MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Fractions

3.8 Common denomination: more adding and subtracting

Teacher guide | Year 5

Teaching point 1:

In order to add related fractions, first convert one fraction so that both share the same denominator (a '*common denominator*').

Teaching point 2:

To subtract related fractions, first convert one fraction so that both share a common denominator.

Teaching point 3:

The common denominator method can be extended to adding and subtracting non-unit related fractions.

Teaching point 4:

To add and subtract *non-related* fractions, the product of the two denominators provides a common denominator.

Teaching point 5:

Converting to common denominators is one of several methods that can be used to compare fractions.

Overview of learning

In this segment children will:

- understand that fractions with different denominators may be *related* or *non-related* and learn to identify each type
- learn the language of 'common denominator'
- explore the relationships between the denominators and numerators when finding common denominators in related and non-related fractions
- learn to add and subtract fractions that are either related or non-related
- learn that common denomination is one of many methods that can be used to compare fractions.

This segment draws heavily on the work that children did in segment *3.7 Finding equivalent fractions and simplifying fractions* on finding equivalent fractions. The first step is to teach children how to find a common denominator to convert to (in segment *3.7*, children were always provided with the numerator or denominator to convert to). Fractions being added or subtracted are considered as either *related* (one denominator is a multiple of the other denominator, e.g. $\frac{1}{3}+\frac{1}{9}$) or *non-related* (neither denominator is a multiple of the other, e.g. $\frac{2}{5}+\frac{3}{4}$). Children are taught a method for finding a common denominator for both related and non-related fractions. Once children are able to find common denominators, this segment focuses heavily on applying this to their prior knowledge of adding and subtracting fractions with the same denominator, both within and across one whole.

When working to find a common denominator of non-related fractions, the initial example questions allow children to derive the lowest common denominator by multiplying the denominators. However, it is important to note that with other calculations, the lowest common multiple of the denominators may not be the product of the two denominators. For example, with $\frac{1}{6}$ and $\frac{1}{9}$, the lowest common denominator is 18, not 54. It is therefore important to acknowledge that some children may derive a lower common denominator in some instances. However, the overall focus at primary level should be on finding *a* common denominator. At secondary school, children will learn a comprehensive method to find the lowest common multiple (and hence denominator) of any two numbers.

The early teaching points are initially supported by diagrams, including number lines, but the level of scaffolding is gradually reduced to encourage children to develop fluency once they have secured conceptual understanding. Try to balance the move to more efficient procedures with sense-checking their calculations. For example, we know that $\frac{1}{3}+\frac{1}{4}$ can't be $\frac{1}{7}$ because $\frac{1}{7}$ is smaller than both the addends. Examples of the types of probing questions you can use to support children as they sense-check, are included throughout the segment.

Towards the end of this segment, children learn that finding a common denominator can also be a useful tool for comparing related and non-related fractions. Children have already assembled a repertoire of ways to compare fractions (including comparing fractions with the same numerator, identifying where fractions sit in the number system, and making proportional judgements about whether a fraction represents a small or large part of the whole). Once they have learnt the common-denominator method, it can be an easy jump for some children to use common denominators to compare fractions without considering whether this is the best strategy. As such, this segment also reviews other reason-based comparison strategies and considers them alongside common denomination, so that children have a range of strategies they can utilise according to the particular numbers involved.

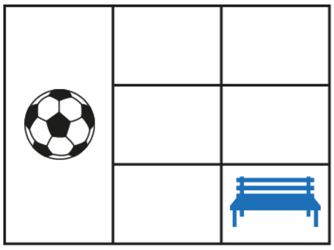
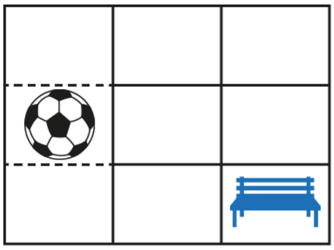
Proportional reasoning has been interwoven through the fractions spine to help children develop 'fraction sense'. This is a focus of the final teaching point.

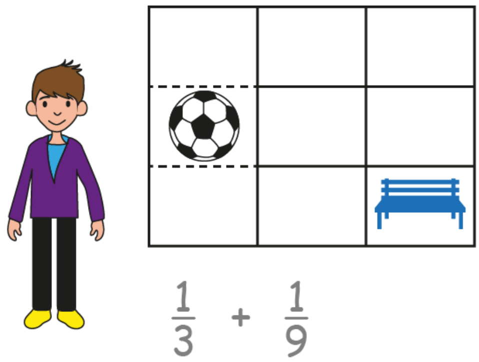
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast*. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.*

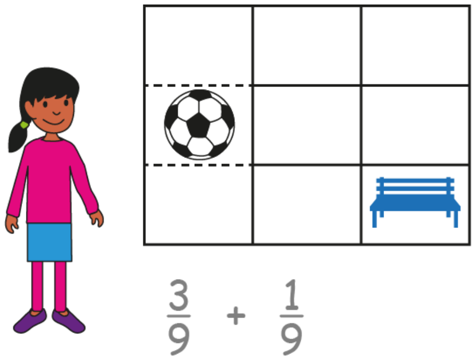
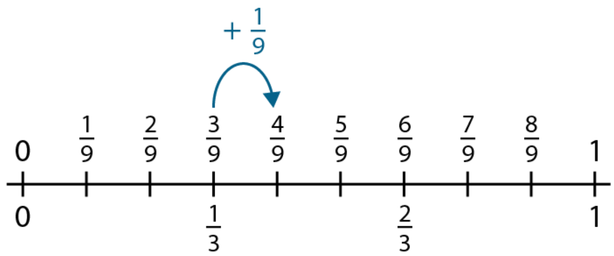
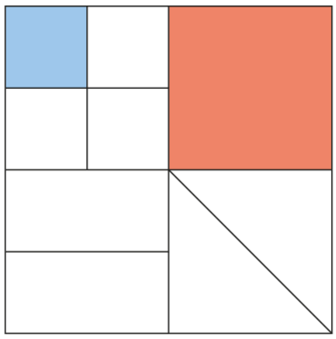
Teaching point 1:

In order to add related fractions, first convert one fraction so that both share the same denominator (a '*common denominator*').

Steps in learning

	Guidance	Representations
1:1	A good starting point for adding and subtracting fractions with different denominators is to work with related fractions. As explained in the overview, related fractions are a pair of fractions where only one fraction needs to be converted to match the denominator of the other. Children are already confident converting a fraction to an equivalent fraction from their work in segment *3.7 Finding equivalent fractions and simplifying fractions*. It is a small step on from that prior learning to look at two denominators, and decide which to convert in order to have common denominators. This teaching point is clearly divided into three important steps that children need to master before moving on: **1.** adding related fractions supported by an image (steps *1:1–1:5*) **2.** understanding what is meant by '*related fractions*' (steps *1:6–1:10*) **3.** adding related fractions without a supporting image (steps *1:11–1:13*). **Step 1: adding related fractions supported by an image** Begin by sharing an image, such as the playground plan opposite. Explain that the playground is divided into different zones, including a ball-game zone (with the football icon) and a quiet zone (with the bench icon). These zones are *not* equal parts of the whole. Ask the children: • *'What fraction of the playground is the ball-game zone?'*	 

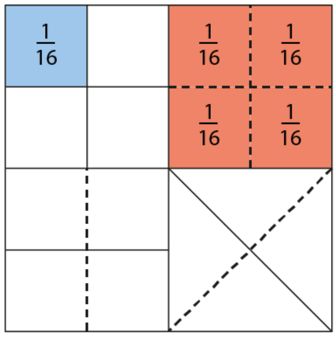
	• *'What fraction of the playground is the quiet zone?'* • *'Do we have equal parts?'* Discuss with the children that at this point, it is not possible to say what fraction of the playground these zones occupy because we do not have equal parts. Some children may recognise that we can see the ball game zone occupies $\frac{3}{9}$ of the playground. Display the second diagram which splits the playground into equal parts and again ask questions, such as: • *'What fraction of the playground is the ball-game zone?'* • *'What fraction of the playground is the quiet zone?'* • *'What fraction of the playground do these zones occupy altogether?'* Agree with children that the quiet zone occupies $\frac{1}{9}$ of the playground, and the ball-game zone $\frac{3}{9}$ of the playground. Altogether they occupy $\frac{4}{9}$ of the playground. This is evident when looking at the representation.	
1:2	Return to the original image shown. Explain that two children, Ella and Will, have written a calculation to show the fraction of the playground that the ball-game zone and quiet zone take up. You could introduce this like a story, as shown opposite. Allow children time to discuss and reason that $\frac{1}{3}$ is equivalent to $\frac{3}{9}$, and therefore both Ella and Will are correct. At this stage, avoid using the procedure of multiplying the numerator by three because the denominator has been multiplied by three. For now, the aim is for children to identify the equivalent	• *'This is Will. He writes this calculation.'* 

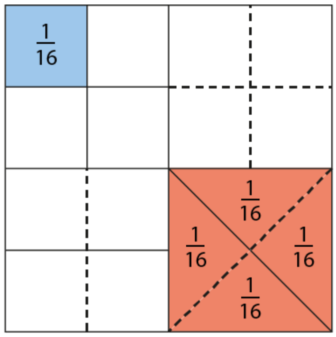
	fractions using the supporting image. Avoid encouraging them to learn a rule without developing their understanding first.	• *'This is Ella. She writes this calculation.'*  *'Whose calculation is correct? Can you explain your thoughts, using the diagram to help you?'*
1:3	Now ask the children if they can think of a way to check their answer with a number line. As with previous segments, the number line offers an important visual representation. Share an image such as the one opposite (children saw similar 'double' number line images in segment *3.7 Finding equivalent fractions and simplifying fractions,* and so should be comfortable with this). This highlights the equivalence between $\frac{1}{3}$ and $\frac{3}{9}$, and is another way to demonstrate why the total of $\frac{1}{3}$ and $\frac{3}{9}$ is $\frac{4}{9}$.	
1:4	Now provide children with another example, such as the one opposite, but this time not set in a real-life context. Ask probing questions, such as: • *'What fraction of the whole does each of the shaded parts represent?'* • *'Can we say what fraction of the whole is shaded altogether?'* Provide children with a copy of the diagram for them to annotate. Ask them to consider how they can make the parts equal. Take suggestions form the children, agreeing that the whole can be split into 16 equal parts. Look at the second diagram on the next page, split into sixteenths, and agree	

that a total of $\frac{5}{16}$ is shaded. Then look back at the previous diagram, asking *'Can we also say that five-sixteenths of the first diagram is shaded?'*

It will be harder for children to recognise that the statement is also true of this first example. However, as we have seen that $\frac{1}{4} = \frac{4}{16}$ and there is an additional $\frac{1}{16}$, then it must also be true that $\frac{5}{16}$ of the first diagram is shaded.

It is worth noting that this particular example presents more of a challenge as the sixteenths do not look the same; they are incongruent. Children have met incongruence in earlier fractions segments, but check that they do understand that these parts are all equal, even though they look different. We can see the four quarters of the large square, and that each of these is divided into four equal parts. Shade two of the parts, as shown in the final diagram opposite, and ask the children to write an expression to show the total of the two shaded amounts. This should help identify any difficulties children have with seeing the equivalence of incongruent parts.

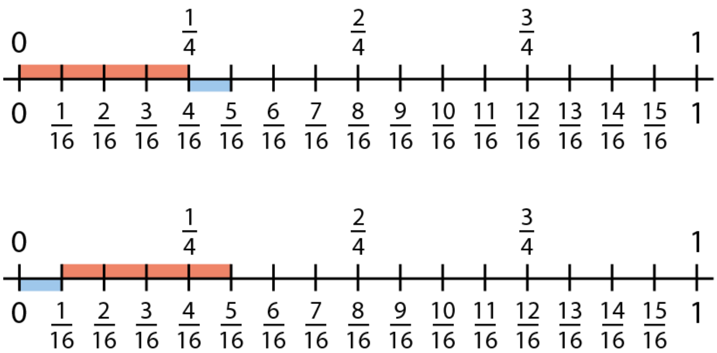


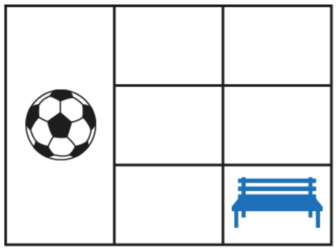


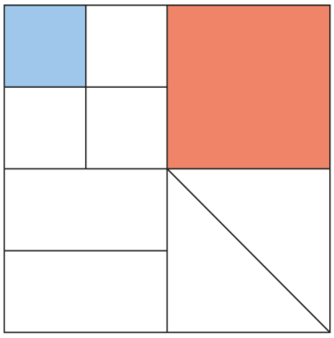
1:5

Return to the first representation in step *1:4* (with $\frac{1}{4}$ and $\frac{1}{16}$ shaded), and challenge children to show what this would look like on a number line. Allow time for children to work on this, and then share the image provided opposite and discuss it as a class.

(Note that it is much easier to show the $\frac{1}{4}$ first on this number line, and then the $\frac{1}{16}$. If the $\frac{1}{16}$ is shown first, it is



	harder to see, as the $\frac{1}{4}$ aligns to the $\frac{5}{16}$ label and not the $\frac{1}{4}$ label on the number line.)	
1:6	**Step 2: understanding what is meant by '*related fractions*'** Display two fractions: $\frac{1}{3}$ and $\frac{1}{9}$. Explain to children that these are called related fractions because one denominator is a multiple of the other. In this case, nine is a multiple of three. At this point, you could introduce children to the generalisation: ***'Related fractions have denominators where one denominator is a multiple of the other.'*** Refer to the diagrams used so far, to show how we are changing one of the fractions so that the denominators are the same. Explain that these are called '*common denominators*'. When something is 'in common' with something else, it has a characteristic that is the same: • $\frac{1}{3}$ and $\frac{1}{9}$ *don't* have a common denominator. • $\frac{3}{9}$ and $\frac{1}{9}$ *do* have a common denominator.	$\frac{1}{3}$ and $\frac{1}{9}$  *'We can change $\frac{1}{3}$ to $\frac{3}{9}$.'*
1:7	Now ask children a question to focus their attention on the written equations, such as: *'What are the two different calculations we can write to express the total fraction which is taken up by the ball-game zone and the quiet zone?'* Conclude with children that it is possible to write the two equations shown opposite. Discuss how they know that both are true. For instance, *'We know that $\frac{3}{9}+\frac{1}{9}=\frac{4}{9}$ is true from our work in segment 3.4 Adding and*	*'What are the two different calculations we can write to express the total fraction which is taken up by the ball-game zone and the quiet zone?'* • $\frac{3}{9}+\frac{1}{9}=\frac{4}{9}$ • $\frac{1}{3}+\frac{1}{9}=\frac{4}{9}$

	subtracting within one whole. We have three one-ninths and one one-ninth, which is a total of four one-ninths.' At this point, you may wish to recap the related generalisation: ***'When adding fractions with the same denominators, just add the numerators.'*** With the second equation, we only know that this is true because the area and number line diagrams were used to relate it to the first calculation. Take some time to focus on the numbers in this second equation. The numerators can't just be added in the same way as the first example, nor indeed the denominators. Before these two fractions with different denominators can be added, the calculation needs to be considered in terms of fractions with a *common denominator*.	
1:8	Return to the second worked example (from step *1:4*) and identify that $\frac{1}{16}$ and $\frac{1}{4}$ are also related fractions. At this point, you might find it helpful to introduce a stem sentence: ***'___ and ___ are related fractions because the denominator, "___", is a multiple of the other denominator, "___".'*** As before, ask the children to write two different calculations to show the total part of the whole which is shaded. Use similar discussion points to those in step *1:7*, including identifying that we are converting one fraction to an equivalent fraction so that both fractions have a *common denominator*.	 $\frac{1}{16}+\frac{1}{4}$ *'$\frac{1}{16}$ and $\frac{1}{4}$ are related fractions because the denominator, "16", is a multiple of the other denominator, "4".'* $\frac{1}{16}+\frac{4}{16}=\frac{5}{16}$ $\frac{1}{16}+\frac{1}{4}=\frac{5}{16}$
1:9	It may be helpful to make the link to something that children can really relate to; in order to add a number of things with different names together, it	

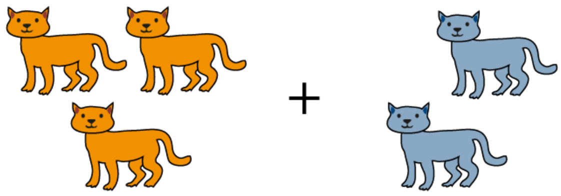
is necessary to find a name that is common to both things. For example, to add a number of cats together, we could say: *'3 cats + 2 cats = 5 cats'*

However, if we try to add, for example *3 cats + 2 dogs = ?* we encounter a problem because we are trying to add a number of things that are not in the same unit. To add a number of cats to a number of dogs, they need to be given a common name, such as *'pets'*:
3 pets + 2 pets = 5 pets

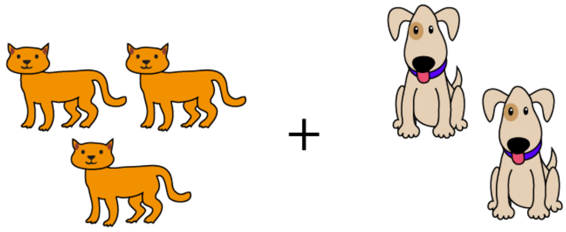
Alternatively, we could also say:
3 animals + 2 animals = 5 animals

Also using the common alternative name 'animals' demonstrates how sometimes there is more than one option for a common name.

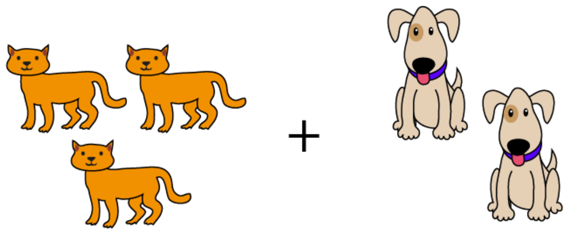
The denominator of a fraction tells us the name of the unit we are working in, and in order to add things which have different names, a common name must first be found. Thus, in order to add fractions with different denominators, we must find a common denominator.



3 cats + 2 cats = 5 cats



3 pets + 2 pets = 5 pets



3 animals + 2 animals = 5 animals

1:10 Provide children with a selection of fraction pairs and ask them to identify which pairs of fractions are related, sorting them into two groups in sorting circles, or into a table, as shown opposite. Ensure the fraction pairs are all unit fractions so that children can focus on the denominators. Again, you may find it helpful to use the stem sentence from step *1:8*: ***'___ and ___ are related fractions because the denominator, "___", is a multiple of the other denominator, "___".'***

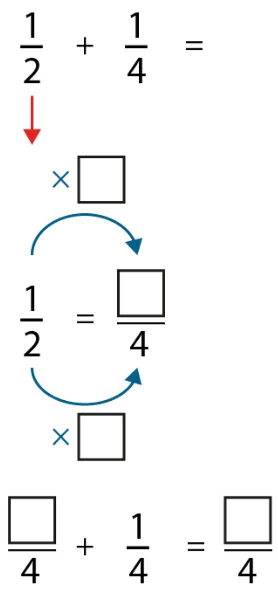
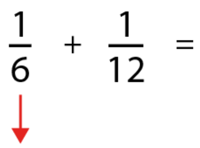
Expand on this task by providing children with individual fraction cards with a unit fraction shown on each. Ask children to find another card with a related fraction. This particular exercise will allow children the opportunity to find more than one related fraction.

$\frac{1}{2}+\frac{1}{4}$	$\frac{1}{3}+\frac{1}{8}$	$\frac{1}{3}+\frac{1}{12}$	$\frac{1}{25}+\frac{1}{5}$
$\frac{1}{6}+\frac{1}{18}$	$\frac{1}{9}+\frac{1}{4}$	$\frac{1}{5}+\frac{1}{8}$	$\frac{1}{10}+\frac{1}{5}$

Related fractions	Non-related fractions

1:11	**Step 3: adding related fractions without a supporting image** In steps *1:7* and *1:8* children were able to write $\frac{1}{3}+\frac{1}{9}=\frac{4}{9}$ and $\frac{1}{16}+\frac{1}{4}=\frac{5}{16}$ because they had a diagram to support them in converting the addends to a common denominator. Once children understand related fractions and can identify pairs that are related, they are ready to add pairs of related fractions *without* a supporting diagram. Take a pair of unit fractions from the previous exercise, such as $\frac{1}{5}+\frac{1}{15}$. Before children dive into finding a common denominator, ask them to *think* about this expression and start to make sense of it. (It is so easy to jump into procedures with adding and subtracting fractions, and while these are of course useful, also take time to help children think about whether what they are doing makes *sense.*) At this point, you might like to pose some questions to help develop their fraction sense, such as: • *'Which of the two addends is larger?'* (This is revising key learning from segment *3.2 Unit fractions: identifying, representing and comparing*, where children learnt how to compare unit fractions.) Agree with children $\frac{1}{5}$ is the larger number. • *'Can you describe how this might look on a number line?'* (Some children will be able to do this more easily than others. For example, they may be able to see that our total is going to be *'a bit bigger than* $\frac{1}{5}$*'*.) • *'Is our total going to be bigger or smaller than* $\frac{1}{2}$*?'* • *'Is our total going to be bigger or*	$\frac{1}{5}+\frac{1}{15}$

	smaller than $\frac{2}{5}$*?'* • *'Is our total going to be bigger or smaller than* $\frac{2}{15}$*?'*	
1:12	Ask the children to remind you why the fractions in an example, such as the one opposite, are related fractions. Again, you may wish to use the stem sentence from step *1:8* to support this: ***'___ and ___ are related fractions because the denominator, "___", is a multiple of the other denominator, "___".'*** As these are related fractions, it is possible to use one of these denominators as a common denominator. Discuss with children how to decide which denominator to choose as the common denominator. $\frac{1}{5}$ can be converted to fifteenths, but $\frac{1}{15}$ cannot be converted to fifths, so we will use fifteenths as the common denominator. Children had lots of practice with converting fractions to equivalent fractions in segment *3.7 Finding equivalent fractions and simplifying fractions*. The only new step here is to identify what the denominator for their equivalent fraction should be. Once the children have identified that they are converting $\frac{1}{5}$ to fifteenths, they should be confident with applying this. The 'horizonal method' is shown in the example opposite, but children should recall from segment *3.7* that they can also work vertically when identifying fractions equivalent to a unit fraction. (In $\frac{1}{5}$, the denominator is five times the numerator, and this will also be the case in an equivalent fraction.) Show pupils how to rewrite the	× 3 $\frac{1}{5} = \frac{3}{15}$ × 3 $\frac{1}{5} + \frac{1}{15} =$ ↓ × 3 $\frac{1}{5} = \frac{3}{15}$ × 3 $\frac{3}{15} + \frac{1}{15} = \frac{4}{15}$

	equation. Model the steps involved, showing how one fraction stays the same and the other is changed, so that both have a common denominator. The example shown offers one possible layout, but choose a way that you feel comfortable with, and that you are confident modelling with your class. As with anything, it is important that the children understand the steps rather than just working through them procedurally, without understanding. Over time, they may make their own adjustments to the layout. You will need to make a judgement about how rigidly you want children to stick with the layout you have taught them.	
1:13	Provide further guided practice in adding related fractions. First, identify the common denominator, then rewrite the equation, and finally add the fractions together. Some initial scaffolding is worthwhile here, in order for children to become fluent in this method. Include examples of adding three fractions. The principles are exactly the same: identifying which denominator from the three related fractions can be used as a common denominator for the other two fractions.	Example 1  Example 2  Example 3 $\frac{1}{4}+\frac{1}{12}+\frac{1}{2}$

1:14

Finally, provide children with varied practice, aiming to embed their understanding of finding a common denominator for related fractions.

Notice that in *Example 1* opposite, the level of scaffolding is gradually reduced over the first few problems. Also note that the fraction to be converted varies in position throughout the exercise.

Include examples presented on bar models (note that the total here is not 'one whole'). See *Example 2*.

Add more than two related fractions, again finding a common denominator, this time converting two of the fractions. See *Example 3*.

Include mixed numbers. See *Example 4*.

Deepen children's understanding by providing a range of reasoning opportunities, such as recognising a mistake. See *Example 5*. Support children to see that the total of one-twentieth must be wrong because it is smaller than both of the addends. Considering where all three of these fractions would be positioned on a number line will help to with this.

Present examples that use comparisons. See *Example 6*.

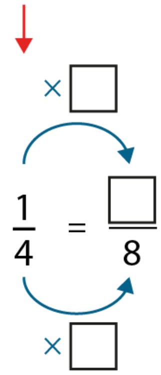
Include practice in reasoning around a statement. See *Example 7*. Here we want children to recognise that these fractions may still be added, even though they have different denominators, but they need converting to a common denominator first.

To consolidate and deepen learning, also provide dòng nǎo jīn examples, such as the examples given.

Example 1 – adding unit fractions with gradual reduction of scaffolding:

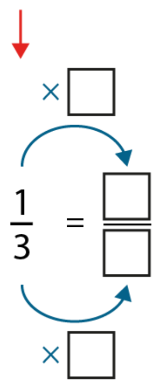
'Fill in the missing numbers.'

$$\frac{1}{8} + \frac{1}{4} =$$



$$\frac{1}{8} + \frac{\square}{8} = \frac{\square}{8}$$

$$\frac{1}{3} + \frac{1}{12} =$$



$$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$$

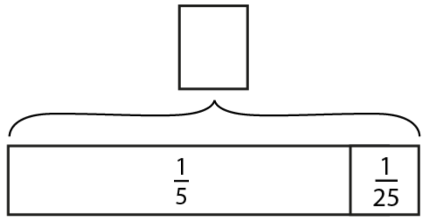
$\frac{1}{6} + \frac{1}{18} =$ $\frac{1}{10} + \frac{1}{5} =$

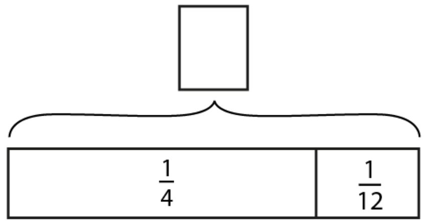
$\frac{1}{7} + \frac{1}{21} =$ $\frac{1}{8} + \frac{1}{24} =$

$\frac{1}{3} + \frac{1}{15} =$ $\frac{1}{9} + \frac{1}{27} =$

Example 2 – finding the total from bar models:

'Write the total of the two parts of the bar model above the bracket.'





Example 3 – adding three unit fractions:

$\frac{1}{2}+\frac{1}{8}+\frac{1}{4}=$

$\frac{1}{3}+\frac{1}{12}+\frac{1}{4}=$

$\frac{1}{3}+\frac{1}{5}+\frac{1}{15}=$

$\frac{1}{9}+\frac{1}{3}+\frac{1}{27}=$

Example 4 – adding mixed numbers:

$3\frac{1}{4}+\frac{3}{8}=$

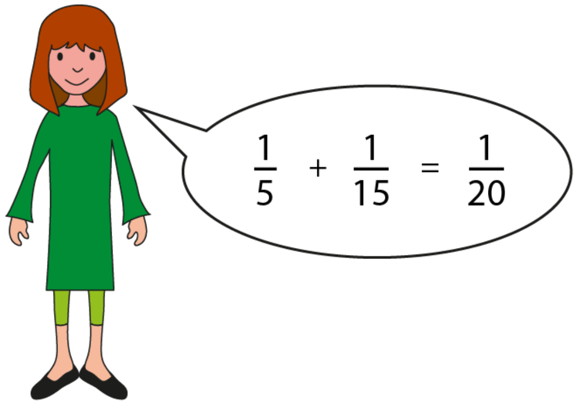
$1\frac{1}{6}+6\frac{5}{12}=$

$\frac{7}{9}+5\frac{2}{3}=$

$2\frac{3}{4}+10\frac{5}{12}=$

Example 5 – explaining errors:

'Can you explain Hannah's mistake?'



- *'How can you use reasoning to convince Hannah that she has the incorrect answer?'*

Example 6 – missing-symbol problems :

'Fill in the missing symbols (<, > or =).'

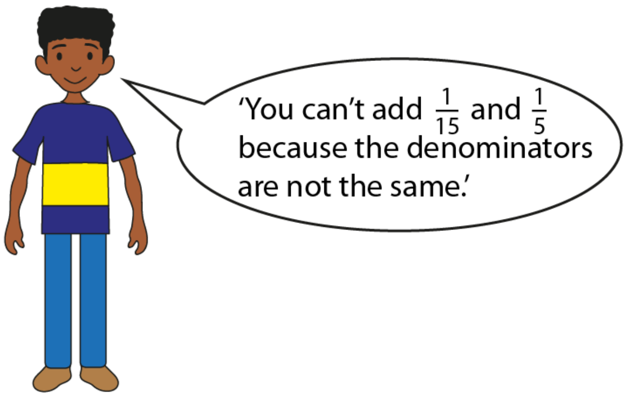
$\frac{1}{5}+\frac{1}{15} \bigcirc \frac{1}{5}+\frac{1}{5}$

$\frac{1}{5}+\frac{1}{15} \bigcirc \frac{2}{5}$

$\frac{1}{5}+\frac{1}{15} \bigcirc \frac{1}{15}+\frac{1}{5}$

$\frac{1}{5}+\frac{1}{15} \bigcirc \frac{2}{5}$

Example 7 – reasoning around a statement:



'Complete the following sentences.'

- Ahmed is partly correct in that ____.
- Ahmed is partly wrong in that ____.

Dòng nǎo jīn:

'Fill in the missing numbers.'

$$\frac{1}{4}+\frac{1}{12}=\frac{1}{\square}$$

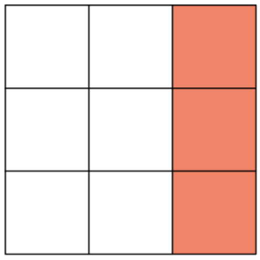
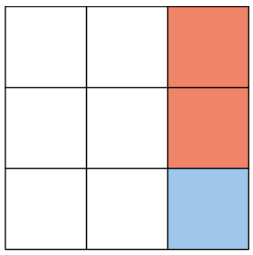
$$\frac{1}{2}=\frac{1}{\square}+\frac{1}{6}$$

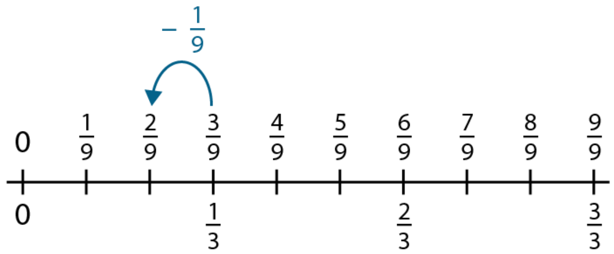
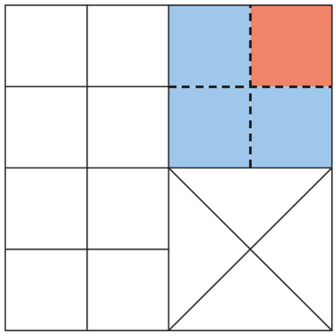
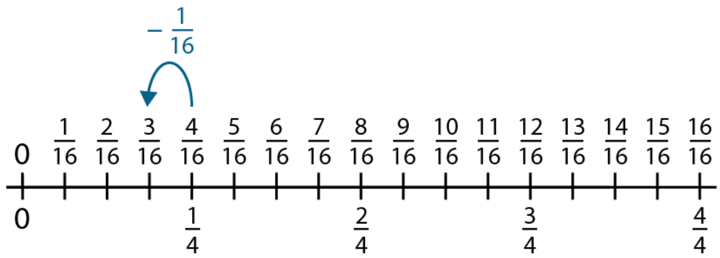
$$\frac{1}{20}+\frac{1}{\square}=\frac{1}{4}$$

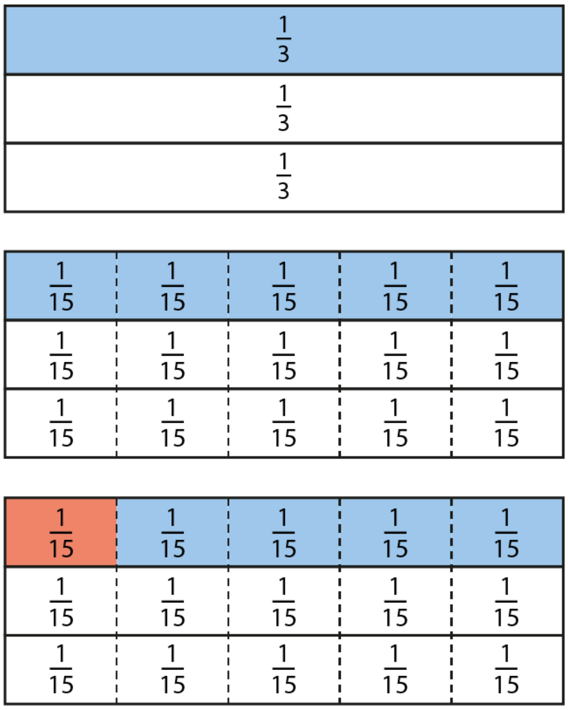
Teaching point 2:

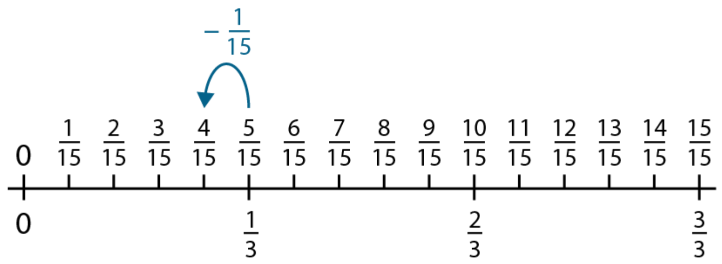
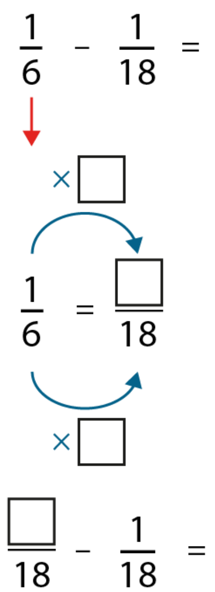
To subtract related fractions, first convert one fraction so that both share a common denominator.

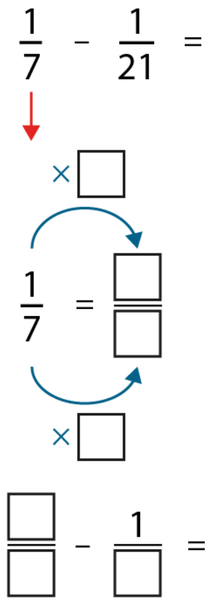
Steps in learning

	Guidance	Representations
2:1	Once children have mastered adding two or more related fractions and can reason around this concept, they can apply these newly acquired skills to subtraction. The sequence of learning from *Teaching point 1*, can be repeated using subtraction. Children should be secure in their understanding of the need to convert to a common denominator and the reasoning behind this, so steps *2:1* and *2:2* are shorter than the comparable steps in *Teaching point 1*.	 $\frac{1}{3} - \frac{1}{9}$
2:2	Again, begin with an area model, as shown opposite, and show one-third of the whole. Display the calculation $\frac{1}{3}-\frac{1}{9}$ and ask questions, such as: • *'Can you see the one-third?'* • *'How could we rewrite one-third so that the denominators of both fractions in our calculation are the same?'* Remind children that these fractions are related, and ask children to rewrite the calculation. Agree that it can be rewritten as $\frac{3}{9}-\frac{1}{9}$. This is now in a form that children can easily solve, giving a difference of $\frac{2}{9}$.	 $\frac{1}{3} - \frac{1}{9}$ $\frac{1}{3} - \frac{1}{9} =$ ↓ ×3 $\frac{1}{3} = \frac{3}{9}$ ×3 $\frac{3}{9} - \frac{1}{9} = \frac{2}{9}$

2:3	Present the same fraction calculation on a number line. As with addition in step *1:3*, ask children to notice that $\frac{1}{9}$ and $\frac{3}{9}$ share the same position on the number line.	
2:4	Repeat steps *2:1* and *2:2* reusing this familiar diagram from *Teaching point 1*. Challenge children to write two calculations. They should write the following: • $\frac{1}{4}-\frac{1}{16}$ • $\frac{4}{16}-\frac{1}{16}$ Children should be able to apply their knowledge from segment *3.4 Adding and subtracting within one whole* (step *2:5*) to solve the second calculation: ***'When subtracting fractions with the same denominators, just subtract the numerators.'*** They can use this generalisation to solve the second calculation and therefore also solve the equivalent calculation of $\frac{1}{4}-\frac{1}{16}$. As before, use the number line representation to reinforce the equivalence of $\frac{1}{4}$ and $\frac{1}{16}$. This will also develop children's understanding of fractions as numbers.	  $\frac{1}{4}-\frac{1}{16}=$ ↓ ×4 $\frac{1}{4}=\frac{4}{16}$ ×4 $\frac{4}{16}-\frac{1}{16}=\frac{3}{16}$

2:5	Progress to looking at how to convert the fractions in an equation without the support of a model. Here the model is used after the calculation to help the children check what they have done. Provide children with a subtraction calculation involving two *related* unit fractions, e.g. $\frac{1}{3} - \frac{1}{15}$. Explain that, as with addition, we need to convert the larger fraction so that both fractions have a common denominator. This means converting $\frac{1}{3}$ into fifteenths. Model how this is done, revising the learning from step *1:12*. Demonstrate how to rewrite the equation, as shown opposite. Model the steps involved, showing how one fraction stays the same and the other is changed, so that both have a common denominator.	$\frac{1}{3} - \frac{1}{15} =$ $\times 5$ $\frac{1}{3} = \frac{5}{15}$ $\times 5$ $\frac{5}{15} - \frac{1}{15} = \frac{4}{15}$
2:6	Once you have solved the calculation by converting to common denominators, look at the models opposite that represent the same calculation. Ask the children to describe the relationship between the two equivalent fractions and these two models. This is to ensure they have a good understanding of why they are doing what they are doing.	

		
2:7	Repeat with other subtraction calculations. For the moment, continue to just use unit fractions such as $\frac{1}{4}-\frac{1}{12}$. Note that in this example, the answer could be simplified. Some children may realise that $\frac{2}{12}$ is equivalent to $\frac{1}{6}$, as this was covered in segment *3.7 Finding equivalent fractions and simplifying fractions, Teaching point 4*. Include guided practice where there are two subtrahends, and all three fractions need to be converted to a common denominator (here, 10).	$\frac{1}{4} - \frac{1}{12}$ ↓ $\frac{3}{12}$ $\frac{3}{12} - \frac{1}{12} = \frac{2}{12}$ $\frac{1}{2}-\frac{1}{10}-\frac{1}{5}=$
2:8	At this point, provide varied practice similar to that in step *1:14*. Children start by identifying the common denominator, then rewriting the equation and finally completing the subtraction. You may choose to use some initial scaffolding provided here in *Example 1*, to help improve fluency with this method. Varied practice may also include: • using a bar model (*Example 3*) • calculating with mixed numbers (*Example 4*).	Example 1 – subtraction with scaffolding: *'Fill in the missing numbers.'* 

<table>
<tr>
<td></td>
<td></td>
<td>


Example 2 – subtracting unit fractions:

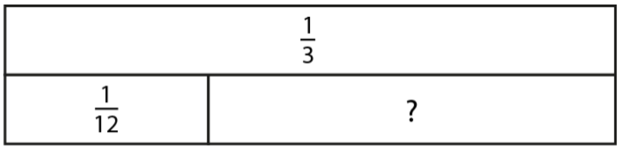
'Complete the fraction subtractions.'

$\frac{1}{8}-\frac{1}{24}=$ $\frac{1}{3}-\frac{1}{12}=$

$\frac{1}{5}-\frac{1}{20}=$ $\frac{1}{6}-\frac{1}{24}=$

Example 3 – writing equations from bar models:

'Write the calculation represented by the bar model.'



Example 4 – subtracting mixed numbers:

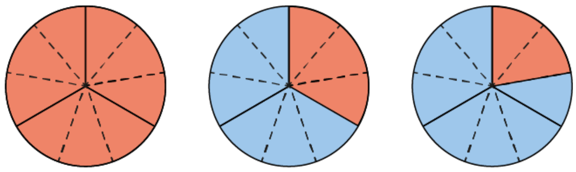
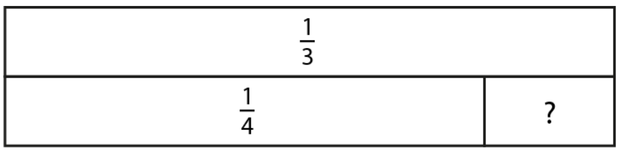
$2\frac{1}{3}-1\frac{1}{9}=$ $6\frac{7}{8}-2\frac{1}{4}=$

$10\frac{2}{5}-3\frac{7}{10}=$ $4\frac{1}{7}-3\frac{13}{14}=$
</td>
</tr>
<tr>
<td>**2:9**</td>
<td>As with adding fractions in *Teaching point 1*, children may progress to subtracting more than one fraction and therefore determining how three different denominators can be converted to a common denominator. As with other segments where they</td>
<td>
$\frac{1}{4}-\frac{1}{8}-\frac{1}{24}=$ $\frac{1}{3}-\frac{1}{4}-\frac{1}{12}=$

$\frac{1}{4}-\frac{1}{5}-\frac{1}{20}=$ $\frac{1}{6}-\frac{1}{18}-\frac{1}{24}=$
</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td>have met subtraction with several terms, these can be calculated by subtraction of the first subtrahend and then of the second, or by adding the two subtrahends and subtracting this total amount from the minuend.</td>
<td></td>
</tr>
<tr>
<td>2:10</td>
<td>By this stage, children have learnt how to add and subtract related unit fractions. Provide further questions, including calculations that combine addition and subtraction, in order to deepen understanding. Include real-life contexts, such as the dòng nǎo jīn problem opposite.

The use of inequalities, such as in the examples opposite, may also provide further depth.</td>
<td>
'Complete the fraction subtractions.'
$\frac{1}{4}+\frac{1}{5}-\frac{1}{20}=$
$\frac{1}{8}-\frac{1}{16}+\frac{1}{4}=$
$\frac{1}{4}-\frac{1}{12}+\frac{1}{3}=$
'Fill in the missing symbols (<, > or =).'
$\frac{1}{3}-\frac{1}{12}$ ◯ $\frac{1}{4}+\frac{1}{12}$
$\frac{1}{5}+\frac{1}{3}$ ◯ $\frac{1}{15}+\frac{1}{5}$

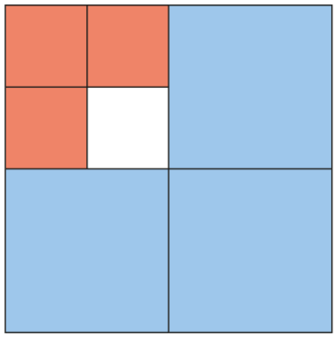
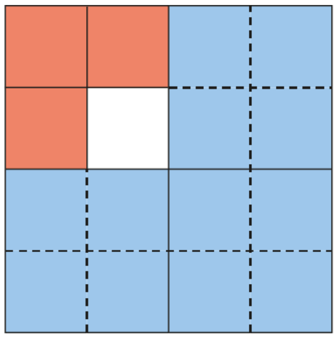
Dòng nǎo jīn:
'Dad makes a chocolate cake. When Sam gets home from school there is $\frac{1}{3}$ of the cake left. Sam eats a piece and there are $\frac{2}{9}$ left. How much has Sam eaten?'

'Find the value of the missing part.'


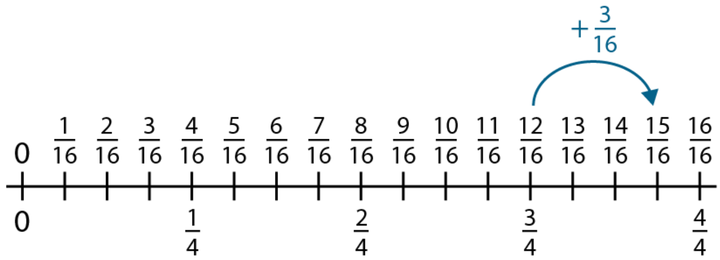
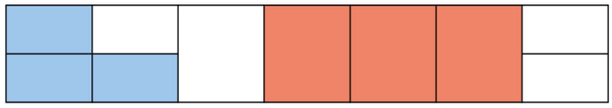
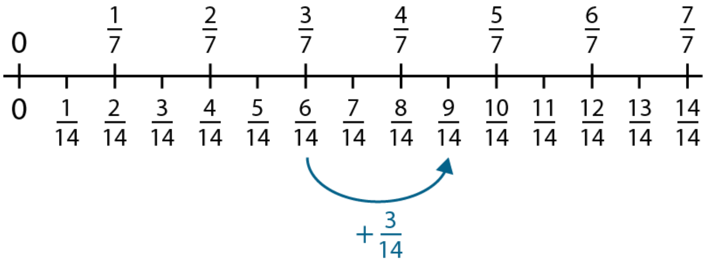
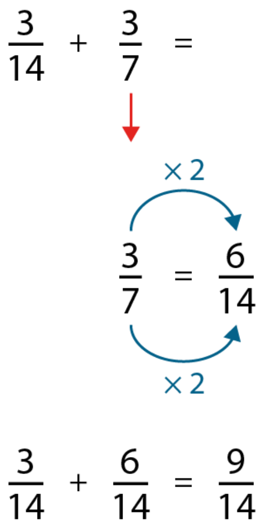
</td>
</tr>
</table>

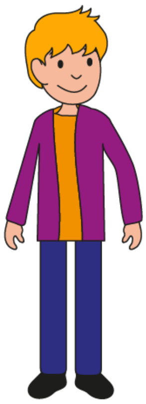
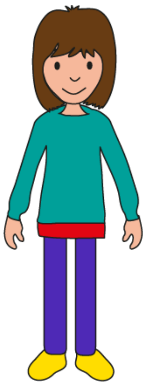
Teaching point 3:

The common denominator method can be extended to adding and subtracting non-unit related fractions.

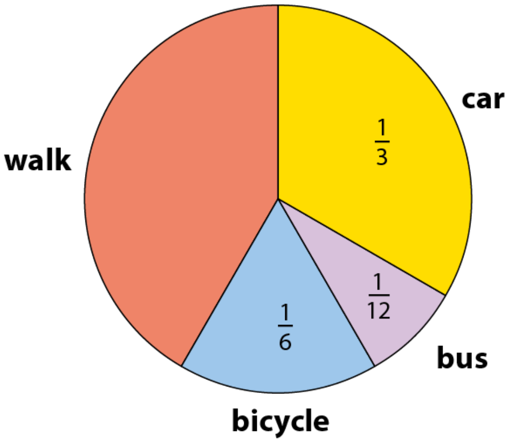
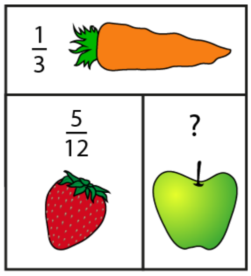
Steps in learning

	Guidance	Representations
3:1	By now, children should be confident in adding and subtracting unit fractions where the denominators are related. They should be able to understand how to identify related fractions, and know that one fraction needs to be changed so that both fractions share a common denominator. They are now ready to apply this learning to adding and subtracting non-unit fractions. *Teaching points 1* and *2* involved small steps towards adding and subtracting related unit fractions and converting to common denominators. Now that you have taken the time to put these foundations in place, it should be relatively easy for children to apply their learning to non-unit fractions, using both addition and subtraction. Begin by showing children a diagram (such as the example opposite) and ask them to identify the addition calculation that is represented. Discuss it together and conclude with children that the calculation represented is $\frac{3}{4}+\frac{3}{16}$. If children struggle to identify the fractions represented, it is important to return to the prompts explored in *Teaching point 1* and ask children: • *'Can you see the three-quarters?'* • *'Can you see the three-sixteenths?'* • *'Do we have equal parts?'* Ask children to identify the common denominator. Which denominator do they need to change to make the	Identifying the fractions:   Rewriting the calculation: $\frac{3}{4} + \frac{3}{16} =$ ×4 $\frac{3}{4} = \frac{12}{16}$ ×4 $\frac{12}{16} + \frac{3}{16} = \frac{15}{16}$

	denominators the same? Agree that both fractions need a denominator of sixteenths. Model how to rewrite the calculation, using the same process as in *Teaching points 1* and *2* and shown again on the previous page.	
3:2	At this point, you may wish to show children the same calculation represented on a number line. Encourage children to notice how $\frac{3}{4}$ and $\frac{12}{16}$ share the same position on the number line.	
3:3	Repeat steps *3:1* and *3:2* with another addition pair of fractions, such as $\frac{3}{14}+\frac{3}{7}$. Show the equation supported by an area model first, and then the related number line.	 
3:4	Work through the written calculation and demonstrate how $\frac{3}{7}$ is rewritten as the equivalent fraction of $\frac{6}{14}$.	
3:5	Ensure children have ample opportunity to apply the learning from *Teaching points 1* and *2* to non-unit fraction calculations for addition and	$\frac{5}{12}+\frac{1}{3}=$ $\frac{3}{10}+\frac{7}{20}=$ $\frac{17}{18}-\frac{2}{9}=$ $\frac{2}{3}-\frac{5}{12}+\frac{3}{4}=$ $\frac{14}{15}-\frac{2}{3}+\frac{2}{5}=$

	subtraction with the aid of area models and number lines. Once they have done this, they should then be ready to rewrite calculations without the support of diagrams. After sufficient guided practice, provide children with independent practice for addition and subtraction of related fractions. Include calculations such as those on the previous page. Notice that the answers to the examples do not cross one whole at this point.	
3:6	Once children have had sufficient practice calculating with related fractions within one whole, progress to calculations that bridge one whole. A good starting point might be to present children with a calculation such as $\frac{3}{4}+\frac{7}{12}$. Show children the workings of Sam and Maisa, as shown opposite, and probe with questions such as: • *'Whose answer is correct?'* • *'What is the same about both answers?'* • *'What is different?'* • *'What has Maisa done to her answer?'* Agree that Maisa has changed her answer from an improper fraction to a mixed number. Note that Maisa's answer can also be simplified further to $1\frac{1}{3}$.	*'Solve this fraction calculation.'* $\frac{3}{4}+\frac{7}{12}$ • *'Sam calculated* $\frac{3}{4}+\frac{7}{12}$ *and got* $\frac{16}{12}$ *as his answer.'* $\frac{3}{4}+\frac{7}{12}=$  $\frac{3}{4}=\frac{9}{12}$ (× 3, × 3) $\frac{9}{12}+\frac{7}{12}=\frac{16}{12}$ • *'Maisa calculated* $\frac{3}{4}+\frac{7}{12}$ *and got* $1\frac{4}{12}$ *as her answer.'* $\frac{3}{4}+\frac{7}{12}$ $\frac{9}{12}+\frac{7}{12}=1\frac{4}{12}$  *'Who is correct, Sam or Maisa?'*

3:7	Provide further practice for addition and subtraction where the answer crosses the whole – first as a class guided session and then independent practice.	$\frac{7}{10}+\frac{5}{20}=$ $\frac{8}{15}+\frac{2}{3}=$ $\frac{5}{7}+\frac{16}{21}=$ $\frac{7}{18}+\frac{5}{6}=$ $\frac{4}{5}+\frac{2}{3}-\frac{2}{15}=$ $\frac{3}{4}-\frac{2}{3}+\frac{11}{12}=$
3:8	Finally, offer varied practice for children to apply their learning. Children should be given opportunities to deepen their understanding by: • solving problems with missing digits (Notice how the questions opposite vary the position of the missing digit from the first to the second term and also from numerator to denominator.) • solving problems including inequality statements • applying addition and subtraction of related fractions to real-life contexts. (Note: in these contexts, children will need to be able to understand the value of the whole as one, which they have previously learnt, can also be written as a fraction with the same numerator and denominator.) • completing linear number sequences and finding mid-points.	Missing-number problems: *'Fill in the missing numbers.'* $\frac{9}{10}+\frac{\square}{5}=1\frac{3}{10}$ $\frac{9}{12}+\frac{\square}{3}=1\frac{5}{12}$ $\frac{6}{\square}+\frac{5}{21}=1\frac{2}{21}$ $\frac{3}{4}+\frac{7}{\square}=1\frac{3}{16}$ $2=\frac{7}{20}+\frac{\square}{\square}+\frac{4}{5}$ Comparisons: • *'Fill in the missing symbols (<, > or =).'* $\frac{3}{5}+\frac{2}{15}\bigcirc\frac{2}{3}$ $\frac{8}{9}\bigcirc\frac{17}{18}-\frac{1}{3}$ $\frac{2}{3}-\frac{1}{4}\bigcirc\frac{5}{12}+\frac{1}{3}$ • *'Fill in the missing numbers. Can you find more than one answer?'* $\frac{5}{7}>\frac{\square}{21}-\frac{3}{7}$ $\frac{\square}{3}>\frac{2}{15}+\frac{1}{5}$ $2<\frac{3}{5}+\frac{19}{20}+\frac{\square}{4}$

		Real-life contexts:
		• *'Class 5 conducted a survey to find out the different ways children travel to school. Here is a pie chart of their findings.'*
		
		
		  • *'What fraction of the children walk to school?'*
		
		• *'Gino wraps some birthday presents. He has a length of red ribbon and cuts it into pieces to decorate the four parcels. He uses:'*
		  • *'$\frac{1}{5}$ of the ribbon to decorate the first present'*
		  • *'$\frac{4}{15}$ to decorate the second present'*
		  • *'$\frac{1}{3}$ to decorate the third present.'*
		  • *'How much ribbon does Gino have left to decorate the fourth present?'*
		
		• *'Farmer Nishi grows fruit and vegetables on her farm. She grows the following:'*
		
		
		  • *'What fraction of the farm is used to grow apples?'*

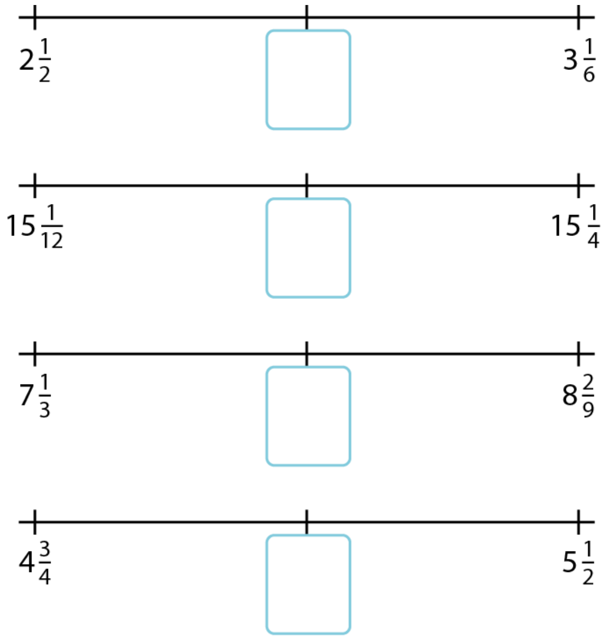
Example 4 – completing sequences:

- *'Complete these linear number sequences.'*

$\frac{1}{2}$	$1\frac{1}{4}$			$3\frac{1}{2}$	

$5\frac{1}{3}$	$4\frac{8}{9}$				$3\frac{1}{9}$	$2\frac{2}{3}$

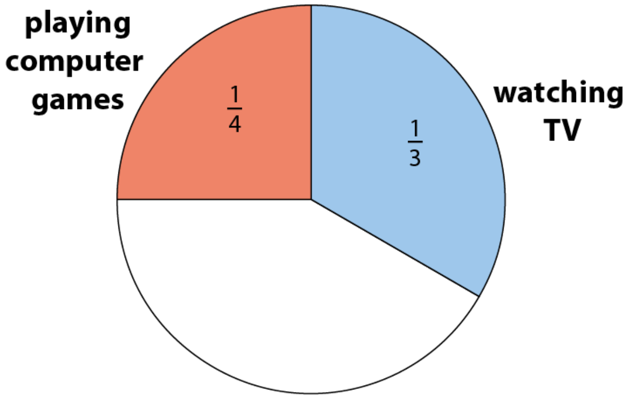
- *'Find the mid-points of these pairs of numbers.'*



Teaching point 4:

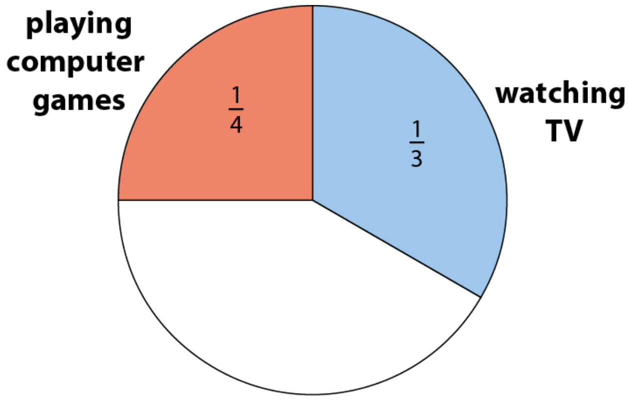
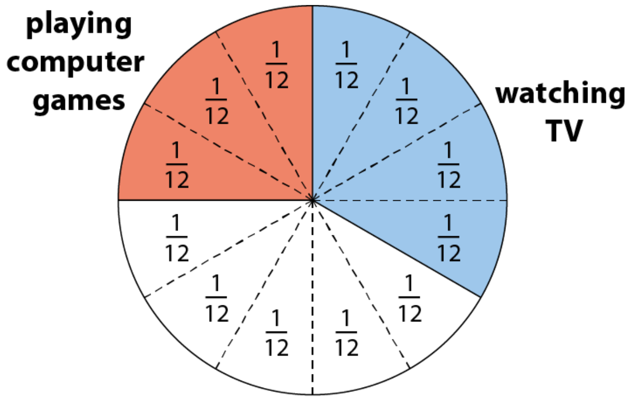
To add and subtract *non-related* fractions, the product of the two denominators provides a common denominator.

Steps in learning

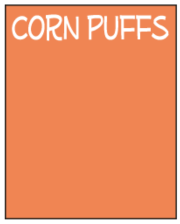
	Guidance	**Representations**
4:1	In this teaching point, children will learn how to recognise fractions where the calculation requires changing both denominators to a common denominator. This teaching point utilises key learning around factors from *Spine 2: Multiplication and Division*, segment *2.21*. Children will learn how to find a common denominator that may be derived from multiplying the denominators. However, it is important they use their proportional reasoning and developing fraction sense to discuss whether the answers are plausible. You may wish to begin by sharing the following scenario opposite with children. Ask *'How might we solve this?'* and allow children time to discuss and take suggestions. Then ask: *'Can you write a calculation to find the total time Will spent playing computer games and watching TV?'* Discuss this until you reach the calculation $\frac{1}{4}+\frac{1}{3}$.	*'Will has made a pie chart to show how he spends his free time in a week. In total, what fraction of the week does Will spend playing computer games and watching TV?'*  $\frac{1}{4}+\frac{1}{3}$
4:2	Extend the scenario by explaining that Will added the fractions and found the following answer: $\frac{1}{4}+\frac{1}{3}=\frac{1}{7}$ ✗ Ask children: • *'What do you think of Will's answer?'* Allow time for them to discuss their ideas. It is important that they consider the plausibility of Will's answer. • *'Can you convince me that Will's answer is wrong?'*	*'Will added the fractions and found the following answer.'* $\frac{1}{4}+\frac{1}{3}=\frac{1}{7}$ • *'What do you think of Will's answer?'* • *'Can you convince me that Will's answer is wrong?'* • *'What would a reasonable answer be?'*

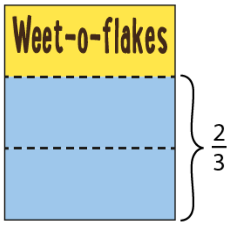
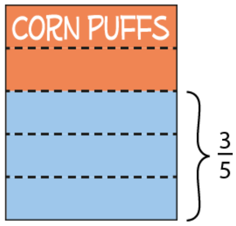
	Children should be encouraged to exhibit their developing fraction sense. In segment *3.2 Unit fractions: identifying, representing and comparing,* children learnt to compare unit fractions. Facilitate the discussion here and encourage them to reason that one quarter is bigger than $\frac{1}{7}$ and $\frac{1}{3}$ is bigger than $\frac{1}{7}$, so the total must be more than $\frac{1}{7}$. • *'What would a reasonable answer be?'* Children could use the pie chart to reason that the total is more than $\frac{1}{2}$.	
4:3	Further extend children's reasoning skills by presenting them with expressions along the lines of those opposite. Ask children to discuss each example and decide which is larger.	*'Fill in the missing symbols (<, > or =).'* $\frac{1}{4}+\frac{1}{3}$ ◯ $\frac{1}{3}+\frac{1}{3}$ $\frac{1}{4}+\frac{1}{3}$ ◯ $\frac{2}{3}$ $\frac{1}{4}+\frac{1}{3}$ ◯ $\frac{1}{4}+\frac{1}{4}$ $\frac{1}{4}+\frac{1}{3}$ ◯ $\frac{2}{4}$
4:4	Once children have had time to really consider the examples and apply their fraction sense to the numbers given, they should be ready to find a common denominator. To start with, you may wish to pose a question such as, *'Are these related fractions?'* Explain that no, they are not, as neither denominator is a multiple of the other. These fractions are called '*non-related'* fractions. Explain that we cannot convert the third to quarters, nor can we convert the quarter to thirds. Therefore, it is necessary to identify a new denominator that is common to both fractions. Look at the fractions in the example opposite. Look for a denominator that is a multiple of four	• *'Are these related fractions?'* $\frac{1}{4}+\frac{1}{3}$ • *'Look for a denominator that is a multiple of four <u>and</u> a multiple of three.'* $\frac{1}{4}=\frac{\square}{\square}$ $\frac{1}{3}=\frac{\square}{\square}$

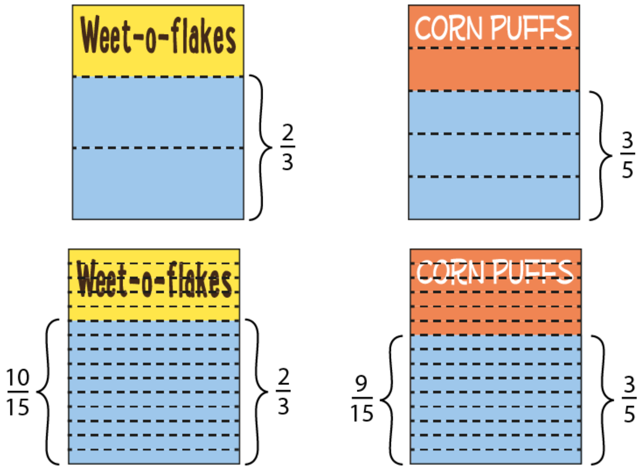
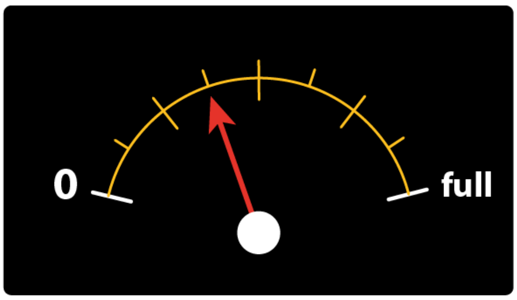
	and a multiple of three. The children have met common multiples in *Spine 2: Multiplication and Division*, segment *2:21*, and should be able to identify twelve as an appropriate common multiple. At this stage, they can work this out by jotting down the factors of each. We don't yet need to identify that a common denominator can be found by multiplying the denominators of the fractions being added, we will progress to that.	
4:5	When twelfths has been identified as a common multiple, model the steps to rewrite the calculation in the same way the related-fraction calculations were rewritten. Children should be fluent in their understanding of the horizontal (and indeed the vertical) relationships between the numerators and denominators in equivalent fractions from their previous work in this segment, and in segment *3.7 Finding equivalent fractions and simplifying fractions*. Think back to the children's original observation from the pie chart that the answer would be a little bit bigger than one-half. *'Is this the case if the answer is $\frac{7}{12}$?'* Yes, it is. The work all the way through this spine on developing fraction sense should have equipped children to see this.	$\frac{1}{4} + \frac{1}{3} =$ $\frac{1}{4} = \frac{3}{12}$ (×3) $\quad \frac{1}{3} = \frac{4}{12}$ (×4) $\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$

4:6	Look back at the pie chart from step *4:1*, comparing it with the same pie chart split into twelfths. Spend some time discussing the pie charts, the equations, and the connections between them.	  $\frac{1}{4} + \frac{1}{3} =$ $\times 3$: $\frac{1}{4} = \frac{3}{12}$ $\times 4$: $\frac{1}{3} = \frac{4}{12}$ $\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$
4:7	Provide a different context for adding non-related fractions. In the example provided opposite, the total is greater than one. Start by asking the children some 'fraction sense' questions about the context: • *'Does the family eat more or less than one-half of a box of Weet-o-flakes?'* • *'What about Corn Puffs?'* The work on estimation that the	*'We buy two equal-sized boxes of cereal. In one week, my family eats $\frac{2}{3}$ of the box of Weet-o-flakes and $\frac{3}{5}$ of the box of Corn Puffs for breakfast. How many boxes of cereal do we eat in total in one week?'*

<table>
<tr>
<td></td>
<td>
children have done should support them with this, but if necessary, prompt them to use this stem sentence to help them imagine how much has been eaten: 'The whole is divided into ___ equal parts, and we have eaten ___ of them.'
Using the picture for support, identify that more than half of both boxes has been eaten, so in total more than one whole box of cereal has been eaten.
Ask children to write down the calculation they will need to do to find out the total that has been eaten.
</td>
<td>

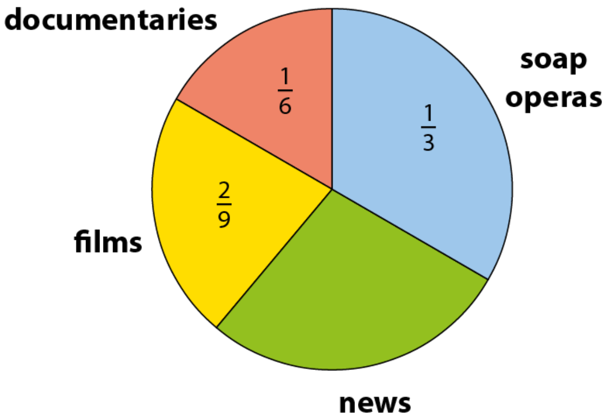
<table>
<tr>
<td>
</td>
<td>
</td>
</tr>
<tr>
<td>'The whole is divided into three equal parts, and we have eaten two of them.'</td>
<td>'The whole is divided into five equal parts, and we have eaten three of them.'</td>
</tr>
<tr>
<td colspan="2">$\frac{2}{3}+\frac{3}{5}$</td>
</tr>
</table>
</td>
</tr>
<tr>
<td>4:8</td>
<td>
Next, ask children questions such as 'Are these related fractions?' No, they are not as neither denominator is a multiple of the other. They are non-related fractions. The thirds cannot be converted to fifths, nor can the fifths be converted to thirds; therefore, a new denominator is needed that is common to both fractions.
Display the fractions given opposite. Explain that they are looking for a denominator that is a multiple of three and a multiple of five. Identify that fifteen is a multiple of both these numbers – again it is fine for children to write out the multiples of each number at this stage.
</td>
<td>
'Are these related fractions?'
$\frac{2}{3}+\frac{3}{5}$
'Look for a denominator that is a multiple of three <u>and</u> a multiple of five.'
$\frac{2}{3}=\frac{\square}{\square}$ $\frac{3}{5}=\frac{\square}{\square}$
</td>
</tr>
</table>

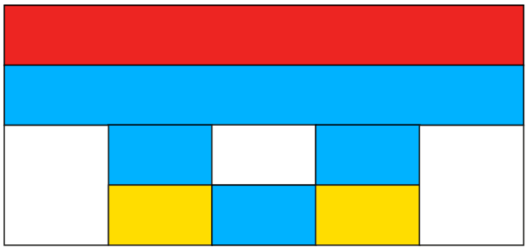
4:9	As with the previous examples, work through this example, showing how to convert both fractions to an equivalent fraction with a denominator of 15, and then add them together. The children have already identified that the total is greater than one, so the improper fraction can be converted to a mixed number.	$\frac{2}{3} + \frac{3}{5} =$ $\frac{2}{3} = \frac{10}{15}$ (× 5) $\quad \frac{3}{5} = \frac{9}{15}$ (× 3) $\frac{10}{15} + \frac{9}{15} = \frac{19}{15}$ $= 1\frac{4}{15}$
4:10	Return to the original image of the cereal boxes and now include images of them marked in fifteenths. Reflect on how converting to a common denominator allows us to add these two fractions. Pose the following statement to the children: *'You can't add fractions with different denominators.'* Ask them to discuss this statement with a partner, and explain whether they agree or disagree with it and why. Build on children's responses until you have identified as a class that they *can* add fractions with different denominators, but they *need to convert to a common denominator* in order to do this.	
4:11	At this point, you could present a subtraction example and again work through the steps that have been outlined above and summarised here: **1.** Identify the calculation. (here: $\frac{3}{8} - \frac{1}{5}$) **2.** Identify the common denominator. (Here it is 40.) **3.** Complete the calculation. (Be sure to include converting the	*'I have $\frac{3}{8}$ of a tank of petrol in my car. I use $\frac{1}{5}$ of a tank driving to see my friend. What fraction of a tank of petrol remains?'* 

	two fractions so that they have a common denominator.) Once the examples have been worked through and discussed, present and examine the following generalisation: ***'To add or subtract fractions with different denominators, first convert to fractions with a common denominator.'***	$\frac{3}{8} - \frac{1}{5}$ $\frac{3}{8} = \frac{\square}{\square}$ $\frac{1}{5} = \frac{\square}{\square}$ $\frac{3}{8} - \frac{1}{5} =$ $\times 5$ $\times 8$ $\frac{3}{8} = \frac{15}{40}$ $\frac{1}{5} = \frac{8}{40}$ $\times 5$ $\times 8$ $\frac{15}{40} - \frac{8}{40} = \frac{7}{40}$
4:12	Look back at the three pairs of fractions that the children have converted to common denominators during the course of this teaching point. Look specifically for patterns within the denominators. The children will notice that the common denominator in these examples is the product of the two original denominators. Discuss why multiplying the two denominators would give us a common denominator. • $4 \times 3 = 12$ • $3 \times 5 = 15$ • $8 \times 5 = 40$ When we multiply two numbers together the product is, by necessity, a multiple of both of them. It has already been determined that our common denominator needs to be a number that is a multiple of both original denominators to allow us to find equivalent fractions. Discuss this thoroughly and come to the generalisation that: ***'We can find a common denominator for two***	$\frac{1}{4} = \frac{3}{12}$ $\frac{1}{3} = \frac{4}{12}$ $\frac{2}{3} = \frac{10}{15}$ $\frac{3}{5} = \frac{9}{15}$ $\frac{3}{8} = \frac{15}{40}$ $\frac{1}{5} = \frac{8}{40}$

	non-related fractions by multiplying their denominators.' (Note that this 'multiplying the denominators' method will also produce a common denominator for related fractions. For example, to calculate $\frac{1}{5}+\frac{1}{10}$, it is possible to convert to a common denominator of fiftieths ($\frac{10}{50}+\frac{5}{50}$). However, in this example it is much simpler to use the denominator of tenths which is the *lowest common multiple*. It is also worth being aware that there are three types of fractions for which children will need to find a common denominator. Methods for the first two have been taught: • related fractions (e.g. $\frac{1}{5}$ and $\frac{1}{10}$) • non-related fractions (e.g. $\frac{2}{3}$ and $\frac{3}{5}$) The third category hasn't been covered explicitly. These are fractions where the denominators share a common factor, but one is not a multiple of the other (e.g. $\frac{1}{6}$ and $\frac{1}{9}$). Here, the lowest common multiple is 18 (rather than 54). At secondary school, children will learn a universal method to find a common multiple for any two numbers, including numbers such as six and nine. At primary level, it is suggested children use the 'non-related fractions' method of finding a common denominator by multiplying the two denominators. However, some children will notice that a smaller common denominator sometimes exists. When they raise this, acknowledge that this is sometimes the case, and if they can see a smaller common denominator, then they can and should use it.)	

4:13	Children now need to practise adding and subtracting non-related fractions, identifying the common denominator each time. Initially, it will be important that you select fractions where the lowest common denominator is the product of the denominators. Start with calculations involving unit fractions. Progress to exercises with non-unit fractions. Where children are calculating with three or more fractions and need to find a common denominator, the most accessible method at primary level is probably to list the multiples of each number and look for a common multiple.	Unit fractions: *'Solve the fraction equations.'* $\frac{1}{5}+\frac{1}{3}=$ $\quad$ $\frac{1}{4}-\frac{1}{5}=$ $\frac{1}{3}+\frac{1}{7}=$ $\quad$ $\frac{1}{3}-\frac{1}{8}=$ $\frac{1}{10}+\frac{1}{3}=$ $\quad$ $\frac{1}{4}-\frac{1}{11}=$ Non-unit fractions: *'Solve the fraction equations.'* $\frac{3}{8}+\frac{2}{3}=$ $\quad$ $\frac{4}{5}-\frac{2}{3}=$ $\frac{9}{10}-\frac{2}{3}=$ $\quad$ $\frac{5}{11}+\frac{1}{4}=$ $\frac{3}{5}+\frac{3}{10}-\frac{1}{3}=$
4:14	Finally, provide varied practice to deepen children's understanding, including: • missing-number problems, including calculations where the answer could be converted to a mixed number • missing-symbol problems, which could include questions where children decide whether to solve by reasoning or by calculating • word problems in a real-life context. You can explore these concepts further using dòng nǎo jīn problems like the ones provided.	Missing-number problems: *'Fill in the missing numbers.'* $\frac{2}{3}+\frac{\square}{5}=\frac{13}{15}$ $\quad$ $1\frac{13}{24}=\frac{7}{8}+\frac{2}{\square}$ $\frac{2}{\square}+\frac{2}{4}=\frac{14}{12}$ $\quad$ $\frac{3}{\square}+\frac{3}{5}=1\frac{\square}{20}$ $1\frac{1}{20}=\frac{3}{5}+\frac{\square}{10}+\frac{1}{4}$ Missing-symbol problems: *'Place <, > or = in the circles to make these inequality statements correct. Tick which ones you can solve using reasoning alone and explain how you know.'* $\frac{2}{7}+\frac{2}{3}$ ◯ $\frac{2}{3}+\frac{2}{8}$ $\frac{2}{3}+\frac{4}{5}$ ◯ $\frac{3}{4}+\frac{1}{3}$ $\frac{2}{3}-\frac{6}{10}$ ◯ $\frac{6}{8}-\frac{2}{3}$

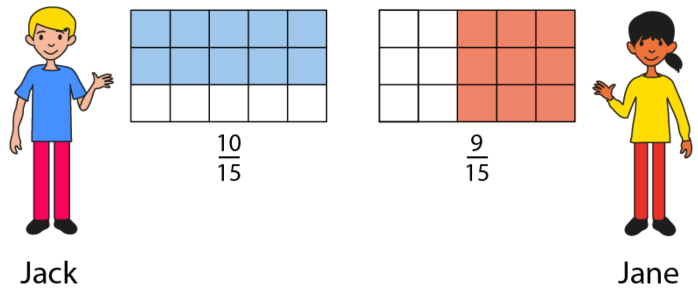
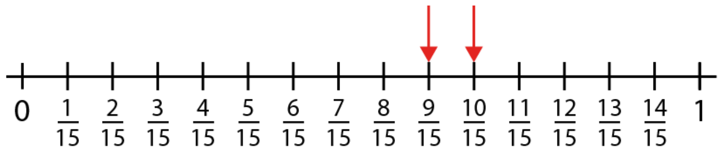
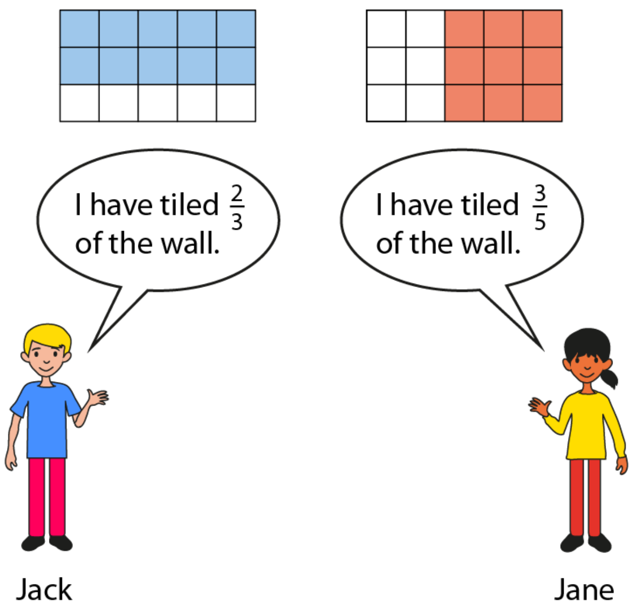
		Word problems:
		• *'Daisy buys a length of ribbon. She uses $\frac{2}{5}$ of the length to make a hair ribbon and $\frac{1}{3}$ of the ribbon to make a bracelet. How much of the length of ribbon does Daisy use in total?'*
		• *'The pie chart shows how much of the evening viewing time is spent on different types of programmes.'*
		  • soap operas: $\frac{1}{3}$
		  • documentaries: $\frac{1}{6}$
		  • films: $\frac{2}{9}$
		
		  • *'What fraction of the viewing time was spent on news?'*

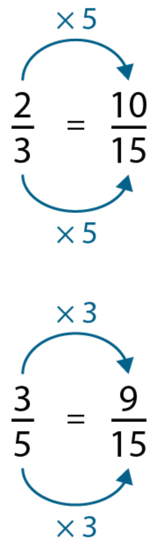
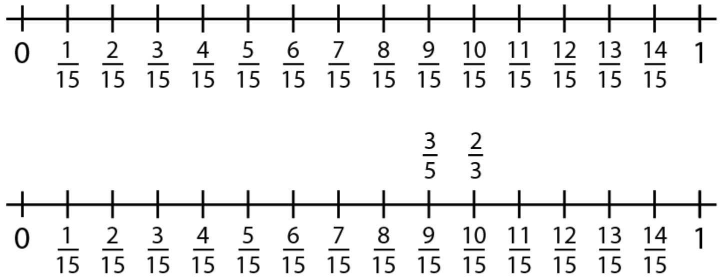
		Dòng nǎo jīn: *'Harry is tiling his bathroom wall. He tiles:'* • '$\frac{2}{5}$ *of the wall with blue tiles.'* • '$\frac{1}{4}$ *of the wall with red tiles.'* • '$\frac{1}{10}$ *of the wall with yellow tiles.'* • *'What fraction of the wall does Harry tile with white tiles?'*  • *'How could you solve this calculation <u>without</u> using fifty-fourths as a common denominator?'* $\frac{1}{6}+\frac{1}{9}$

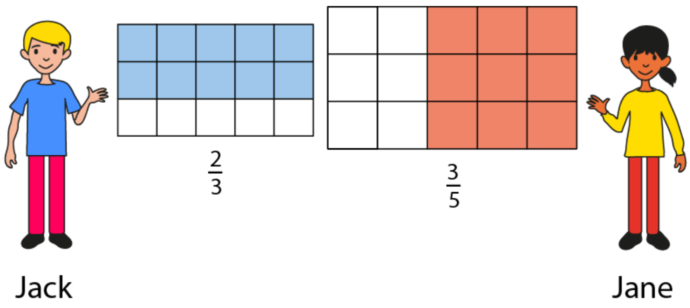
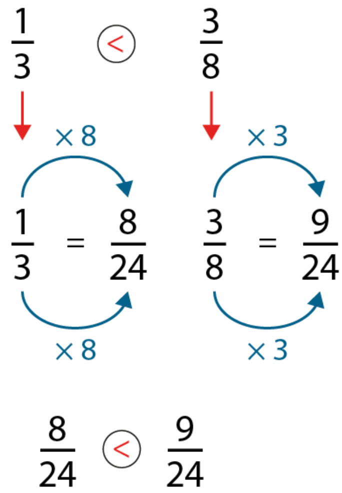
Teaching point 5:

Converting to common denominators is one of several methods that can be used to compare fractions.

Steps in learning

	Guidance	Representations
5:1	Once children have had extensive practice in finding equivalent fractions and finding common denominators, progress to looking at how this can also be used to support the comparison of fractions. You may find it easiest to begin by providing a contextualised story, supported by representations, such as the example shown opposite. Children might respond saying that ten tiles is more than nine tiles, so therefore Jack has tiled more. Try to encourage children to reason using the language of the fraction of the wall that has been tiled, rather than simply the number of tiles.	*'Jack and Jane are tiling identical-sized walls. Jack has tiled $\frac{10}{15}$ of his wall and Jane has tiled $\frac{9}{15}$ of her wall. Who has tiled the larger area?'*   • *'Jack has tiled more, because he has tiled $\frac{10}{15}$ of the wall, and Jane has only tiled $\frac{9}{15}$ of the wall. $\frac{10}{15}$ is greater than $\frac{9}{15}$.'*
5:2	Reveal the example opposite and refer to the speech bubbles which show Jack's claim that he has tiled $\frac{2}{3}$ of the wall and Jane's claim that she has tiled $\frac{3}{5}$ of the wall. Ask *'Are Jack and Jane's claims correct?'* Children can discuss this with a partner and then explain their reasoning to the class. Explanations that the children might give to prove their claims include: • Referring to the part and whole: On Jack's wall the whole is divided into three equal parts (the rows) and Jack has tiled two of them. On Jane's wall the whole is divided into five	

	equal parts (the columns) and Jane has tiled three of them. • Using equivalent fractions: Agree that $\frac{2}{3}$ and $\frac{10}{15}$ are two ways of describing the same fraction; both the numerator and the denominator have been scaled-up by five. Agree that $\frac{3}{5}$ and $\frac{10}{15}$ are also two ways of describing the same fraction; both the numerator and the denominator have been scaled-up by three. Both Jack and Jane's claims are correct.	
5:3	Ask children which they found easier to compare, $\frac{10}{15}$ and $\frac{9}{15}$, or $\frac{2}{3}$ and $\frac{3}{5}$? Conclude that fractions are much easier to compare when the denominators are the same. This is returning to key learning from *segment 3.3 Non-unit fractions: identifying, representing and comparing, Teaching point 7* where they learnt the generalisation: ***'When we compare fractions with the same denominator, the greater the numerator, the greater the fraction.'*** Explain to children that just as it is possible to add or subtract fractions with different denominators by converting to a common denominator, the same approach can be used here, converting to a common denominator to compare fractions. Return to the original pair of fractions and ask again, *'So, which is larger, two-thirds or three-fifths?'* Conclude this step by asking the children to position $\frac{2}{3}$ and $\frac{3}{5}$ on number lines. This will continue to emphasise that fractions are numbers, and that the numbers $\frac{2}{3}$ and $\frac{10}{15}$ have exactly the same value, as	• *'Which is easier to compare?'* $\frac{10}{15}$ ◯ $\frac{9}{15}$ $\frac{2}{3}$ ◯ $\frac{3}{5}$ • *'Place the following numbers on the number line.'* $\frac{10}{15}$ $\frac{9}{15}$ $\frac{2}{3}$ $\frac{3}{5}$ 

	do the numbers $\frac{3}{5}$ and $\frac{9}{15}$.	
5:4	Return to the two images opposite. What if the walls that Jack and Jane were tiling were different sizes? Who has tiled the larger share now? Although the *area* that Jack has tiled is now smaller than Jane's, he has still tiled a larger *share* (proportion) of his wall than Jane has of hers. This is a challenging concept for children, but is the essence of proportional reasoning. Fractions are about the relationship *between* the whole and the part: what proportion of the whole is the part? $\frac{2}{3}$ is greater than $\frac{3}{5}$, so Jack has tiled a greater proportion of his wall than Jane has of hers.	
5:5	Take a new fractions pair such as $\frac{1}{3}$ and $\frac{3}{8}$ and model how to use this method to compare them. • One denominator is 3, and the other is 8. Multiply these together to get a common denominator: 24. • Convert both fractions to fractions with a denominator of 24. Once we have a common denominator it is easy to compare the two fractions: $\frac{8}{24} < \frac{9}{24}$ so $\frac{1}{3} < \frac{3}{8}$. Make sure that you also look at a related pair of fractions, for example $\frac{1}{6}$ and $\frac{3}{12}$. As for addition and subtraction of fractions, where the fractions are related only one of them needs to be converted. Here, 12 is a common denominator, and converting $\frac{1}{6}$ to $\frac{2}{12}$ shows that $\frac{1}{6} < \frac{3}{12}$.	

5:6

At this point, children should apply this method to compare other pairs of fractions. Focus on selecting pairs of fractions where the pairs cannot easily be compared using reasoning. $\frac{1}{4}$ and $\frac{1}{3}$ for example, could be compared using our knowledge of unit fractions, so it wouldn't be necessary to convert them to a common denominator.

Some suggested pairs to compare are provided opposite. Ensure children return to the original pair of fractions when comparing each time, and do not simply compare the converted pair.

- *'Fill in the missing symbols (<, > or =). Convert each pair to a common denominator.'*

$\frac{5}{7}$ ◯ $\frac{2}{3}$	$\frac{6}{10}$ ◯ $\frac{3}{5}$
$\frac{7}{9}$ ◯ $\frac{3}{4}$	$\frac{5}{7}$ ◯ $\frac{6}{8}$
$\frac{2}{3}$ ◯ $\frac{7}{10}$	$\frac{2}{6}$ ◯ $\frac{3}{9}$
$\frac{3}{11}$ ◯ $\frac{1}{3}$	$\frac{1}{5}$ ◯ $\frac{2}{11}$

- *'Order these fractions from largest to smallest.'*

  $\frac{2}{5}$ $\frac{3}{10}$ $\frac{1}{4}$

5:7

Although the common-denominator method will allow children to compare any two fractions, it is often not necessary to use this method. It can be easy to jump to using common denominators to compare fractions without considering whether they can be compared using reasoning. It is important to teach children that there is more than one way in which fractions can be compared. Throughout this spine, the use of proportional reasoning has been interwoven in order for children to develop 'fraction sense' and this is drawn on now.

Look at the inequality statement $\frac{1}{3} < \frac{3}{4}$. One way to compare these fractions is to convert to a common denominator, in this case twelfths. Ask children to look carefully at the numbers $\frac{1}{3}$ and $\frac{3}{4}$. They should discuss with a partner what other ways that they can think of to convince you that $\frac{1}{3}$ is less than $\frac{3}{4}$. There are various answers they could give, which include:

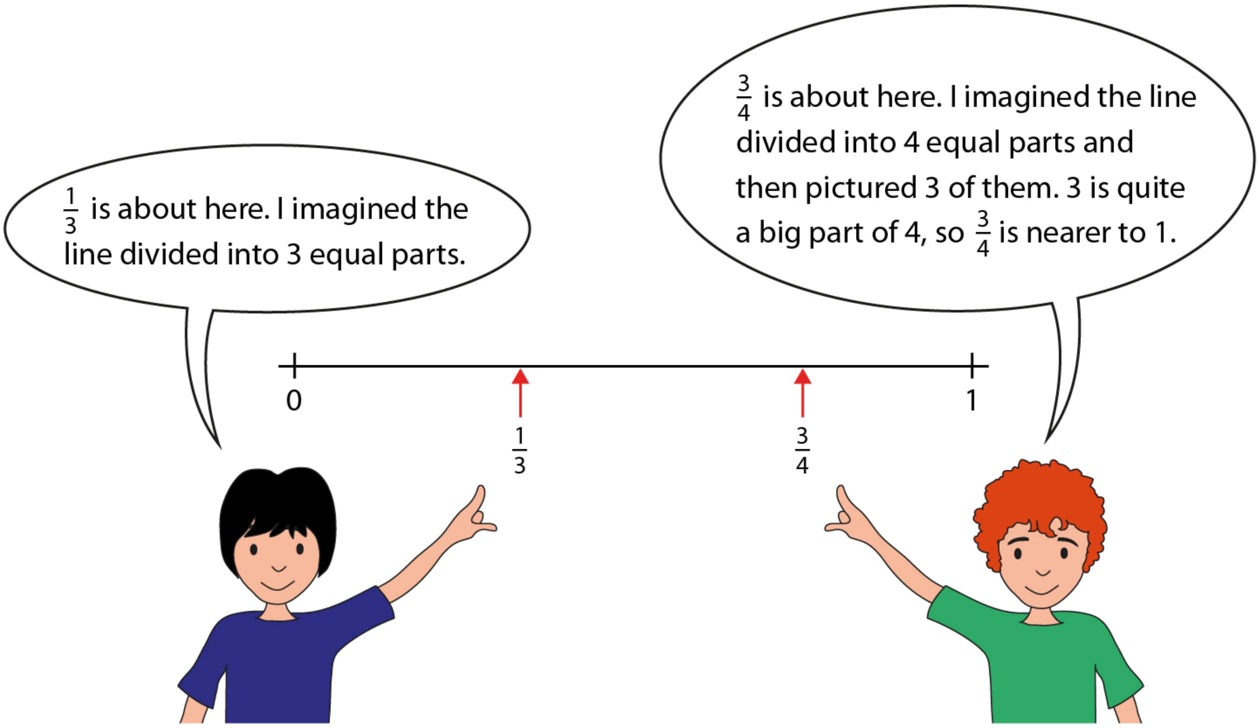
- reference to a benchmark fraction, e.g. $\frac{1}{2}$; $\frac{1}{3}$ is less than $\frac{1}{2}$ and $\frac{3}{4}$ is more than $\frac{1}{2}$, so $\frac{1}{3}$ must be less than $\frac{3}{4}$
- thinking about their position on a number line; $\frac{1}{3}$ is nearer to zero and $\frac{3}{4}$ is nearer to one
- thinking about the size of the numerator in relation to the denominator; three is a bigger part of four than one is of three, so $\frac{3}{4}$ is greater than $\frac{1}{3}$
- drawing a bar model; where children use area models like the bar model to compare fractions, emphasise that they need to have the same whole (this has been covered in previous segments).

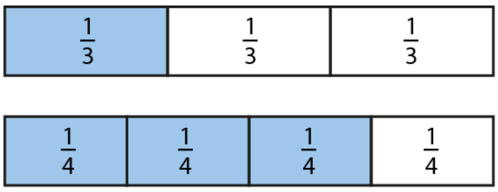
Strongly encourage the children to start to visualise these numbers on a number line. Look at an unmarked 0–1 number line and estimate the rough position of both numbers. By now, children should have moved beyond needing to literally divide the number line into four equal parts and count up three of them to position $\frac{3}{4}$. (If children need support with estimating the position of fractions on unmarked number lines, refer back to the pouring work from earlier segments, such as segment *3.7 Finding equivalent fractions and simplifying fractions*, as a bridge into this.)

$$\frac{1}{3} < \frac{3}{4}$$

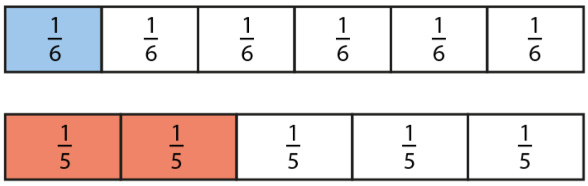
$$\downarrow \qquad \downarrow$$

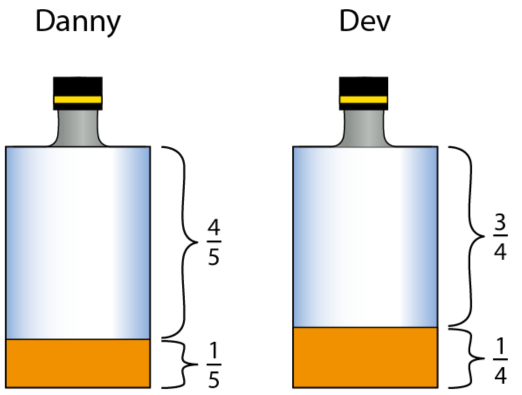
$$\frac{4}{12} < \frac{9}{12}$$





5:8	Ask children if both converting to equivalent fractions and using reasoning were useful strategies here. • *'Did converting to a common denominator help us compare?'* • *'Did using reasoning by drawing and placing on a number line help us compare?'* • *'Which strategy is best here?'* At this point, it is important for children to understand that sometimes, using reasoning may be a more efficient method for comparing.	
5:9	Be aware that in step *5:7*, some children may have mistakenly concluded that because the numerals are bigger in $\frac{3}{4}$ than in $\frac{1}{3}$, this must mean that $\frac{3}{4}$ is greater than $\frac{1}{3}$. This reasoning does not hold up. As has been explored several times through this spine, it is the *multiplicative relationship between the numerator and denominator* that determines the value of a fraction, rather than the absolute size of the numerator and denominator. Ask children to compare another pair of fractions which will prompt further discussion of this, such as $\frac{5}{12}$ and $\frac{2}{3}$. These fractions have been chosen because both the numerator and denominator are bigger numbers in $\frac{5}{12}$ than in $\frac{2}{3}$ but $\frac{5}{12}$ is smaller than $\frac{2}{3}$. Use similar points of reasoning to those in step *5:7* to prove this (it can also be proven by converting to a common denominator).	$\frac{5}{12}$ ◯ $\frac{2}{3}$

5:10	In segment *3.3 Non-unit fractions: identifying, representing and comparing, Teaching point 8*, as well as comparing fractions with the with the same denominator, children learnt how to compare fractions with the same numerator. Look at the three groups of fractions opposite, and discuss how they could be ordered. To help with this, you may choose to refer back to this generalisation from segment *3.3*: ***'If the numerators are the same, then the bigger the denominator, the smaller the fraction.'*** Make sure that children can still explain why that is the case: the more parts that a whole is divided into, the smaller each part will be. And if it is known that, for example, $\frac{1}{11}$ is smaller than $\frac{1}{8}$, then it follows that $\frac{3}{11}$ is smaller than $\frac{3}{8}$.	*'Order each set of fractions from largest to smallest.'* $\frac{10}{8}$ $\frac{7}{8}$ $\frac{5}{8}$ $\frac{3}{8}$ $\frac{8}{8}$ $\frac{4}{8}$ $\frac{2}{8}$ $\frac{1}{6}$ $\frac{1}{5}$ $\frac{1}{8}$ $\frac{1}{7}$ $\frac{1}{10}$ $\frac{1}{9}$ $\frac{3}{3}$ $\frac{3}{8}$ $\frac{3}{11}$ $\frac{3}{100}$ $\frac{3}{5}$ $\frac{3}{2}$
5:11	Reinforce this point by taking another pair of fractions for children to compare, such as $\frac{2}{5}$ and $\frac{1}{6}$. Challenge children to compare these fractions using reasoning alone, allowing them time to work in pairs and consider how they are going to justify their response. Listen to the children's ideas and extract that $\frac{1}{5}$ is greater than $\frac{1}{6}$ and therefore $\frac{2}{5}$ is greater than $\frac{1}{6}$. Use the diagram opposite to support the discussion.	

5:12	This *'same-numerator'* method can also be used to compare fractions which are 'one part less than a whole'. Look at the example opposite and discuss with children how they could approach trying to solve it. Common denominators are an obvious option, but it is also possible to compare $\frac{4}{5}$ and $\frac{3}{4}$ by using reasoning. One way to do this is to prompt the children to think about how much drink is left and compare unit fractions of fifths and quarters. This reasoning can be applied to other fraction pairs that are one part less than the whole, for example $\frac{7}{8}$ and $\frac{8}{9}$.	*'Danny and Dev had identical bottles of juice. Danny drank $\frac{4}{5}$ of his juice. Dev drank $\frac{3}{4}$ of his juice. Who drank more of their juice?'*  • *'If Danny drank $\frac{4}{5}$ of his juice, then he has $\frac{1}{5}$ left.'* • *'If Dev drank $\frac{3}{4}$ of his juice, then he has $\frac{1}{4}$ left.'* • *'$\frac{1}{5}$ is less than $\frac{1}{4}$, therefore Danny has less juice left. That means he drank more.'* $\frac{1}{5} < \frac{1}{4}$ so $\frac{4}{5} > \frac{3}{4}$
5:13	Provide children with the opportunity to sort pairs of fractions into two categories: pairs to compare using conversion to a common denominator, and pairs to compare using reasoning. There are no set rules for sorting these. For example, $\frac{4}{10}$ and $\frac{5}{12}$ is one that many children might use the common denominator method for. Others might recognise that $\frac{4}{10}$ is $\frac{1}{10}$ less than $\frac{1}{2}$, and $\frac{5}{12}$ is $\frac{1}{12}$ less than $\frac{1}{2}$. This therefore means that $\frac{4}{10}$ is less than $\frac{5}{12}$. Other fractions, like $\frac{1}{10}$ and $\frac{7}{8}$ are much easier to compare by reasoning.	

Whereas $\frac{3}{4}$ and $\frac{7}{9}$ might be a pair best suited to a common denominator approach.

Ensure there is ample time for extensive discussion around the sorting of the fraction pairs, as this will deepen children's understanding of comparing fractions. This work develops learning from previous segments, in particular segments:

- *3.2 Unit fractions: identifying, representing and comparing*
- *3.3 Non-unit fractions: identifying, representing and comparing*
- *3.7 Finding equivalent fractions and simplifying fractions.*

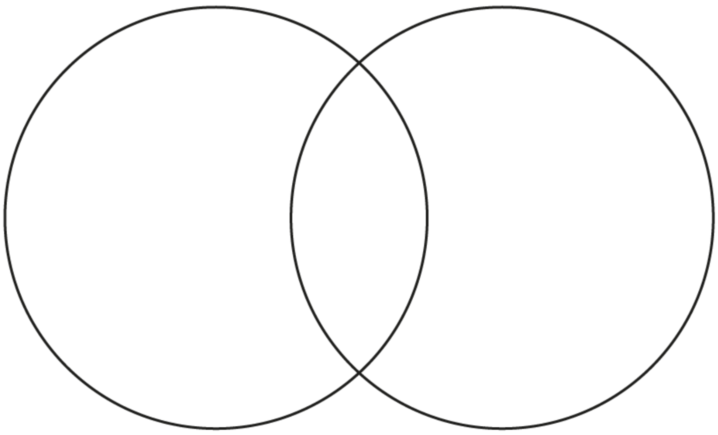
Draw supporting diagrams as needed, including number lines and bar models, to help the children access some of the more sophisticated reasoning.

Sorting:

'Sort the pair of equations according to the method you would use to compare them.'

$\frac{7}{8}$ and $\frac{5}{6}$ $\quad$ $\frac{1}{3}$ and $\frac{1}{4}$ $\quad$ $\frac{5}{11}$ and $\frac{3}{5}$ $\quad$ $\frac{1}{10}$ and $\frac{7}{8}$

$\frac{4}{10}$ and $\frac{5}{12}$ $\quad$ $\frac{2}{5}$ and $\frac{3}{8}$ $\quad$ $\frac{6}{9}$ and $\frac{6}{10}$ $\quad$ $\frac{3}{4}$ and $\frac{7}{9}$

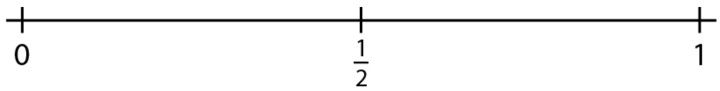


5:14 Spend some time with children exploring pairs of fractions and creating a 'toolkit' to focus their reasoning around how to compare them. The following points and examples may be helpful.

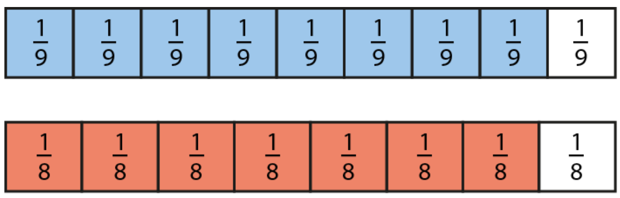
- Consider where the fraction sits on a number line. Is it more or less than $\frac{1}{2}$? *(Example 1)*
- Compare the numerators to the denominators. Consider:
  - *'How much of the whole is this?'*
  - *'Is it the large part or the small part?'*
- If both fractions are one part less than the whole, then it is possible to work backwards. For example, when comparing $\frac{8}{9}$ and $\frac{7}{8}$, discuss that $\frac{8}{9}$ is $\frac{1}{9}$ less than the whole and $\frac{7}{8}$ is $\frac{1}{8}$ less than the whole. We also know

Example 1:

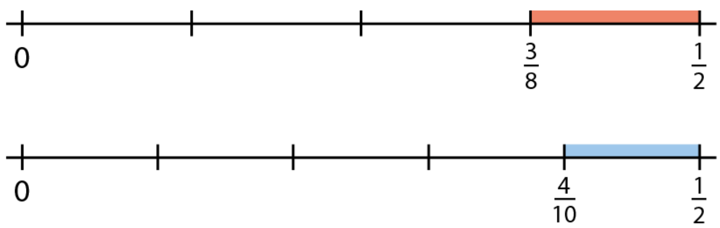
'Which side of $\frac{1}{2}$ does the fraction sit?'



Example 2:



Example 3:



	that $\frac{1}{9}$ is less than $\frac{1}{8}$, therefore $\frac{8}{9}$ is greater than $\frac{7}{8}$. Use *Example 2* to support children's reasoning. • Use $\frac{1}{2}$ as a benchmark to compare how far away from $\frac{1}{2}$ each fraction is. For instance, $\frac{3}{8}$ and $\frac{4}{10}$ (*Example 3*). • Visualise what each fraction looks like compared to the whole. At several points through the spine, children have made judgements about the relative size of numerators to denominators. Ask: *'Is it quite a big part of the whole or quite a small part of the whole?'* • Look for common denominators or common numerators. For instance, if comparing $\frac{3}{4}$ and $\frac{5}{8}$, emphasise how $\frac{3}{4}$ is equivalent to $\frac{6}{8}$, so $\frac{3}{4}$ is greater than $\frac{5}{8}$. • Consider whether the fraction is a proper fraction or an improper fraction. Is it less than, equal to, or greater than one?	
5:15	Provide varied practice for children in comparing and ordering fractions using reasoning. You could include questions that cover a range of question types and styles. Some examples are provided opposite: • missing-symbol problems similar to those in step *5:9* • real-life contextual problems • number sequences that challenge children to compare and order fractions based on their relative sizes • dòng nǎo jīn questions to consolidate and deepen children's learning, focusing on:	Missing-symbol problems *'Fill in the missing symbols (<, > or =).'* $\frac{5}{6}$ ◯ $\frac{4}{7}$ $\frac{8}{9}$ ◯ $\frac{7}{11}$ Real-life contextual problems *'Sabijah and Will are in a running race. Sabijah has run $\frac{9}{10}$ of the race. Will has run $\frac{8}{9}$ of the race. Who is further ahead?'*

- ordering groups of fractions which may require more than one type of reasoning (for example, with the first dòng nǎo jīn question opposite, use the common denominator to identify that $\frac{5}{16} < \frac{7}{16}$, and similarly, by using the common numerator, identify that $\frac{5}{18} < \frac{5}{16}$; the order must therefore be $\frac{5}{18} < \frac{5}{16} < \frac{7}{16}$)
- missing-number problems involving identifying and ordering missing fractions.

Sequencing:

'For each sequence, tick the correct box to show if the fractions in each pattern are increasing, decreasing or neither.'

	Increasing	Decreasing	Neither
$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$	□	□	□
$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$	□	□	□
$\frac{3}{2}$ $\frac{4}{3}$ $\frac{5}{4}$ $\frac{6}{5}$	□	□	□
$\frac{6}{13}$ $\frac{7}{12}$ $\frac{8}{11}$ $\frac{9}{12}$	□	□	□

Dòng nǎo jīn:

- *'Order each set of fractions using the < symbol.'*

  $\frac{7}{16}$ $\frac{5}{18}$ $\frac{5}{16}$

  $\frac{3}{7}$ $\frac{2}{9}$ $\frac{3}{6}$

  $\frac{2}{7}$ $\frac{1}{9}$ $\frac{4}{5}$

  $\frac{4}{10}$ $\frac{2}{12}$ $\frac{5}{9}$

  $\frac{2}{5}$ $\frac{2}{6}$ $\frac{3}{4}$

- *'Think of a number that can go in each box so that the fractions are arranged in size order.'*

  $\frac{1}{3}$ $\frac{\square}{5}$ $\frac{1}{\square}$ $\frac{4}{7}$ $\frac{3}{\square}$

- *'Use these number cards to make two different fractions that complete the inequality statement.'*

  1 2 3 4 5 6

  $\frac{1}{2} < \frac{\square}{\square} < \frac{\square}{\square} < 1$