Using Numicon to teach division as grouping using informal written methods, with a group of lower achieving Year 5 pupils.

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**Summary**

This project is a critical inquiry into the use of Numicon, to examine whether the resource assists children to feel secure, practically, in their understanding of division and to observe whether they are more likely to take risks to improve their learning focus. The study explores whether students can overcome misconceptions about grouping, using informal written methods. Numicon uses a series of structured shapes to represent numbers, encouraging children to build up a mental image to support their understanding.

A group of under achieving pupils in Year 5 were assessed on their understanding of division. Their results suggest that they view the concept of division as being primarily about sharing. It revealed they needed to develop an integrated meaning for division – to understand division as sharing as well as grouping, repeated subtraction, ratio and the inverse of multiplication.

Throughout my case study, I refer to theories of learning and explore the notion put forward by Utall et al (2009: 156) that children acquire much of their mathematical knowledge by manipulating concrete objects; and that children must be able to connect concrete to more symbolic representations as they advance through school.

The importance of physical apparatus is often over looked in Key Stage 2, with many teachers of the opinion that such resources are unnecessary. This research
contends that, in contrast, the more challenging a concept such as division becomes, the more understanding is needed and to achieve this, more and not less resources need to be used.

The findings suggest that the social interaction between pairs, groups and between teacher and pupil is of critical importance. The children showed greater confidence through using Numicon, their performance improved, and they demonstrated a more positive attitude to mathematics. All names and places have been changed to maintain anonymity.

**Introduction & Rationale**

The study took place in an inner city primary school. Most pupils are from minority ethnic backgrounds and almost two thirds of the pupils speak English as an additional language. The school was graded “good” by Ofsted in 2008.

A closer examination of the most recent pupil tracking data (2010-11) showed that in Year 5, 17% of children were under-attaining (below level 3). Of these, 10 pupils were significantly underachieving and their attainment and rate of progress was cause for concern. During the one year period from Year 3 summer term 2009 to Year 4 summer term 2010, 4 of the 10 children had made only one sub-level of progress. These 4 pupils (2 boys and 2 girls, Child A, B, C & D) were amongst the lowest attaining pupils in the cohort with level 2b and upon this basis, I selected these students to form the focus of my case study.

At my school, amongst the low achieving pupils, division has been considered a weak area of the curriculum. Children often only consider division to be sharing and fail to appreciate that the concept covers grouping, repeated subtraction, ratio and the inverse of multiplication (Frobisher et al, 1999). Anghileri (2001) argues that division is more complex than addition and subtraction, because it requires knowledge of multiplication facts, the ability to estimate, as well as being able to use addition and subtraction within the solution.

My intention was to determine what prior knowledge or misconceptions my Year 5 group held about division and consider how I could help these students to develop an understanding of division using informal written methods. For the children to
understand a mathematical concept, I was mindful of Haylock and Cockburn’s (2008) assertion that they would need to build up a complex network of connections between language, symbols, concrete materials and pictures.

Numicon had been introduced to Key Stage 1 in September 2009, as part of the School Development Plan. This resource had been a success in helping Year 2 children catch up with their peers and senior management decided to extend the use of Numicon throughout the school, from September 2010. Feedback from KS1 supported the notion that it was not so fiddly that it distracted the children but was interesting enough to keep them engaged and support the development of visualisation. As one TA I spoke to commented, “It’s a really motivational, tactile resource.” As part of this initiative, Numicon was introduced to the whole staff on a school training day in October 2010.

As a result of the training I could see that Numicon had the potential to provide a system of using small numbers to help transfer the children’s learning to another context and thereby raise the children’s self esteem. The shapes (as shown in figures 1 & 2) represent number in a series of structured patterns, and the apparatus lends itself to multi-sensory teaching, integrating the visual, auditory and kinaesthetic modalities (Dunn et al, 2010). According to Dunn et al (2010), Numicon can help children see relationships between numbers and to see numbers as objects, not just the end of a count.

*Figures 1 & 2: Numicon Plates – 1-10*
The social constructivist model of learning influenced my study, where children are seen as active learners who seek to make sense of their world and try to build on what they know and believe when presented with new mathematical experiences (Dunn et al, 2010). Bruner’s (1977) constructivist model is particularly relevant in specifying 3 developmental stages: enactive (handling physical objects), iconic (mental imagery) and symbolic.

Bruner (1977) argues that learning through the three modes of representation is more meaningful when learners are autonomous and take responsibility for their own learning. Discovery learning, confirms my original analysis that children’s understanding develops when they can exchange viewpoints and think critically about their own ideas in relation to other people’s views (Dunn et al, 2010). This necessitates a critical role for language, where drawing on the work of Vygotsky, Pritchard (2008) argues that the interaction between the learner and the teacher is crucial in the process of intellectual development. My decision to use Numicon, was influenced by Delaney (2010: 82) who observes that a resource which “facilitates demonstration and interaction mediates discussion in powerful ways”. It offers the possibility for teachers to support their task with appropriate actions and in turn, to watch and listen to pupils interacting with the same resource, revealing their inherent
understanding of important concepts as well as any potential misconceptions (Delaney, 2010).

Some learning theories imply that pupils develop expertise by starting with a task and breaking it down into a sequence of smaller and manageable tasks (Wood, 1988). This study notes that careful scaffolding is required, with problem solving and action to be the foundation, so that the child becomes a genuine participator, not a spectator (Bloomfield, 1998). By using Numicon, I was mindful that research cited by Moyer (2001) shows that pupils who use concrete apparatus outperform pupils who do not use them. My decision to use Numicon, was guided by noting how research into the dual representation aspect of manipulatives means that careful consideration has to be given to the type of manipulative used (Uttal et al, 2009). The relatively simple design of the Numicon plates offers the opportunity for reflection and abstraction to be facilitated, allowing the children to focus and reflect on their actions (Uttal et al, 2009).

Prior to preparing my division unit, to identify misconceptions, my students completed a pre-assessment. This revealed that they all viewed division as sharing but some had the ability to use halving to find fractions of amounts. Child C was only able to draw the division symbol and failed to complete any of the example questions. The pupil’s confidence when answering division questions was low, with 100% grading themselves 3 out of 4, where 1 is the most confident. Their feelings about division revealed they were unsure, with Child B speaking for the group when she said, “I try but I find it hard.” Taking into account Thompson’s view (2010), my goal became to plan for the children to have a deeper understanding of division, by using grouping (repeated subtraction). Whilst I was seeking for the children to be able to develop a sufficient level of confidence with division, so that they became aware of patterns of divisibility, for example, finding a ¼ by halving and halving again.

My primary intention within this study was to explore the use of Numicon, to overcome misconceptions about the grouping aspect of division. My secondary intention was to promote a learning environment that encouraged students to feel
secure in their understanding, take risks with their learning and form meaning from their activities.

**Methods**

The school training day highlighted an important qualifier; that for Numicon to be effective, older children would need to be competent with foundation activities. This meant allowing the children initially in the autumn term, to develop their own meaning for the number shapes, through free exploration. To imbed this understanding, I made time for my class to undertake activities, such as placing the plates 1-10 to represent the number line and looking for the pattern of odd and even numbers. Later, I used the interactive Numicon number line, to model problems such as having to share the cost of a restaurant bill - £36, between 4 people. These ‘How many in...?’ real life activities, began to show the children the links between Numicon and division.

Evidence for this study was gathered with the group of 4 children, over a period of 3 weeks, during the main class teaching sessions. This comprised teacher observational notes, the completion of a range of division questions from the Abacus Evolve Framework, the children’s written recordings, a post study pupil questionnaire and interview analysis.

**Session 1**

The first session focused on the children applying tests of divisibility. Using the Numicon number line on the interactive whiteboard, children were asked questions such as ‘Can 165 be divided by 2?’ Child B observed that this was not possible, because 165 is an odd number (Appendix B: 2). When I asked the child to explain, B collected sixty and five from the Numicon plates and said, “I know this because 5 is an odd number.” This activity showed that the group could divide by 2, using partitioning and their knowledge of odd and even number patterns, using Numicon. The session confirmed the importance of language, showing that to deepen the children’s understanding, the teacher has the role of stimulating dialogue and maintaining its momentum (Pritchard, 2008).
**Session 2**

This lesson focused on moving the children on, to have a secure understanding of dividing by 4. The children were asked to investigate with 3 digit numbers, if the last 2 digits were divisible by 4. Child A correctly found that of the six numbers (Appendix C), only 676 is divisible by 4. When asked to show if 431 is divisible by 4, the child took 3 tens, a 1 plate and arranged plates of 4 over the top, to see how many groups would fit into 31. When I asked Child A if he found it helpful to use Numicon, he replied, “I enjoy it Sir, I can see what’s left over.” The fact that 2 of the children struggled with these questions, until Child A’s demonstration showed that ‘manipulatives are not magic’ and for them to be used effectively, children must know the materials well enough to use them automatically (Moyer, 2001: 176).

**Sessions 3 & 4**

To build on their knowledge of divisibility, these sessions involved dividing tens and units by units, using informal written methods. This began by setting the learning in a meaningful context by using a whole class book reading example (Pritchard, 2008). This introduced the children to chunking, using repeated subtraction for the first time. Initially, when I asked the group to check this answer using Numicon, they stared blankly and were clearly unsure. However, once I reminded them to find the total first by using the plates, Child B then said, “Oh I now need to take away 40, using groups of 4.” Although the group were clearly worried by having to use subtraction, once they saw that they could take away a number of groups in one go and check visually, their anxiety began to fade.

The children used Numicon to help estimate the cost of telephone calls, according to how many groups there were. When asked to divide 67 by 3, Child A was unsure, until Child B said, “Hang on, we know that 10 x 3 is 30.” Child B then proceeded to group 3 lots of 10 plates together and then repeat this, leaving 7. After practice, the children’s group work revealed that Numicon works in this context by visually breaking the problem down into smaller parts, which facilitates grouping, using the children’s existing knowledge, allowing the group to estimate and check their
answers. By this stage, it was also becoming clear that Numicon was helping to foster a rich mathematical discussion, encouraging effective peer collaboration.

Session 5:

The children had grown in confidence, they were now able to divide hundreds, tens and units, by units. They were able to verbalise the questions and it was becoming clear that by using Numicon, the group were beginning to participate more readily in discussions within the whole class setting (Dunn et al, 2010). The evidence shows that the children were learning to develop a tally of the multiples involved, as their jottings showed estimation and a ‘running commentary’ (Thompson, 2010). By now, the group was beginning to develop an integrated meaning of division, with the linkages to multiplication and subtraction becoming apparent.

The post study questionnaire conducted a week later was completed 4 weeks after the pre-test, without using Numicon and contained the same questions and an extended interview session. This revealed that the children now realised that division, as well as representing sharing, can involve halving, subtraction and multiplication. The children were increasingly confident, using informal jottings and visual images in their calculations, suggesting they were able to form meaning in their own interpretations.

The pupil’s confidence had risen considerably, using a scale of 1-4, where 1 is very confident, 2 graded themselves as very confident and 2 quite confident. Their feelings about division are now marked by self-belief, with 100% smiley faces. When asked if they use their times tables knowledge, Child B said, “It helps a lot. I really understand, I use different methods.” Whilst Child C, who on the pre-test had been unable to answer several questions, revealed that, “I use my times tables and Numicon to help.”

Conclusions and Recommendations

The intention of this research unit was to critically explore the use of Numicon, to overcome students’ misconceptions about the grouping aspect of division and to
observe any improvement in children’s confidence with division. Clearly, the children made good progress in their learning, as evidenced by session 5 and the post-test questionnaire. Their learning was following along a pathway from the concrete stage, as suggested by Bruner (1977), with structured imagery helping to connect children’s different mathematical experiences. However, it is difficult to determine the effect that the manipulative alone had on this success. Since each activity involved collaborative work scaffolded by teacher intervention, precise language and then explicit sharing of strategies, one cannot claim that the use of Numicon alone was effective in developing this understanding. However, it can be stated that, by using Numicon, in addition to the scaffolding and mathematical language provided by the teacher and collaborative support from peers, substantial improvement was made by all pupils in both confidence and learning.

This research contends that when using a manipulative, the role of the teacher is crucial in initiating the construction of a set of obligations and expectations (Bloomfield, 1998). When the children had the chance to undertake familiarisation activities, as in this study, this helped to develop trust, enthusiasm and persistence which facilitated the pupils taking risks in their learning. The social interaction between the teacher and the children is of paramount importance. The Williams Review (2008: 245) makes a strong case for working in groups and this study shows that when children used Numicon, they “learned best” when they actively constructed their own understanding, with the freedom to make mistakes which could be corrected in a supportive group setting. Pritchard (2008) argues that the discussion between pairs, groups and between teacher and pupil is ‘essential for the effective development of understanding’.

In summary, there is a strong case for extending the time that children are given for experiencing a variety of division activities covering all the meanings of division before they are introduced to formal symbolic representation (Frobisher et al, 1999). This study has shown that the more challenging a concept becomes, the more understanding is needed, and to achieve this in Key Stage 2, more physical apparatus should be used. Numicon is a powerful resource which can be effective, particularly with under-achieving pupils. For this potential to be realised, it needs to be imbedded across the school and in particular, in Years 3 and 4. Teachers need
to make the time to experiment with resources and consider how these meet the needs of all their learners. After all, as in all teaching, the success depends on how the resources are used, not just on the resources themselves (Dunn et al, 2010).
References


http://O-lib.mylibrary.com.library.edgehill.ac.uk [accessed 4 April 2011]


(Examples of pupil's work can be made available to illustrate this article)