



Hungary for Calculation

By David Dowling of Luton Borough Council

Abstract/Summary

In Hungarian primary schools the foundations of mathematics are established through manipulating objects and shapes to prepare for mathematical work later on. There is a critical emphasis on the use of accurate mathematical vocabulary from an early age. The spatial awareness and visual imagery that children develop underpins their ideas about number and arithmetic.

We aimed to use this Hungarian methodology as inspiration when developing our approaches to calculation within the new National Curriculum. We aimed to work collaboratively to plan lessons and activities for each year group that model progression in calculation. As a result we were able to create a shared approach to calculation and develop and share activities and resources.

The teachers involved in the project demonstrated a growing passion of teaching for understanding. They became more confident in not just using manipulatives, visual imagery and key vocabulary, problems and challenges, but also more analytical in evaluating the impact and effectiveness of the resources and activities that they used.

The lessons trialled were characterised by increased engagement, enjoyment and understanding – particularly with low attainers and those who were usually anxious about or disaffected with maths. Pupils were becoming confident and independent about choosing tools, methods and approaches.

Background

A number of the teachers involved have been or are currently involved with the MaST programme and the Hungarian approach fits in well with those explored in the MaST programme. The schools involved were keen to develop an approach to calculation in readiness for the implementation of the new National Curriculum Programmes of Study.

Surrey Street Primary is a school a high level of EAL, deprivation and mobility. It was judged by Ofsted as 'good' in 2011. Currently the school is expanding from a single form entry school to a three form entry school. The percentage of children with little or no English is also increasing which will make raising attainment even more challenging.

Ramridge Primary School is very keen to embrace the new National Curriculum and have started to use this to aid planning from Years 1 to 4. They are concerned about gaps some children will face with the changes in the curriculum and are determined to provide the best maths teaching so these children do not miss out on key elements. Many children come from a low economic background.

Denbigh Primary School was judged as 'Satisfactory' by Ofsted in January 2012. It was originally a Junior school but became a primary school in 2011. Almost all pupils are of minority ethnic heritage. Most pupils speak English as an additional language.

Icknield High School is a school with good levels of attainment and progress at KS4. It recently had a 'Good' judgement from Ofsted. It has a large intake of around 280 pupils each year from a number of primary schools. The school is keen to develop its approaches to calculation in KS3 as part of its numeracy strategy and in order to prepare for the New KS3 Programme of Study.

Icknield Primary is a large school judged as 'good' by Ofsted. It has good levels of attainment. It is a feeder school for Icknield High.

Aims of the Collaborative Teacher Project

We aimed to develop approaches to calculation to support the introduction of the New National Curriculum. We also wanted to develop our own understanding of the use of visual imagery in developing pupils understanding of calculation and number sense. We were keen to develop approaches which could be used to support professional development across our schools.

With the publication of the new KS1 and 2 programmes of study we were fearful that mathematics learning could become very procedural, Our aim was to ensure that pupils could become fluent with procedural approaches but also develop the conceptual understanding and visual imagery that underpins it. This approach can be seen as a challenge to some primary teachers who are not as confident with approaching calculation in this way. We wanted to model how calculation could be developed throughout primary by capturing lesson ideas, resources and activities that modelled the methodology behind the Hungarian approach.

We aimed to create a bank of lesson ideas, resources and planning tools that allowed us to plan coherently for the development of calculation in our schools that could also be shared with others.

Details of those involved in the Collaborative Teacher Project

Lyn Adams, Assistant Headteacher and Lead Applicant at Surrey Street Primary School

Lisa Phee, Year 2 teacher (Joint lead on calculation policy at KS1) at Surrey Street Primary School

Greg Iszchak, Year 5 teacher (Joint lead on calculation policy KS2) at Surrey Street Primary School

Amy McNulty, Year 6 teacher and Numeracy Lead at Denbigh Primary School

Joanne Lunn, Year 6 teacher and Literacy coordinator at Ramridge Primary School

Emma Pearson, Numeracy Lead at Icknield Primary School

Kyle Barry, NQT at Icknield High School

David Dowling, Maths Consultant and Improvement Agent at Luton Borough Council

A description of the Collaborative Teacher Project

We took the following themes from Hungarian approaches to Primary mathematics:

- Manipulating objects
- Developing mental and visual imagery
- Number 'acrobatics' to develop number sense
- The use of accurate mathematical language
- Active engagement in learning and collaborative learning
- Emphasis on the fluency of mental maths
- Development of understanding for calculation

The teachers involved in the project met together for an introductory meeting at which we agreed the principles for good teaching of calculation. We explored the new KS1, KS2 and KS3 programmes of study to landmark what expected progress in calculation is. We developed our understanding of what is expected within each year.

At the first planning session teachers developed three progression grids, one for addition / subtraction, one for multiplication / division and one for place value. This was our starting point in modelling progression and we used these to 'hang' lessons, activities and resources onto. Each

teacher went away to trial out some approaches to some of the concepts in the progression maps and explore what types of resources would support the project further, bringing examples of work and their reflections to the next planning session. Two further planning sessions followed a similar format exploring calculation expectations across year groups and the strands identified.

The first part of the project was focused on trying out ideas and we then meet together for an interim meeting where we shared our experiences so far. At this point teachers began to develop a direction and vision for the project. We split into four pairs, each working in more detail on one of the three strands with one pair focusing on year 5 and 6 across all three strands.

The next three planning sessions involved the plan, teach review cycle, peer observations and team teaching. Impact on learning was collected through examples of pupils work, peer observations and pupil questionnaires.

The final meeting pulled together the wealth of materials that were created and gave teachers time to present their findings to the group.

What has been learned from the project?

Greg Iszchak, Surrey Street Primary

The main thing I have learned from this project is the importance of providing choice. Children need to be able to choose from a range of resources and strategies on a regular basis so that making and evaluating their choices becomes second nature. Children therefore need training in using a range of resources and methods so that their choices can be successful. Generalisation is also key. Children need to be able to evaluate their choices and make statements about the efficiency and suitability of a method for a particular type of problem. Children are far more confident trying new approaches, especially when asked afterwards what's the same and different about a new method. They are more confident making generalisations and critiquing the efficiency of methods.

Amy McNulty, Denbigh Primary

When planning calculation lessons for my class I tried to focus on developing mental and visual imagery, use of accurate mathematical vocabulary and the development of understanding for calculation. Having a top set of year 6 children proved tricky as their calculations were of a high standard, yet I was concerned that their fluency in using written methods was more of an instrumental understanding rather than of a relational one. Therefore, use of the physical resources and representations of multiplication and division, helped to deepen their understanding and make an abstract concept more concrete. When questioning the children I ensured I put a focus on their understanding. I used open ended pedagogical questions such as 'What do you notice?', 'What is the same and what is different?' and 'Prove to me how you know.' Developing my questioning and teaching for understanding have been key learning points for me. Even when children are fluent with calculation procedures, it is still important to engage them with thinking around why and how the methods they use work.

Joanne Lunn, Ramridge Primary

The project has helped me understand that children need a conceptual understanding of maths. It is not enough to set them a set of questions to solve. Children need to question and investigate concepts. By using images and a variety of resources children will engage and discuss more effectively in mathematics. (Using area images to develop understanding of the order operations are carried out and using number lines when adding and subtracting directed numbers). Planning needs to include vocabulary and associated questions that are used to develop understanding. Activities carried out in pairs and small groups are critical in encouraging talk and pupils own questioning.

Kyle Barry Icknield High School

I have focussed on the transition from physical to abstract imagery with the year 7 and 8 groups I teach. As a result, I have instructed students to draw images before reinforcing concepts with a more abstract version. In addition, I have also emphasised the importance of accurate mathematical language and notation / conventions to ensure students improve their fluency across the various strands of numeracy. At this stage in my career, I have found the whole process very beneficial and has helped reinforce the importance of reflection. Through collaborating across the Key Stages, I have had the opportunity to listen and bounce ideas off Primary level experts. As a result, I am more confident in working with low attaining students on

basic numeracy. During the peer observation, I developed my competence at assessing the quality of learning in a lesson and had the unique opportunity to engage with students during an active lesson.

Lynda Adams, Surrey Street Primary

I embarked on this project with the aim of working with colleagues to develop a way of teaching formal algorithms, which have a greater emphasis in the new curriculum, in a way which would continue to develop conceptual understanding. This project reinforced the importance of making connections between a wide range of visual representations, concrete manipulatives and mathematical language. Whilst I now feel more confident in being able to make connections between the informal methods which I had always previously taught and some formal algorithms, I still struggle to make the connections for myself for subtraction (using decomposition rather than using a range of number line based methods) and division (chunking versus short and long division). Personally, the greatest benefit from this project has been working more closely with colleagues from other schools. This is something which I intend to continue after the end of the project.

Lisa Phee, Surrey Street Primary

My understanding is that the approach is multifaceted. Teaching a concept in a variety of ways, using a variety of materials that is led by the pupils. The teacher helps the children to decide how to make the correct choice for efficient methods. The teacher needs to be confident in their approach to teaching concepts and that the method that may be the easiest or most convenient, may not be the best approach for their children. The Hungarian approach allows the children to lead their learning and use a variety of appropriate resources to scaffold the learning and understanding. My next step is to start removing what might be deemed as 'mandatory' resources from less confident children and encouraging them to choose the resources they might need in order to solve a particular problem.

Emma Pearson, Icknield Primary

By using physical models, children have been able to have a greater insight into understanding numbers, patterns and their relationships to one another. Working with children who are significantly off track in year 5, I have been able to represent abstract concepts in a number of ways and support these with visual images which have then enabled the children to have a deeper understanding of written algorithms and abstract methods. By using these resources e.g. diennes, place value manipulatives and arrow cards, children have increased their active engagement within the lesson, worked collaboratively and also developed an acute 'number sense' and deepened their understanding of many concepts. The approach has permitted the use of open questioning throughout the whole lesson so that children are constantly thinking and challenging their own ideas. I developed the use of What if? Show me? What do you notice? What's the same/different? in each lesson and found that it was easier to incorporate these into lessons without having to specifically plan. Hungarian approaches also enabled me to develop cognitive conflict within my lessons where I would ask the children to 'prove something' or give them a misconception as a 'good mistake' and ask them to convince a partner whether this was correct or incorrect. Similarly, the use of the correct mathematical terminology and reasoning was improved. Being paired with a teacher from a secondary phase has allowed me a further insight into the transition process from primary to secondary school. As a result of this, I will be able to tailor teaching, especially in year 6 to ensure that there is continuity in the strategies used and the teaching methods from year 6 to year 7.

Impact on teachers' practice

The teachers involved have begun to change their style of planning, encouraging the use of a variety of resources which allow children the time to explore the resources and methods they use. Teachers are more confident about allowing children to choose the resources they need, and decide for themselves which resource / manipulative is best in a particular case. Resources in the classroom have 'no hidden home' as one teacher put it. They are always out on the tables so that children's options are in front of them. Lessons becoming child led with misconceptions are thoroughly discussed. Building on patterns has become more common place, for example $34+17$; $3.4+1.7$ etc. so that children can start making connections of their own.

The idea of 'number acrobatics' has become an integral part of teachers practice, which promotes the use of known facts in flexible and effective ways using games to develop fluency in basic number facts. The flexibility for children to choose which game to play and with whom, and

what resources they might find helpful has engaged children. The strategy has had a noticeable impact on the instant recall of many of the children in the class, especially the lower achieving children and those with special educational needs.

Developing mental and visual imagery, the use of accurate mathematical vocabulary and the development of understanding for calculation has become more apparent in planning and day to day teaching. There has been a real focus on deepening children's understanding through developing their mental and visual imagery. Teachers have taken greater care with developing fluency in written methods but through more of an instrumental understanding rather than of a relational one. Therefore, use of the physical resources and representations of multiplication and division, helped to deepen understanding and make an abstract concept more concrete.

The use of open ended pedagogical questions such as 'What do you notice?', 'What is the same and what is different?' and 'Prove to me how you know.' has developed, as has reflecting and acting on pupils responses to these questions. Teachers have developed cognitive conflict within lessons where children are asked to 'prove something' or give them a misconception as a 'good mistake' and ask them to convince a partner whether this was correct or incorrect.

Impact on others

Usually disengaged children have become more engaged since using concrete materials. Furthermore, children have the ability to concentrate for longer periods of time; most children were focused for the whole of each session. The children were also more responsive, eager to answer more questions, enthusiastic about their learning and confident enough to solve a problem or show an answer more readily.

As mentioned previously, the lower achieving and SEN children benefited from work on 'number acrobatics'. When the class were surveyed about the change in their numeracy lessons to a more resource-based approach, it was also the lower achieving and SEN who described the greatest positive impact.

Some pupils didn't respond well to the use of the resources or visual models, higher achieving children weren't really motivated when using this approach. They thought they didn't need to use them as they already had an established understanding of multiplication and division. It was interesting though to discover how poor their initial representations for multiplication were.

Those children who were less confident in attempting formal written methods, found the use of physical resources and visual representations useful. Many commented on the fact they didn't know that multiplication, for example, could be represented in many ways. The majority thought that the links shown between the grid method and the expanded column method supported their understanding. It is easy to assume that the children you teach were taught using models and images prior to being taught by you. It is easy to take for granted that children have a secure visual representation of multiplication.

One very high achieving pupil thanked the teacher for 'such a good lesson' (children are often the best judge of how successful the lesson was.) Behaviour tended not to be an issue as all children were on task, often teachers assume that the use of new teaching strategies will result in negative changes to behaviour. Lower attaining pupils made great steps in understanding and the pride those children felt was great to see. This has led to these children being more confident across the board.

The use of correct mathematical language had an immediate impact on the students, as soon as the teacher was using the correct terminology e.g. integer, the children would quickly reciprocate this. Children would continue to use the correct vocabulary, at first when prompted, but then independently in subsequent lessons.

In one school, visits have been made to lessons by a range of staff including foundation stage and year 3 and 4 teachers. These teachers have been able to understand how these stages in learning and resources used can have impact on their year group and consequently have started to use similar methods. Teachers have fed back that their children have quickly grasped concepts

that have previously been difficult to explain. Similar methods of teaching have cascaded through the school and teaching assistants have also started to use manipulatives when working with small groups of children and individuals. In other schools, the dissemination of these strategies is a critical next step.

Advice to teachers who may want to try something similar

Teachers need to have confidence in letting children explore and make their own decisions. Allow them to lead lessons and the methods in many cases and encourage them to evaluate how they solved the problem. Allow them to compare methods and choose which is the most efficient. But giving children choices is more beneficial if children are given the chance to generalise afterwards. If children are not prompted to reflect on the success of their choices, they won't fully appreciate them.

Using a wide range of visual representations and concrete resources certainly develops children's confidence and makes the learning more fun. Children are also always asking to play more games so linking these to 'number acrobatics' is an effective way for children to learn.

Being a part of a collaborative project is something that we would definitely recommend to other teachers. Not only do you have the opportunity to work closely with like-minded people but you also get to take away new approaches (and not just the ones set out by the project). Time allocated to collaborative work in pairs or triads, including observations and team teaching was a very beneficial aspect of the project.

Teachers involved in the project found it useful to talk to other leaders in maths about their concerns with the new curriculum and their strategies to help implement it. It was also reassuring to discover we have similar approaches in certain areas. As a leader, it has also given me the confidence to go back to my school and suggest ways to implement the new curriculum as I know other teachers in my local authority are also taking these routes.

References and resources produced or used

The Hungarian approach was the basis for work by CIMT resulting in the MEP project. <http://www.nationalstemcentre.org.uk/elibrary/collection/410/mathematics-enhancement-programme-mep-primary-extension>

<http://cdn.cfbt.com/~media/cfbtcorporate/files/research/2012/r-enhancing-primary-mathematics-2012.pdf>

A video exploring Hungarian methodology can be found here: <http://www.teachersmedia.co.uk/videos/hungary-primary-maths>

And related research here: <http://www.bsrlm.org.uk/IPs/ip31-1/BSRLM-IP-31-1-04.pdf>

A set of progression maps for calculation has been produced.

A bank of resources and lesson notes has been collated. These use activities from some of the following:

- Success@arithmetic <https://everychildcounts.edgehill.ac.uk/ecc-for-schools/success>
- Improving Learning in Mathematics (Standards Unit, Shell Centre) http://tlp.excellencegateway.org.uk/resource/su_mat_5822/screens/math_004_003_005/page.html
- Various National Strategies Primary strategy resources including Springboard 5
- Nrich website