and *4:5*, continue to encourage children to describe the steps in laying out and applying the algorithm, paying attention to where to record the *regrouping in calculating both partial products:*

- *'First, write the larger factor: 27.'*
- *'Then write the smaller factor below, lining up the digits: 23.'*
- *'Now multiply by the ones digit to give a partial product: to multiple twenty-seven by three…'*
- *'…multiply seven ones by three to give twenty-one ones…'*
- *'…<u>and regroup: twenty-one ones is equal to two tens and one one; write "2" above the tens column and "1" in the ones column</u>…'*
- *'…then multiply two tens by three to give six tens…'*
- *'…and add the two tens from regrouping to give eight tens; write "8" in the tens column.'*
- *'Then, place a zero to show that it's ten times the size.'*
- *'Next, multiply by the tens digit to give a partial product: to multiply twenty-seven by twenty…'*
- *'…multiply seven ones by two tens to give fourteen tens…'*
- *'…<u>and regroup: fourteen tens is equal to one hundred and four tens; write "1" above the hundreds column and "4" in the tens column</u>…'*
- *'…then multiply two tens by two tens to give four hundreds…'*
- *'…and add one hundred from regrouping to give five hundreds; write "5" in the hundreds column.'*
- *'Finally, add the partial products together: 81 add 540 is equal to 621.'*

Step 1 – write the factors

	100s	10s	1s
		2	7
×		2	3

Step 2 – multiply the ones digit by the ones digit and regroup

	100s	10s	1s	
		2		
		2	7	
×		2	3	
			1	3 × 7 ones = 21 ones = 2 tens + 1 one

Step 3 – multiply the tens digit by the ones digit and add the regrouped tens

	100s	10s	1s	
		~~2~~		
		2	7	
×		2	3	
		8	1	3 × 2 tens = 6 tens 6 tens + 2 tens = 8 tens

Step 4 – place a zero to show that it's ten times the size

	100s	10s	1s
		~~2~~	
		2	7
×		2	3
		8	1
			0

Step 5 – multiply the ones digit by the tens digit and regroup

	100s	10s	1s	
	1	~~2~~		
		2	7	
×		2	3	
		8	1	
		4	0	2 tens × 7 ones = 14 tens = 1 hundred + 4 tens

Step 6 – multiply the tens digit by the tens digit and add the regrouped hundred

	100s	10s	1s	
	~~1~~	~~2~~		
		2	7	
×		2	3	
		8	1	
	5	4	0	2 tens × 2 tens = 4 hundreds 4 hundreds + 1 hundred = 5 hundreds

Step 7 – add the partial products

	100s	10s	1s	
	~~1~~	~~2~~		
		2	7	
×		2	3	
		8	1	27 × 3
	5	4	0	27 × 20
	6	2	1	
	1			

4:8

Once more, give children the opportunity to practise setting out and completing calculations using the long algorithm. Include some problems from real-life contexts, for example:

- *'Tilda has worked out that it costs 68 p per day to feed her dog Humphrey. How much money must she spend on dog food next April?'*
- *'Marbles come in packs of eight and there are six packs in each box. If Rafal bought twenty-four boxes and one extra pack, how many marbles does he have altogether?'*
- *'The diameter of a 10 p coin is about 25 mm and the diameter of a 5 p coin is 18 mm. Becca has lined up £6.70 in 10 p coins and £2.45 in 5 p coins. How long is her line?'*

As with previous practice, children may use the expanded layout initially, but by now should be able to move on to using the compact layout without place-value headings relatively quickly.

In the final example opposite, Lesa has written regrouping the wrong way around so that the tens are in the ones place and the ones are in the tens place, and so on.

- *'Use long multiplication to do these calculations.'*

  $22 \times 78 =$ $68 \times 24 =$ $46 \times 68 =$ $68 \times 68 =$

- *'Lesa has done this calculation but has the incorrect answer. Explain her mistake.'*

		~~1~~	~~5~~		
		5	7		
×		3	5		
	3	0	3		57 × 5
1	6	2	0		57 × 30
1	9	2	3		

4:9

Once children are confident multiplying any two-digit number by any other two-digit number, build on this by multiplying three-digit numbers by two-digit numbers (e.g. 28×312).

Follow the same process as earlier (for example, step *4:7*), asking children to describe the instructions at each step in laying out and applying the algorithm. You may find that, by now, children begin to shorten the descriptive language they use to reason through the algorithm as they become more confident with the strategy. Point out that we always write the three-digit number first and then the two-digit number.

Begin with the expanded layout until children are confident moving to the compact layout without place-value headings. You may wish to compare the expanded layout with the compact layout for the first few examples.

Multiplication algorithm – expanded layout:

Step 1 – write the factors

	1,000s	100s	10s	1s
		3	1	2
×			2	8

Step 2 – multiply the ones digit by the ones digit and regroup

	1,000s	100s	10s	1s
(regrouped)			1	
		3	1	2
×			2	8
				6

8 × 2 ones = 16 ones
= 1 ten + 6 ones

Step 3 – multiply the tens digit by the ones digit and add the regrouped tens

	1,000s	100s	10s	1s
(regrouped)			~~1~~	
		3	1	2
×			2	8
			9	6

8 × 1 ten = 8 tens
8 tens + 1 ten = 9 tens

Step 4 – multiply the hundreds digit by the ones digit and regroup

	1,000s	100s	10s	1s
(regrouped)			~~1~~	
		3	1	2
×			2	8
	2	4	9	6

8 × 3 hundreds = 24 hundreds
= 2 thousands + 4 hundreds

Step 5 – place a zero to show that it's ten times the size

	1,000s	100s	10s	1s
(regrouped)			~~1~~	
		3	1	2
×			2	8
	2	4	9	6
				0

Step 6 – multiply the ones digit by the tens digit

	1,000s	100s	10s	1s	
			~~1~~		
		3	1	2	
×			2	8	
	2	4	9	6	
			4	0	2 tens × 2 ones = 4 tens

Step 7 – multiply the tens digit by the tens digit

	1,000s	100s	10s	1s	
			~~1~~		
		3	1	2	
×			2	8	
	2	4	9	6	
		2	4	0	2 tens × 1 ten = 2 hundreds

Step 8 – multiply the hundreds digit by the tens digit

	1,000s	100s	10s	1s	
			~~1~~		
		3	1	2	
×			2	8	
	2	4	9	6	
	6	2	4	0	2 tens × 3 hundreds = 6 thousands

Step 9 – add the partial products

	1,000s	100s	10s	1s	
			~~1~~		
		3	1	2	
×			2	8	
	2	4	9	6	312 × 8
	6	2	4	0	312 × 20
	8	7	3	6	
		1			

Multiplication algorithm – compact layout:

$$\begin{array}{cccccc} & & & \not{1} & \\ & & 3 & 1 & 2 \\ \times & & & 2 & 8 \\ \hline & 2 & 4 & 9 & 6 \\ & 6 & 2 & 4 & 0 \\ \hline & 8 & 7 & 3 & 6 \\ \hline & & 1 & & \end{array}$$

4:10 Provide children with the opportunity to practise the algorithm for multiplication of a three-digit number by a two-digit number. Use varied practice, including real-life contexts and mixed operations, such as the examples opposite and below:

- *'A school buys milk in cartons of 284 ml. If each child in the reception class of thirty-two children needs one carton of milk, how much milk is that altogether?'*
- *'It takes two hundred and four stickers to fill a sticker album. Twenty-eight children each buy enough stickers to fill their album and one more sticker each for their teacher. How many stickers is that altogether?'*

Ensure that children are able to calculate using the compact layout, without place-value headings.

Ask children to apply their learning through a dòng nǎo jīn question where they will need to reason and estimate to help them solve the problem. For the example shown opposite, the answer is *213 × 45*.

'Use long multiplication to do these calculations.'

327 × 32 =	54 × 706 =	837 × 25 =
504 × 44 =	12 × 375 =	365 × 56 =

Dòng nǎo jīn:

'Thomas uses these five number cards to create a multiplication equation. What was his equation?'

1 2 3 4 5

□ □ □ × □ □ = 9,585

4:11 When children are confident multiplying three-digit numbers by two-digit numbers, move on to multiplying four-digit numbers by two-digit numbers (e.g. *28 × 3,126*). Follow the same process as earlier (for example, step *4:7*), with children repeating the

	instructions at each step in laying out and applying the algorithm. As in step *4:9*, children may shorten their descriptive language now the algorithm is familiar to them but make sure this doesn't lead to mistakes, particularly where regrouping is necessary. Emphasise again that we write the larger, four-digit, number first and then the smaller, two-digit, number below. As before, begin working through examples with the expanded layout until children are confident working with the compact layout without place-value headings.	

4:12 Multiplication algorithm – expanded layout:

Step 1 – write the factors

	10,000s	1,000s	100s	10s	1s	
		3	1	2	6	
×				2	8	

Step 2 – multiply the four-digit number by the ones digit, regrouping where necessary

	10,000s	1,000s	100s	10s	1s	
(regrouping)		~~1~~	~~2~~	~~4~~		
		3	1	2	6	
×				2	8	
	2	5	0	0	8	3,126 × 8

Step 3 – place a zero to show that it's ten times the size

	10,000s	1,000s	100s	10s	1s	
(regrouping)		~~1~~	~~2~~	~~4~~		
		3	1	2	6	
×				2	8	
	2	5	0	0	8	3,126 × 8
					0	

Step 4 – multiply the four-digit number by the tens digit, regrouping where necessary

	10,000s	1,000s	100s	10s	1s	
(regrouping)			~~1~~			
(regrouping)		~~1~~	~~2~~	~~4~~		
		3	1	2	6	
×				2	8	
	2	5	0	0	8	3,126 × 8
	6	2	5	2	0	3,126 × 20

Step 5 – add the partial products

	10,000s	1,000s	100s	10s	1s	
(regrouping)			~~1~~			
(regrouping)		~~1~~	~~2~~	~~4~~		
		3	1	2	6	
×				2	8	
	2	5	0	0	8	3,126 × 8
	6	2	5	2	0	3,126 × 20
	8	7	5	2	8	

Multiplication algorithm – compact layout:

			~~1~~		
		~~1~~	~~2~~	~~4~~	
		3	1	2	6
×				2	8
	2	5	0	0	8
	6	2	5	2	0
	8	7	5	2	8

4:13 Complete this teaching point by giving children varied practice multiplying four-digit numbers by two-digit numbers, such as the examples opposite and below. Include problems with real-life contexts and mixed operations.

- *'A telephone exchange receives an average of 3,047 calls each week. How many calls does it receive in one year?'*
- *'A school has twenty-five bottles of juice left after a party. One bottle has 725 ml left in it, while the others are full, with 1,725 ml in each. How much juice is left?'*
- *'Cereal bars come in boxes of twenty-four. A supermarket buys 2,005 boxes, but one box is dropped and seventeen of the cereal bars are ruined. How many cereal bars are left to sell?'*
- *'A book has an average of 3,542 words in one chapter and three hundred words in the introduction. How many words are there in the book if it has twenty-three chapters?'*

Once children are confident with these types of calculations, develop their reasoning by asking them to complete the empty boxes in a calculation, such as the missing-number problems shown opposite.

Develop this reasoning further with a dòng nǎo jīn question. The example opposite requires children to think

Long multiplication:

'Use long multiplication to do these calculations.'

$4{,}027 \times 32 =$ $\quad 32 \times 4{,}207 =$ $\quad 4{,}270 \times 32 =$

$33 \times 4{,}444 =$ $\quad 25 \times 5{,}252 =$ $\quad 2{,}368 \times 36 =$

Missing-number problems:

'Fill in the missing digits.'

		1	2	□	4
×				2	□
		7	7	0	4
	2	5	6	8	0
	3	3	3	8	4

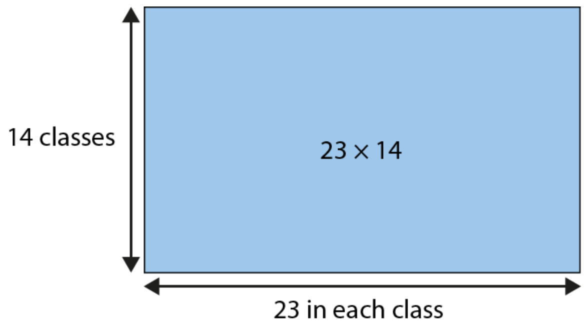
			7	0	□
×				□	4
		2	8	2	8
	6	3	6	3	0
	6	6	4	5	8

	about the positions of the digits and how that impacts the size of the product. Estimation can be used to make a 'first best guess'.	Dòng nǎo jīn: □ □ □ × □ □ = △ *'Using the digits 0–4 once each, create a calculation where:'* 6,000 < △ < 12,000 *'Can you create more than one solution?'*

Teaching point 5:

Multiplication where one of the factors is a composite number can be carried out by multiplying one factor and then the other factor.

Steps in learning

	Guidance	Representations
5:1	This teaching point draws on the learning from segment *2.20 Multiplication with three factors and volume, Teaching point 4* (associative law) and segment *2.21 Factors, multiples, prime numbers and composite numbers, Teaching points 2* and *3* (composite and prime numbers). Begin with a review of the latter. Remind children that composite numbers have more than two factors, whereas prime numbers have exactly two factors. Using counters in array formations, ask children to convince you whether numbers are prime or composite.	*'Is each number prime or composite? Convince me using counters or a drawing.'* 15 18 23 24 27 29 32 36
5:2	Next, begin with a real-life context, for example: *'If there are fourteen classes of twenty-three children in a school, how many children are there in the school?'* Explain to children that this problem could be solved using long multiplication but that we are going to look at using factors to help solve it more efficiently. Begin by showing an area model to represent the equation *23 × 14*. Demonstrate how the '14' can be factorised into seven groups of two, as shown on the next page, to give the same result: $23 \times 14 = 23 \times 2 \times 7$ The calculation on the right of the equals symbol can be solved using short multiplication by first calculating *23 × 2* and then multiplying that product by seven.	*'If there are fourteen classes of twenty-three children in a school, how many children are there in the school?'* 

Explain that another way to solve this problem is to multiply by the seven first, also shown opposite:

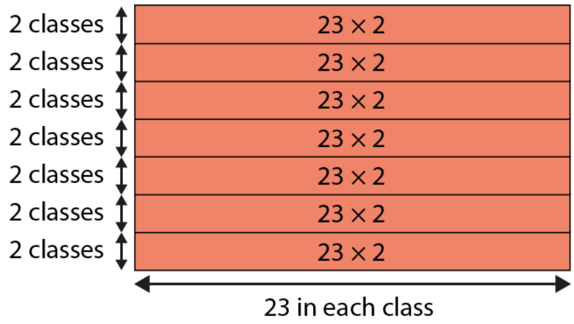
$23 \times 14 = 23 \times 7 \times 2$

Draw attention to the fact that you can associate the '23' and the '7', and multiply these first and then multiply that product by two. Again, this uses short multiplication. It does not matter which part of the equation we solve first, the product of *23* × *14* is the same. Ask children to decide which they think is the more efficient solution.

This leads us to the generalised statement: ***'When multiplying, you can write a composite number as factor × factor and use the associative law to make the calculation more efficient.'***

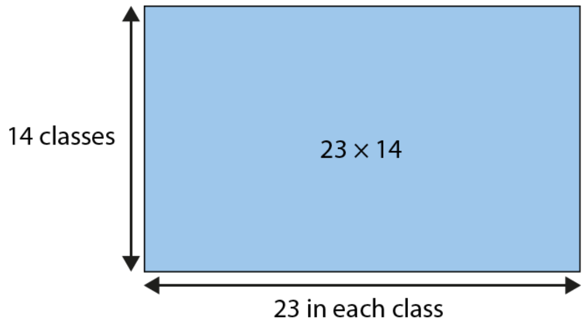
Draw children's attention to the connection with *Teaching Point 1*. We are using the same mathematical rules here but this time looking at the full range of factors to find the most efficient, or elegant, solution.

- *'We can represent this as seven groups of twenty-three-times-two.'*

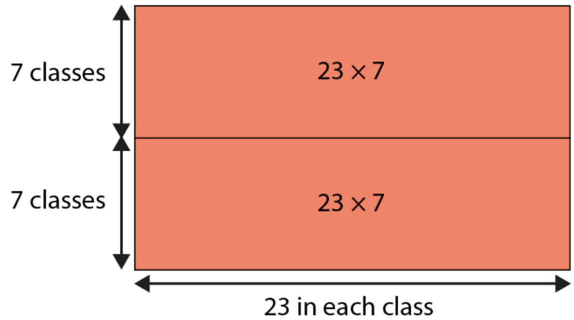


$$
\begin{aligned}
23 \times 14 &= 23 \times 2 \times 7 \\
&= 46 \times 7 \\
&= 322
\end{aligned}
$$

Alternatively:



- *'We can represent this as two groups of twenty-three-times-seven.'*



$$
\begin{aligned}
23 \times 14 &= 23 \times 7 \times 2 \\
&= 161 \times 2 \\
&= 322
\end{aligned}
$$

5:3

Give children the opportunity to practise identifying factors through examples such as those shown opposite. Where no factors are given, they will need to identify which is the composite number first.

Missing-number problems:

'Fill in the missing numbers.'

$31 \times 16 = 31 \times 4 \times \square$

$14 \times 8 \times \square = 32 \times 14$

$25 \times 15 = 25 \times \square \times \square$

$\square \times \square = 17 \times 4 \times 8$

$19 \times 35 = \square \times \square \times \square$

$\square \times \square \times \square = 23 \times 27$

True/false-style problems:

'Use a tick or a cross to show whether each equation is correct or not.'

$24 \times 26 = 8 \times 26 \times 4$

$5 \times 23 \times 3 = 23 \times 15$

$16 \times 32 = 4 \times 8 \times 32$

Dòng nǎo jīn:

'How many different ways can you make this true?'

$18 \times 24 = \square \times \square \times \square$

5:4

Next, compare this strategy with long multiplication by presenting children with an example such as: *'Water bottles come in boxes of fifteen bottles. A school has bought three hundred and twenty-four boxes. How many water bottles does the school have?'*

Ask children to look at Oliver's method and Tanima's method, and decide which they think would be the most efficient. Which do they prefer?

Ask children to practise some examples, such as those shown opposite and on the next page, first using long multiplication and then

- *'Water bottles come in boxes of fifteen bottles. A school has bought three hundred and twenty-four boxes. How many water bottles does the school have?'*

  Oliver's method:

  324 boxes of 15 bottles

		~~1~~	~~2~~		
		3	2	4	
$\times$			1	5	
	1	6	2	0	324×5
	3	2	4	0	324×10
	4	8	6	0	

	factorising the two-digit factor and using short multiplication. Ask them to compare the methods and decide which is the most efficient in each case.	Tanima's method: 324 boxes of 15 bottles $324 \times 15 = 324 \times 3 \times 5$ $\begin{array}{r} 3\ 2\ 4 \\ \times \quad\ \ 3 \\ \hline 9\ 7\ 2 \\ \hline {\scriptstyle 1} \end{array} \qquad \begin{array}{r} 9\ 7\ 2 \\ \times \quad\ \ 5 \\ \hline \mathbf{4\ 8\ 6\ 0} \\ \hline {\scriptstyle 3\ 1} \end{array}$ • *'Calculate each product, first using long multiplication and then factorising the two-digit factor and using short multiplication. For each one, which method is the most efficient?'* $425 \times 42 =$ $\quad$ $318 \times 28 =$ $\quad$ $542 \times 16 =$ $125 \times 15 =$ $\quad$ $222 \times 16 =$ $\quad$ $497 \times 36 =$
5:5	The next step is to look more closely at which factors to choose. Referring back to the dòng nǎo jīn example from step *5:3* (*18 × 24*), investigate with children how many ways the equation can be solved using factors. They should come up with the following: $18 \times 2 \times 12$ $\quad$ $18 \times 12 \times 2$ $18 \times 3 \times 8$ $\quad$ $18 \times 8 \times 3$ $18 \times 4 \times 6$ $\quad$ $18 \times 6 \times 4$ $24 \times 2 \times 9$ $\quad$ $24 \times 9 \times 2$ $24 \times 3 \times 6$ $\quad$ $24 \times 6 \times 3$ Ask children to work in pairs to solve each equation and discuss how efficient they each were. Is there one solution that is more elegant than the others? Now give children some multiplication problems where both factors are two-digits for further practice. Where both factors are composite numbers, they will need to choose which factor to factorise and how. Once complete, ask children to share their answers. Discuss with them which strategies were more or less efficient.	*'Solve each equation by factorising one of the factors and using short multiplication.'* $37 \times 18 =$ $\quad$ $21 \times 15 =$ $\quad$ $16 \times 18 =$ $36 \times 24 =$ $\quad$ $27 \times 28 =$ $\quad$ $19 \times 42 =$

<table>
<tr>
<td>5:6</td>
<td>Once children are confident using factors for multiplication, complete this teaching point by giving them the opportunity for varied practice, such as the examples opposite and below. Include problems with real-life contexts and mixed operations.

Children should assess each problem individually and choose the most efficient strategy to find a solution, looking for composite numbers and identifying whether factorising would make the solution more elegant.

At this point, they may draw upon other strategies from previous segments (for example, use of the distributive law from segment 2.10 Connecting multiplication and division, and the distributive law). The key point is to justify why the strategy is more efficient or elegant than another.

• 'Sohail buys nine packs of twenty-four stickers. Jasmine says that she only needs to buy six packs of thirty-six stickers to have the same number as Sohail. Explain why she is correct.'

• 'A football pitch is 105 m long and 68 m wide. What is the total area of the pitch?'
• 'It takes 55 ml of paint per metre to paint the perimeter line of the same football pitch. How much paint does the groundsman need to buy?'

• 'A shop sells ribbon in 1.5 m lengths. Mr Rodgers bought twenty-eight lengths of ribbon for his class to make streamers, but asked the shop to cut 5 cm off each length first. How much ribbon did he buy?'

• '325 ml bottles of water are arranged on a shelf in three rows of five bottles. How many litres of water are there?'</td>
<td>Choosing the most efficient strategy:
'Solve each equation using the most efficient strategy.'

$37 \times 19 =$ $315 \times 15 =$ $42 \times 24 =$
$18 \times 546 =$ $36 \times 321 =$ $972 \times 16 =$
$22 \times 514 =$ $302 \times 24 =$

True/false-style problems:
'Use a tick or a cross to show whether each equation is correct or not.'
$45 \times 10 + 45 \times 7 = 17 \times 9 \times 5$
$162 \times 30 + 162 \times 6 = 162 \times 30 \times 6$
$232 \times 3 \times 8 = 232 \times 20 + 232 \times 4$

Dòng nǎo jīn:
'Baker A packed seven crates of buns. In each crate, there are four layers of twelve buns. Baker B arranges the same number of buns in six crates.'
• 'How many layers of buns could there be in each of Baker B's crates?'
• 'How many other ways are there of arranging the same number of buns?</td>
</tr>
</table>

	• *'There are thirty children in a class. They are carrying out a sponsored jog where each child must jog for 15 minutes, one after the other. If two children are away, how long will the sponsored jog take?'* Finally, you can ask children to apply their knowledge of factorising to solve the dòng nǎo jīn problem (previous page) where there is more than one possible solution.	