

# RECME

Researching Effective CPD in  
Mathematics Education

## A perspective on the literature: CPD for teachers of mathematics

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A perspective on the literature: CPD for teachers of mathematics .....	1
1. Introduction .....	1
1.1 Why a review of literature? .....	1
1.2 How did we approach the review? .....	2
1.3 Overview of the review .....	2
2. What do we mean by CPD?.....	3
2.1 Describing the full landscape of formal and informal teacher learning.....	3
2.2 Formal professional development: why?.....	5
3. The complexity of professional knowledge .....	7
3.1 What is professional knowledge, and how is it generated? .....	7
3.2 Cultural and personal knowledge .....	9
3.3 The transfer of knowledge .....	10
4. Professional knowledge for teachers of mathematics .....	11
4.1 Three aspects of mathematical knowledge for teaching .....	11
4.2 Professional development programmes and mathematics knowledge for teaching.....	12
5. Change and Professional Development.....	13
5.1 Sustained change; organising professional development programmes.....	13
5.2 What sort of change is improvement?.....	15
5.3 Evaluating Professional Development and Change .....	16
5.4 How do we evaluate change?.....	17
5.5 Teachers evaluating change in their own classrooms.....	18
6. Important characteristics of 'effective' professional development programmes: synthesising the literature .....	20
6.1 Introduction.....	20
6.2 Context: schools, classrooms, on-site and off-site.....	21
6.3 Content: teacher knowledge, learning and mathematics .....	22
6.4 Process.....	24
7. Conclusions.....	24
7.1 Summary and recommendations.....	26
7.2 Areas that are under-researched .....	29
References.....	30



## A perspective on the literature: CPD for teachers of mathematics

### 1. Introduction

#### 1.1 Why a review of literature?

The National Centre for Excellence in the Teaching of Mathematics (NCETM) was set up to provide effective strategic leadership for mathematics-specific continuing professional development (CPD) with the overall aims of:

- raising the professional status of all those engaged in the teaching of mathematics;
- improving institutional performance, including raising standards, by supporting targeted workforce development in order that the mathematical potential of learners be fully realised.

The National Centre seeks to promote CPD opportunities for teachers that are cumulative and sustained over the career of a teacher. The Centre promotes a varied blend of approaches to effective CPD, taking advantage of the resources and new communication tools made available on the portal.

The NCETM is committed to understanding the nature of effective CPD for teachers of mathematics and has commissioned a large-scale project — Researching Effective CPD in Mathematics Education. The NCETM also supports a range of inquiry-based CPD initiatives.

This review aims to synthesise existing literature in order to inform:

- policy makers who are engaged in specifying programmes of CPD for teachers of mathematics
- practitioners who are engaged in CPD for teachers of mathematics
- researchers who are investigating the characteristics of effective CPD for teachers of mathematics

The NCETM operates through a combination of face-to-face activity led by a team of Regional Coordinators based in different parts of England, and through its online portal communities and interactive professional learning framework.

*'CPD is about personal learning journeys - over time - that are of course about growing as a teacher of mathematics.'*  
(Celia Hoyles, Director of NCETM)

Research on CPD for teachers of mathematics has grown over the years. Working groups on professional development are part of most of the international research conferences in mathematics education.

## 1.2 How did we approach the review?

We began the review with the characterisation of continuing professional development for teachers of mathematics provided by the Advisory Commission on Mathematics Education <sup>1</sup>

'By a Continuing Professional Development (CPD) programme, we mean a sustained development programme: this could comprise different sets of professional development and some training put together so as to progress over time to reflect as teacher's needs. Thus a CPD programme in mathematics typically will continue over years, planned by the teacher in collaboration with a head of department, a Head Teacher or a mathematics co-ordinator or others with expertise, with the aims of enhancing the knowledge, skills and enthusiasm of the teacher' (p 6).

However, from the outset we also aimed to consider the ways in which more informal and serendipitous learning could contribute to continuing professional development.

Research on mathematics teacher education has grown over the last fifteen years and there is now a relatively large body of work in this area. See, for example, Jaworski, et al. 1999; Lin and Cooney 2001 <sup>2</sup>.

We initially considered research on professional development for teachers of mathematics, selecting the research from peer-reviewed journals, international handbooks, edited books, conference proceedings, and policy documentation. As the process of reviewing developed we also considered more general research on professional development, teacher change, professional knowledge, and networked e-learning.

The NCETM supports teachers of mathematics across all the phases from early years to adult education and we aimed to source literature related to each of these phases.

## 1.3 Overview of the review

We start the review in Section 2 by outlining our understanding of what constitutes professional development for teachers in order to set the context for the subsequent sections. The argument developed in Section 2 emphasises the importance of both formal and informal workplace learning. This is followed in Section 3 by a discussion of the way in which professional knowledge is a complex mix of cultural and personal knowledge that is mostly tacit and difficult to communicate to others. In Section 4 we go on to discuss the nature of professional knowledge for teachers of mathematics. It is this knowledge that professional development seeks to change, and in Section 5 the discussion then turns to a consideration of change and growth in mathematical knowledge for teaching. In Section 6 we present a synthesis of what the literature says about the important characteristics of professional development for teachers of mathematics. In Section 7, the concluding section, we summarise the review and suggest areas of CPD that are under-researched.

For example

- [The International Conference for the Psychology of Mathematics Education](#)
- [The International Congress on Mathematics Education](#)
- [The Conference of European Research in Mathematics Education](#)

The review includes the following sections:

- What do we mean by CPD?
- The complexity of professional knowledge;
- Professional knowledge for teachers of mathematics ;
- Change and professional development;
- Important characteristics of 'effective' CPD;
- Summary and recommendations.

1 - ACME 2002

2 - Jaworski, et al. 1999; Lin and Cooney 2001

## 2. What do we mean by CPD?

### 2.1 Describing the full landscape of formal and informal teacher learning

There is increasing awareness that teacher learning takes place, and should take place, in a variety of settings. For example, the UK schools inspectorate, Ofsted, drew attention to the importance of both on-site and off-site learning opportunities for teachers:

'the narrow perception that professional development always involves off-site activity, such as attendance at a course [ ] is gradually being replaced by a wider and more comprehensive view of CPD' (p 11).<sup>3</sup>

The report goes on to suggest that provision of CPD is seen by teachers to include a range of opportunities, such as 'whole school training days, team planning opportunities, joint teaching, peer observation, work shadowing, residential working groups, and local and national conferences and networks' (p 11).

Whereas the learning opportunities listed above are likely to have been planned, there is increasing awareness of the importance of un-planned informal learning:

'Informal learning should no longer be regarded as an inferior form of learning whose main purpose is to act as the precursor of formal learning; it needs to be seen as fundamental, necessary and valuable in its own right, at times directly relevant to employment and at other times not relevant at all'. (Coffield, 2000, p 8)

However understanding the complex nexus of formal and informal experiences that contribute to a teacher's professional development presents a challenge. Importantly, as Eraut's typology of learning in the workplace shows<sup>4</sup> (Figure 1), the majority of learning opportunities take place either as a by-product of the work of the professional or through learning activities located in work; the first two columns provide examples of learning within the workplace, which could be planned or unplanned, explicit or implicit. By contrast the third column provides examples of planned and structured learning processes. This typology suggests that learning within a short course or working for a qualification is likely to be only a small part of the overall learning opportunities available to teachers and that teachers predominantly learn through practice.

The full landscape of teacher learning includes a 'patchwork of opportunities – formal and informal, mandatory and voluntary, serendipitous and planned'

The ways in which these opportunities are woven together varies from teacher to teacher.

*'Knowing about teaching and becoming a teacher evolve and are deeply interwoven in ongoing activity in the practice of teaching. Knowledge about teaching is not acquired in courses about teaching, but in ongoing participation in the teaching community in which such courses may be a part.'* (Adler, 1996, p 2)

3 - Ofsted 2002. This report by Ofsted investigated teachers' continuing professional development (CPD) activities within agendas of school improvement. The inspectors looked at the ways in which these activities were managed and evaluated their effectiveness.

4 - Eraut 2004b

**Figure 1: A typology of workplace learning**

<b>Work processes with learning as a by-product</b>	<b>Learning Activities located within work or learning processes</b>	<b>Learning processes at or near the workplace</b>
Participation in group processes	Asking questions	Being supervised
Working alongside others	Getting information	Being mentored
Consultation	Locating resource people	Being coached
Tackling challenging tasks and roles	Listening and observing	Shadowing
Problem solving	Reflecting	Visiting other sites
Trying things out	Learning from mistakes	Conferences
Consolidating, extending and refining skills	Giving and receiving feedback	Short courses
Working with clients	Use of mediating artefacts	Working for a qualification
		Independent study

In general literature on professional development does not sufficiently take into account the extent or importance of informal workplace learning opportunities (as exemplified in Figure 1), but anecdotal evidence, particularly gained from the working groups we have run at the British Society for Research into Learning Mathematics (BSRLM), suggests that these informal workplace learning opportunities are very much valued by teachers. Informal learning includes:

'conversations with colleagues, passing glimpses of another teacher's classroom on the way to the photocopying machine, tips swapped in the coffee lounge ..... the daily experience of the classroom.' (Wilson and Berne, 1999, p 174).

However, although teachers and teacher educators value informal learning in the workplace, without feedback, critical reflection<sup>5</sup> and structured support it is unlikely that teachers will be able to capitalise on such learning. In addition, as suggested by Doerr (personal e-mail communication), this sort of informal learning can result in

'a hodge-podge of professional development opportunities that end up needing to be integrated into some coherence at the level of the individual teacher, which sometimes happens, but often doesn't ....it's the patchwork quilt of incoherent opportunities that are potentially problematic (in my view) and all too common (at least in USA).'  
(November, 2007)

Thus, while informal learning is clearly important and is discussed in the literature, it seems that frequently it is not fully optimised to address aspects of teacher knowledge and understanding that need to be improved or developed in some way.

We suggest that research on professional development for teachers of mathematics needs to pay more attention to informal learning.

*As Fraser et al<sup>6</sup> point out, 'The nature, extent and role of informal incidental opportunities in teachers' professional learning are currently under-researched and therefore remain unclear.'* (p 166)

5 - For discussion of the importance of critical reflection on informal learning see Day 1999; Eraut 2004a; Eraut 2007a.

6 - Fraser, et al. 2007

## 2.2 Formal professional development: why?

By contrast to the informal learning opportunities which occur in schools as a by-product of working as discussed above, formal professional development opportunities are planned frequently in response to a perceived need to address teachers' knowledge and understanding<sup>7</sup>. These needs sometimes arise from innovations such as 'reform classrooms' in the USA, the National Numeracy Strategy<sup>8</sup> (NNS) in the UK, new examination curricula, and the changing content of curricula such as the introduction of computers into mathematics classrooms.<sup>9</sup> Poor student performance, particularly in international comparisons such as TIMSS<sup>10</sup> and PISA<sup>11</sup> also frequently drives professional development programmes, as does the under-qualification of many teachers of mathematics and numeracy<sup>12</sup>. (In the UK, the under-qualification of teachers of mathematics has been identified in all sectors, for example in the Smith report<sup>13</sup> (secondary), the Williams report<sup>14</sup> (primary and early years) and Lucas, 2007<sup>15</sup> [FE])

Formal, planned professional development should respond to the perceived or actual needs of teachers, students and schools.

Other formal professional development programmes can be motivated by the desire of individuals or departments to extend their academic qualifications or to open up career opportunities.

Formal professional development programmes can be mandatory for teachers, and are frequently delivered by external experts. In the UK, examples include courses run by the National Numeracy Strategy at primary level, courses run by the Key Stage 3 strategy<sup>16</sup> and certification programmes for teaching basic skills to adults. Some sort of accreditation or recognition (for example, an award or certificate) is sometimes attached to attending and completing the course, but this is not always the case.

7 - Jaworski, et al. 1999; Millett and Johnson 2000

8 - The National Numeracy Strategy was introduced by the government in the UK in 1999/2000 to address perceived low numeracy skills in primary schools (for more detail see Brown, et al. 2000).

9 - For example, Artigue 1998; Balacheff 1988; Bottino and Furinghetti 1996.

10 - Stiegler and Hiebert 1997. The Trends in International Mathematics and Science Study (TIMSS) collects educational achievement data at the fourth and eighth grades (Year 5 and Year 9 in the UK) to provide information about trends in performance over time. See <http://www.standards.dfes.gov.uk/primary/features/primary/timss/>

11 - Adams and Wu 2002. The Programme for International Student Assessment (PISA) is an internationally standardised assessment given to 15-year-olds in schools.

12 - Coben and Chanda 1997; Jaworski, et al. 1999

13 - Smith 2004

14 - Williams 2008

15 - Lucas 2007

16 - The Key Stage 3 Strategy was introduced to address concerns relating to students in their first three years of secondary school, and was based on the model developed for primary teachers. For more details see Stobart and Stoll 2005.

## Different kinds of planned professional development

In terms of the more planned formal professional development opportunities the literature reports on a wide range of different models, providing various classification frameworks<sup>17</sup>. Here we choose to categorise these in ways that relate to the way the CPD is organised. The categories we use are not necessarily mutually exclusive and some programmes will span these categories<sup>18</sup>.

- *Teachers working together.* Teachers within a school (or within several schools) jointly develop their professional development, sometimes led or initiated by teacher educators. Examples include peer review and coaching<sup>19</sup>, mentoring<sup>20</sup>, study groups and communities of practice<sup>21</sup>, lesson study<sup>22</sup> and some sorts of 'self-study'<sup>23</sup>. This category also includes the professional development of teachers through the use of specific resources<sup>24</sup> and reports of learning through teaching<sup>25</sup>. These approaches can be seen to share a common 'bottom across'<sup>26</sup> approach.
- *Courses or workshops which concentrate on particular aspects of mathematics or mathematics for teaching.* These might focus on, for example, specific mathematical topics, the integration of computing into mathematics classrooms or children's thinking. They may also be associated with the implementation and revision of policy such as 'reform' in the USA or the National Strategy in the UK.<sup>27</sup> These approaches are sometimes described as 'top-down'.
- *Teachers and researchers collaborating in classroom 'enquiry'* with teachers choosing their own research questions, implementing an innovation in the classroom. The researcher and the teacher review and discuss the processes and results.<sup>28</sup>

Planned professional development takes three main forms:

- teachers working together within schools or wider areas;
- courses often run by external experts;
- teachers working as researchers in their own classrooms.

17 - For example, Kennedy 2005 discusses models of CPD in terms of training, award-bearing, deficit, cascade, standards-based, coaching/mentoring, community of practice, action research and transformative.

18 - Most of the literature we looked at concerned the learning of individual teachers, but we recognise that organisational and departmental change are important. For a discussion of the need for institutional change see Cobb and Smith 2008 and for discussion on departmental change see Knight 2002.

19 - Becker 2001; Showers and Joyce 1996; Swafford 2001

20 - Halai 1998; Smith, et al. 2003

21 - Borko 2004; Loucks-Horsley 1995

22 - Brown, et al. 2002; Fernandez 2002; Isoda, et al. 2007; Lewis 2000; Lewis and Tsuchida 1999; Saito, et al. 2006

23 - Collopy 2003; Dinkelman 2003; Rogers, et al. 2007b; Williamson 2006

24 - Adler 2000; Ball and Cohen 1996; Clarke 1997; Collopy 2003; Davis and Krajcik 2005; Empson and Junk 2004; Hagedorn 2004; Kaahwa 2002; Remillard 2000; Remillard 2005

25 - Margolinas, et al. 2005

26 - Fraser, et al. 2007

27 - Barabash, et al. 2003; Borba 2005; Chapman 1999; Chapman 2001; Chapman 2003; Fennema, et al. 1996; Franke, et al. 1998; Franke, et al. 2001; Haydar 2005; Kunzman 2003; Leikin 2003; Moreira and Noss 1995; Nickerson and Sowder 2002; Parker, et al. 2005; Perks and Prestage 2002; Schifter 1998; Schifter and Fosnot 1993; Smart and Hoyles 2005; Smith, et al. 2003; Swafford, et al. 1999; Watson 2006; Whitenack, et al. 2000

28 - Askew and Burns 2005; Garcia, et al. 2007; Jaworski 2003; Jaworski 2004; Knight, et al. 2000; Krainer 1999; Leikin 2005; Leikin and Rota 2006; Scherer and Steinbring 2006; Smith, et al. 2003; Steinberg, et al. 2004; Sutherland, et al. 2008

### 3. The complexity of professional knowledge

This section addresses the issue of professional knowledge, from the perspective of professional communities and of the individual within these communities. We reviewed a number of papers in this area<sup>29</sup> and turned again to the work of Eraut<sup>30</sup>, which contributed to our understanding of the complex issues surrounding professional development and learning. In particular Eraut argues that sharing knowledge within a profession is constrained by the significant role played by tacit knowledge of practices within a profession.

#### 3.1 What is professional knowledge, and how is it generated?

Eraut argues that the emphasis within a profession is social purpose and in this respect a profession is better understood as an applied field rather than as a discipline. This distinction is important because the knowledge base of an applied field is different from the knowledge base of a discipline.

Whereas the knowledge base of a discipline is predominantly the codified knowledge of publications and books, the knowledge base of an applied field draws on knowledge from related disciplines, for example in mathematics education it draws on (at least) psychology, linguistics, sociology and mathematics.

A profession also generates its own theories and knowledge through:

- empirical research
- the elaboration of practitioner maxims and practical principles and
- the preferred view or ideology of the profession.

The majority of the literature we reviewed falls within the first category above, that is *empirical research* published in peer-reviewed journals. These papers provide some understanding of the current trends and innovations as well as comment on the effectiveness of particular models of professional development in given circumstances. They tend to focus on planned professional development, and not on the more informal workplace learning.

The second category, *the elaboration of practitioner maxims and practical principles*, for teachers of mathematics might be the national curriculum in the UK, or the Numeracy Strategy documents<sup>31</sup>, which Eraut terms 'mediating artefacts'. The majority of the knowledge associated with such an artefact is embedded in the conversations and actions that take place around it and as such is situated at the boundary of codified and uncoded knowledge. The professional learning associated with participating in these conversations and actions is mostly informal, and does not usually involve learning codified knowledge.

For teachers of mathematics, professional knowledge draws on (at least) psychology, linguistics, sociology and mathematics. It also generates its own knowledge through research, practitioner know-how and the ideology of the profession.

29 - For example, we read work by Dinham and Stritter 1986; Whitrock 1989

30 - Eraut 2001; Eraut 2004b; Eraut 2007a

31 - DfEE 1999

The third category *the preferred view or ideology of the profession* is normally a justification of its purposes and practices in terms of moral principles, views of society and occupational beliefs about the effectiveness of various practices'.<sup>32</sup>

For example we suggest that, in some countries, 'constructivist theories' of learning have become embedded in curriculum documents to the extent that they have become the 'dominant ideology'<sup>33</sup>.

In the USA, the 'mathematics reform agenda' has become an ideology of the profession and the vast majority of the American professional development research literature that we reviewed was concerned with working with teachers to the 'reform mathematics' ideology or agenda<sup>34</sup>.

In the literature we reviewed, the ideology behind programmes of professional development was frequently not made explicit and seldom questioned.<sup>35</sup>

Eraut suggests that there is a tendency in professions to construct 'ideological' theories that are attractive, but are very difficult to implement. He also suggests that there is often 'a significant gap between the theories of practice taught by former practitioners based on how they would have liked to have practised, and the activities performed by current practitioners'<sup>36</sup>. This point may have important implications for the professional development of teachers because the gap described by Eraut may mean that teachers participating in professional development initiatives see the professional development as irrelevant, unrealistic or not appropriate for them in some other way.

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32 - Eraut 2007b

33 - See also Jaworski 1999 for a discussion of this issue.

34 - See <http://www.ncetm.org/>

35 - Exceptions include Butler, et al. 2004; Fischman 2001; Kaahwa 2002; Kuntze and Reiss 2005; Ross, et al. 2006.

36 - Eraut 2007b, p 6

## 3.2 Cultural and personal knowledge

Within the context of professional knowledge, Eraut also differentiates between cultural and personal knowledge<sup>37</sup>.

*Cultural knowledge* takes two main forms, the codified knowledge of the academic community and the know-how of the professional practice, which is not necessarily codified. Whereas codified knowledge is discussed in terms of its truth value, uncoded knowledge is usually discussed in terms of its ownership, location and history.

*Personal knowledge* is the knowledge that a person brings to a situation that enables them to act and practice in the situation. Such personal knowledge can incorporate codified knowledge, know-how in the form of skills and practices, understandings of people and situations, accumulated memories of cases and episodic events, practical wisdom and self-knowledge, attitudes, values and emotions. Personal knowledge is mostly uncoded and tacit and therefore difficult to uncover.

This distinction between cultural and personal knowledge is important because a teacher's practice is informed by both their personal knowledge and the cultural knowledge built up within the profession.

In classroom situations, teachers perform and make decisions related to the personal and cultural knowledge they can access. Eraut suggests that decision making in real-life contexts is often characterised by problems that are ill-structured, information that is incomplete, ambiguous, or changing, goals that are shifting, ill-defined or competing, constraints on time, high stakes, and decisions that occur in multiple event-feedback loops.

A focus on decision-making in practice leads to Eraut's claim that it is the personal knowledge that people draw on when making decisions that is more important than any codified knowledge that might be available.

This point has implications for programmes of professional development; first in acknowledging the importance of personal knowledge and second in addressing the challenge of revealing it.

Whereas CPD tends to focus on cultural knowledge, the literature suggests that very little of this is usable as it stands. It has to be transformed into a form that suits the user's practical context and purpose.

Professional knowledge is a complex mix of cultural and personal knowledge. The practice of professionals draws predominantly on personal knowledge that is tacit and thus difficult to communicate to others.

*'Learning to use formal knowledge in practical situations is a major learning challenge in its own right – it is not a natural consequence of learning knowledge on its own and practice independently of any critical questioning of its appropriateness and effectiveness.'*  
(Eraut, 2007b, p 14)

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37 - Eraut 2007b

### 3.3 The transfer of knowledge

Transforming knowledge learned in formal situations so that it becomes usable in practice is not straightforward. In complex situations (such as teaching) this transfer is very difficult.

'Nearly all the taught components of professional and vocational education are intended for future use at work; but the evidence that this happens as intended is often disappointing.

(Eraut, 2007a, p 12)

In developing explanations for why transfer is so difficult, Eraut draws on the work of Salomon and Perkins<sup>38</sup> who make a distinction between forward-reaching and backward-reaching approaches to transfer and transformation. The forward-reaching approach is more likely to occur in a training context and involves anticipating that certain kinds of 'cultural' knowledge will be useful in the future. The backward-reaching approach is more likely to occur in practice and involves searching for relevant knowledge already learned. For a teacher the backward-reaching approach is difficult, because a teacher's work predominantly involves teaching in the classroom where it would be very difficult to take time out to search for relevant knowledge already learned.

The backward-reaching approach could take place within the planning and preparation times of teachers, but they are still likely to need support to transform knowledge learned in different contexts in order to be able to use it to think and act within the classroom.

'The pain of change lies in the loss of control over one's own practice, when one's tacit knowledge ceases to provide the necessary support and the emotional turmoil is reducing one's motivation. Hence the need for time and support is an order of magnitude greater than that normally provided.' (Eraut, 2004b, p 261)

This is further complicated by the routines teachers develop in their classrooms. As Eraut discusses, professionals often develop 'coping routines' in order to survive and cope with pressures within a practice. The benefits of 'coping routines' are that they reduce cognitive load and enable a professional to pay more attention to the situation at hand<sup>39</sup>. Further, such routines can increase productivity at the expense of quality. However, they also lead to 'knowledge becoming less explicit and less easily shared with others, i.e. more tacit'.<sup>40</sup> As such these routines are very resistant to change.

The transfer of knowledge between formal learning situations and everyday practice is difficult to achieve and is often hindered by the coping routines teachers develop in the workplace that are resistant to change.

38 - Salomon and Perkins 1998

39 - Eraut 2007a

40 - Eraut 2007b p 14

## 4. Professional knowledge for teachers of mathematics

In the previous section we have raised what we consider to be important issues related to professional knowledge and have suggested some of the implications for professional development. We now focus more specifically on professional knowledge and in particular professional development for teachers of mathematics.

### 4.1 Three aspects of mathematical knowledge for teaching

Clearly the knowledge domain of mathematics is a key aspect of professional knowledge for teachers of mathematics. In reference to school mathematics, ACME<sup>41</sup> made the strong case that the technical and interconnected nature of mathematics means that there is a special requirement for CPD for those who teach mathematics. This is also true for the teaching of numeracy to adults.<sup>42</sup>

In 1986 Les Shulman introduced the idea of ‘pedagogical content knowledge’ to represent ‘the blending of content and pedagogy into an understanding of how particular aspects of subject matter are organized, adapted, and represented for instruction’<sup>43</sup>. This categorisation, which moved away from a separation of subject or content knowledge (mathematics) and management of teaching, has had an important influence on professional development programmes.

Nowadays there is general agreement in the literature that teachers of mathematics need to know about three aspects of knowledge<sup>44</sup> namely:

- knowledge about mathematics
- knowledge about students’ mathematical conceptions (which includes the *development* of conceptions<sup>45</sup>)
- knowledge about ways of teaching mathematics.

‘The knowledge that teachers need consists of at least the mathematical understanding of the idea, and understanding of how children’s thinking might develop and a knowledge of pedagogical strategies in relationship to both the mathematical development and the psychological development’  
(Lesh and Doerr, p131).

Teacher knowledge is embedded in the ‘teacher-curriculum relationship’<sup>46</sup>, which is perhaps particularly important for teachers of mathematics because of the strong association between mathematics teachers and curriculum materials<sup>47</sup>. The curriculum can be seen as one of Eraut’s mediating artefacts and as such it embodies current practitioner maxims and theories.

Mathematical knowledge for teaching includes:

- mathematics;
- students’ conceptions in mathematics;
- ways of teaching mathematics.

41 - ACME 2002

42 - Coben and Chanda 1997

43 - Shulman 1986 p 9

44 - See, for example, Bromme 1994; Cooney 1999; Smith 2004,

45 - Shulman 1986

46 - Remillard 2000

47 - Ball and Cohen 1996; Remillard 2005

These three aspects of mathematical knowledge will have both personal and cultural dimensions, and much of this knowledge will be uncoded and tacit. Further, all three aspects of 'mathematical knowledge for teaching' are likely to be both interrelated and closely linked to practice, and embedded in routines and not normally available for scrutiny. What is important about this point is that teacher knowledge is a complex domain, and that, in developing an appreciation of this domain we need to pay attention not only to the three aspects listed above, but to the relationships between them and to those aspects of them that remain hidden.

## 4.2 Professional development programmes and mathematics knowledge for teaching

Some professional development programmes pay more attention to teachers' developing understanding of *mathematical knowledge*, others to teachers' developing understanding of *students' conceptions* of mathematical knowledge and yet others to *ways of teaching mathematics*.

Wood<sup>48</sup> illustrates the distinction between the first two of these in her discussion of two well established professional development programmes in the USA. In the first programme, Cognitively Guided Instruction<sup>49</sup> the predominant focus is on children's mathematical thinking and this programme centres on the sharing of research-based knowledge on students' conceptions of early mathematics.<sup>50</sup>

In the second programme, Learning Mathematics for Teaching<sup>51</sup> the predominant focus is on teachers' understanding of the mathematics their students are learning.

Programmes attending mainly to ways of teaching mathematics include examples well known to many teachers of mathematics in the UK, such as the Primary National Strategy Training, which advises on many aspects of how mathematics should be taught, including the structure of lessons, style of teaching and so on.

Interestingly there is no agreement in the literature about the most effective way of structuring professional development so that teachers learn about the *interrelated* aspects of mathematical knowledge for teaching.

The challenge of CPD for teachers of mathematics is to work on all aspects of mathematical knowledge for teaching, taking into account the complex relationship between cultural and personal knowledge.

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48 - Wood 1999

49 - Carpenter, et al. 1999; Franke, et al. 1998

50 - Other programmes using this approach include those described by Chamberlin 2005; Empson and Junk 2004.

51 - Schifter 1998; Schifter and Fosnot 1993

## 5. Change and Professional Development

In the previous sections, we discussed the professional knowledge of teachers of mathematics. We now turn to the question of professional development, which is related to changes in teacher knowledge. However, it is also related to changes in teachers' awareness and beliefs and in the ways in which they work with students. These changes should, in turn, enhance the mathematical learning of students<sup>52</sup>.

This section is concerned with change, and we begin by making the point that, if we are investigating change, we need to be clear about what change counts as improvement, how sustained the change is and about how we understand that change. As discussed already the majority of literature we have reviewed is concerned with organised and planned professional development but we suggest that the question of change is as relevant for unplanned and serendipitous learning as it is for planned programmes of professional development.

*'Professional development programmes are systematic efforts to bring about change in the classroom practices of teachers, in their attitudes and beliefs, and in the learning outcomes of students.'*  
(Guskey, 2002, p 381).

### 5.1 Sustained change; organising professional development programmes

Importantly, changes should be sustained and ongoing<sup>53</sup> and the argument in the literature is that, without change in teacher knowledge and beliefs, any change in classroom practice is unlikely to be sustained<sup>54</sup>. However, the literature is divided as to how to organise professional development in order to achieve changes in teacher knowledge and beliefs.

There are some who argue that change in knowledge and beliefs is a necessary precursor to changes in classroom practice, which should, in turn, lead to changes in student learning<sup>55</sup>. Others argue that changes in classroom practice and student learning is a precursor to changing teacher's knowledge and beliefs, putting forward the notion that it is not the professional development itself that provokes change, but the experience of successful implementation of change that will lead to changes in teachers' knowledge and beliefs.

This notion underpins the design of some professional development programmes in which teachers are asked to act in new ways even if they do not necessarily believe these will be effective

'It is important to ask teachers to suspend disbelief and act in new ways' (Swan, 2006, p 178)

This model of change is based on the view that change is primarily an experientially based learning process for teachers which is strongly dependent on their perceptions of the benefits of changes for their students.

A third perspective, however, can be seen as accommodating both the above. For example, Franke et al suggest that:

Professional development programmes should pay attention to both changes in beliefs and knowledge and changes in practice in order to understand more about the potential interrelated nature of these change processes.

*'Teacher change depends on the recognition that there is an intimate relationship between beliefs and pedagogy.'*  
(Swan, 2006, p 19)

52 - Guskey (Guskey 2000) suggests that a hierarchy of changes with student learning at the deepest level of the hierarchy. He proposes that the design of CPD should start with understanding what changes in student learning are wanted and then work out what changes in teacher practice, knowledge and beliefs are needed.

53 - See Cordingley et al., 2003

54 - Cooney 2001; Guskey 2002

55 - See for example, Loucks-Horsley & Matsumoto, 1999, Schifter, 1998, Shulman, 1986.

'... changes in beliefs and practices occur in a mutually interactive process. Teachers' thoughts influence their classroom practices. Their reflections on these activities and the outcomes of changed practice influence the teachers' beliefs about mathematics learning and teaching. Changes in attitudes and behaviours are iterative. Therefore well conceived professional learning experiences should consistently address both, knowing that change in one brings about and then re-inforces change in the other'.  
(Franke, et al., 1998, p 673)

The key point emerging from this discussion is the importance of taking into account the dynamic inter-relationships between knowledge, beliefs and practice and of recognising that all three aspects have to change in order to achieve sustained change in student learning. This perspective has the important implication that 'using learning from a CPD event in practical situations involves a great deal of further learning, usually much more than happened at the CPD event'.  
(Eraut, 2001, p 10).

## 5.2 What sort of change is improvement?

Change, per se, is not necessarily desirable. The question is, what sort of change can be viewed as improvement? We suggest that this is a central question for professional development programmes, not least because evaluating the effectiveness of the programme will depend on the extent to which these changes can be observed.

What sort of change counts as improvement depends on the philosophical underpinning of the professional development, which is, in turn, based on a view of what sorts of learning experiences for students are desirable and therefore what sorts of professional development the CPD would like to encourage. At any one time, this view is likely to be linked to the dominant theories about teaching and learning mathematics (as we discussed in Section 3.1).

Take as an illustrative example the massive professional development programme associated with the implementation of the National Numeracy Strategy in the UK. Undoubtedly, change has taken place; the strategy has been adopted in most schools. However, whether the mere adoption of the strategy could be viewed as an improvement is perhaps open to question. As Brown et al<sup>56</sup> 2003b note, there is very little evidence of ‘deep change’ in the teaching methods and approaches of primary teachers. We are not necessarily suggesting that the changes that have taken place do **not** count as improvement; rather we want to use this example to illustrate the question of what change counts as improvement.

Within much of the professional development literature there tends to be agreement that ‘what counts as improvement’ is reflected in a shift towards more student-centred approaches to teaching mathematics, away from more teacher-centred approaches<sup>57</sup>.

Our view is that it is not always helpful to polarise teaching and learning in these ways<sup>58</sup> and we suggest that it is important for organisers<sup>59</sup> and teachers involved in professional development to recognise that the programmes they are involved in are likely to be drawing on dominant ideologies that have become enshrined in policy. It is important to engage critically with such ideologies and to ‘unpack’ them, because as we discussed in Section 3, it is often very difficult to implement the implications of such theories.

Finally, programmes of professional development need to be clear about ‘what counts as improvement’ for them and the students.

It is important for professional development programmes to be explicit about ‘what change counts as improvement’.

*‘The NNS has been an expensive programme of systemic reform, costing more than £400 million over the first five years and untold hours of teachers’ time. Yet, in spite of politician’s claims, there is little evidence that it has been ‘an indisputable success’ as judged by a rise in attainment.’*  
(Brown, et al. 2003b, p 670)

56 - Brown, et al. 2003a

57 - These approaches are sometimes characterised as ‘constructivist’. There are some who adopt a more ‘cognitive constructivist’ theoretical approach (based on the work of Piaget) and others whose approach is more ‘social constructivist’ (based on the work of Vygotsky). See Paul Ernest Ernest 1996 for a discussion of these theoretical notions (or visit his notes on the Internet <http://www.people.ex.ac.uk/PErnest/pome12/article8.htm>). Research reporting on the adoption of these approaches in mathematics classrooms includes Chamberlin 2005; Cobb, et al. 1990; Simon and Schifter 1991.

58 - For further discussion of this see Sutherland 2007.

59 - In the literature those responsible for designing CPD are referred to as facilitators, teacher educators and organisers.

### 5.3 Evaluating Professional Development and Change

In general, it seems that insufficient attention is given to the evaluation of professional development.

'Evaluation is as basic to professional development as it is to education. Unfortunately, as is so often the case in education, systematic evaluations of professional development programs are rarely undertaken. ... Millions of dollars have been provided in the name of faculty professional development, but the quality of these programs goes virtually unchallenged'. (Rose, 1976, p 1)

CPD is most frequently evaluated in terms of the reactions of teachers to their CPD experiences, commonly through the use of so-called 'happy sheets'<sup>60</sup>. However, these methods are criticised for three main reasons; the first is that they are superficial, the second is that they evaluate 'the quality and relevance of the experience and not its outcomes'<sup>61</sup> and third that such evaluation only provides a snapshot in time, whereas professional development is an ongoing process which develops over time.

As discussed above, CPD may lead to changes in a) teacher beliefs and knowledge<sup>62</sup>, b) teacher practice<sup>63</sup> and c) student learning. Evaluation of CPD needs to address these three outcomes in addition to the reactions of the teachers participating in the CPD. Moreover, it is also important to evaluate change at the level of the department and school or college.<sup>64</sup>

Further, as Goodall et al suggest it is also important to research an 'antecedent' level, which includes 'motivations behind and reasons for the professional development programme/activity, why the particular programme was chosen, or why it was developed in a particular way, policy backgrounds and other factors affecting choice and development of the program' (Goodall, et al. 2005, p 36). This relates to the philosophies underpinning professional development programmes, and the need to be aware of what they are.

In addition to the opinions of teachers, evaluations of CPD should take into account the changes that take place in the knowledge, attitudes, beliefs and practices of teachers, as well as changes in students' learning.

60 - Lawless and Pellegrino 2007

61 - Goodall, et al. 2005, p 27

62 - Harland and Kinder 1997 suggest that these changes could include 'informational outcomes', 'new awareness', 'value congruence' (i.e. alignment of teacher beliefs with the values of the CPD), 'affective outcomes' 'motivational and attitudinal outcomes', 'knowledge and skills'.

63 - This could include Harland and Kinder's (1997) 'material and provisionary outcomes', and 'impact of practice' such as intentionality and planning of mathematics activities, organisation and management of activities and the nature of interactions between students and teacher.

64 - See Guskey 2000 for a discussion of this.

## 5.4 How do we evaluate change?

The literature is generally critical about evaluation of CPD, but we suggest that the lack of systematic and rigorous evaluation can be partly explained by the difficulty of finding suitable instruments or tools with which to conduct evaluation. In most of the research literature, teacher change in knowledge and beliefs or in classroom practice is 'measured' in some way. Changes in student learning are measured less often.

Perhaps the easiest of these to investigate is change in practice and in the research this is reported most frequently. For example, teachers may keep journals, dairies or logs which document changes in practice<sup>65</sup>, and researchers may interview and observe teachers to investigate how their practice has changed<sup>66</sup>.

There are also reports of instruments to measure change in teacher knowledge and beliefs. As we discussed earlier, measuring changes in teacher knowledge is difficult because of the complexity of this knowledge,<sup>67</sup> and because providing evidence of change requires information about the knowledge and beliefs of teachers both before and after their professional development.

The literature includes some examples of 'auditing' teachers' professional knowledge<sup>68</sup> and one approach has been developed by the Californian Study of Instructional Improvement<sup>69</sup>, which used a survey instrument to measure the mathematical knowledge used in teaching, which they suggest includes two main domains: 'specialised knowledge of content' which relates to knowledge of students' understanding and of teaching mathematics, and 'common knowledge of content' such as accurate computation of numerical calculations and solving word problems.

An approach developed in the UK for both trainee and practising teachers involves the use of the 'knowledge quartet'<sup>70</sup>; a framework which proposes the use of four dimensions which can be used by teachers to understand their professional knowledge; foundation, transformation, connection and contingency. The first category consists of knowledge, beliefs and understanding acquired *in the academy*, in preparation (intentionally or otherwise) for teaching. The second is 'knowledge-in-action' which is seen in planning to teach and in teaching. The third category concerns knowledge about how to structure the mathematical curriculum for maximum coherence. These three categories can be seen as relating to Eraut's cultural knowledge. The final category is the knowledge needed to react to classroom events which occur spontaneously and are not possible to plan for which can be seen to relate to Eraut's personal knowledge. The research reports that teachers find this framework useful in thinking about changes in their own knowledge.

*'Whatever the learning model and context, the purposes, processes, and outcomes of CPD are problematic because of the dynamic interaction with teachers' own implicit and explicit, conscious and unconscious learning and development needs which themselves are always 'filtered' by personal, school and environmental factors.'*  
(Goodall, et al. 2005, p 28)

Change is most often reported in terms of observable changes in classroom practice. It is more difficult to understand changes in teacher knowledge although there are reports of approaches that seem to be successful.

65 - Halai 1998; Hill, et al. 2005

66 - Arbaugh, et al. 2006; Chapman 1999; Clarke 1997; Franke, et al. 2001; Steinberg, et al. 2004; Tzur, et al. 2001

67 - Doerr and Lesh 2003.

However, there are programmes which aim to develop specific content knowledge, and some research reports about how the change in the teachers' knowledge is 'assessed'.

For example, Hough, et al. 2007 reported on the use pre- and post- programme assessment of teachers' understanding of algebra using mindmaps.

68 - Hill and Ball 2004, Rowan, et al. 2001, Ryan and Williams 2007

69 - The Study of Instructional Improvement (SII) is a program of comprehensive research that seeks to understand the impact of school improvement programs on instruction and student performance in elementary schools. See <http://www.sii.soe.umich.edu/>

70 - Rowland, et al. 2005; Turner 2008

## 5.5 Teachers evaluating change in their own classrooms

Whereas we consider it important to develop robust research instruments for evaluating research at a global level, within this section of the review we focus on research instruments that can be developed by teachers to evaluate the effectiveness of their own CPD at the level of their work with students in the classroom. This is because we believe this is an area which has been under-researched, and which is likely to be the main driver for achieving one of the aims of the NCETM, namely to improve institutional performance, which includes raising standards, by supporting the development of teachers of mathematics in order to realise the mathematical potential of learners. Related to this a recurring view in the literature is that effective teacher development should provide opportunities for teachers to develop insight into their students' thinking<sup>71</sup>.

We draw on the work of Doerr and Lesh<sup>72</sup> who argue that the development of 'teacher instruments' can be used to reveal the models and implicit theories that teachers use in the classroom. The idea is to 'shift the emphasis from repairing the deficiencies in teacher's mathematical knowledge to using what teachers already know to express, test, revise and refine their content knowledge and to extend that knowledge to more powerful forms for classroom teaching.'  
(Doerr and Lesh, 2003, p 130).

Doerr and Lesh suggest that it is important to create CPD activities in which all three areas of potential change, teachers' knowledge, teachers' practices and student learning, are revealed. There are ways in which Doerr and Lesh emphasise the importance of establishing communities where teachers' interpretations can be viewed from different perspectives and in which potential change can be revealed. They also suggest that teachers can produce resources that can be used to provide evidence of students' learning. The value of such resources is that they can be used in multiple settings and shared with other teachers. The following are examples of resources that could be valuable for 'disturbing current ways of thinking and for revealing mismatches between the interpretations and the experienced realities of other teachers' (p 136):

*A library of students' responses to particular mathematical activities.* This could incorporate a teacher's interpretations of students' responses as they relate to the aims of particular activities.

*Teacher procedures for assessing the strengths and weaknesses of students' work.* This could draw on the literature on Assessment for Learning<sup>73</sup> and include discussion of both formative and summative assessment.

*Tools for supporting students to assess the quality of their own work.* We suggest that peer assessment could also play a role. Most of the research about these practices is set in the context of higher education<sup>74</sup>, but the literature could inform the development of these tools.

Teacher instruments or resources can be used by teachers in their own classrooms to reveal models and implicit assumptions they use, in order to help them understand their own practice. We find this idea important from the perspective of the NCETM because such resources could be shared and re-used via the portal.

71 - Ginsburg and Seo 1999; Lesh and Doerr 2003; Schorr and Lesh 2003

72 - Doerr and Lesh 2003

73 - For example, Black 2003; Wiliam, et al. 2004

74 - Literature on self- and peer-assessment includes Boud 1995; Dochy, et al. 1999; Shepard 2000, Black 2003; Boud, et al. 1999; Topping 1998

*Observations about students' work in progress.* For example this could focus on students' learning of a particular mathematical topic over time and incorporate discussion of similarities and differences between students' approaches.

*Modification of a learning task* in light of students' engagement with and performance on the task. This could reflect the process of designing, testing and re-designing a learning task.

*Use and evaluation of information and communication technologies (ICT) for learning a particular area of mathematics.* There is a wide literature on the use of ICT to teach specific mathematical areas<sup>75</sup>, and this could provide a background against which to develop such tools.

Whereas we consider it important to develop robust research instruments for evaluating research at a global level, within this section of the review we focus on research instruments that can be developed by teachers to evaluate the effectiveness of their own CPD at the level of their work with students in the classroom. This is because we believe this is an area which has been under-researched, and which is likely to be the main driver for achieving one of the aims of the NCETM, namely to improve institutional performance, which includes raising standards, by supporting the development of teachers of mathematics in order to realise the mathematical potential of learners. Related to this a recurring view in the literature is that effective teacher development should provide opportunities for teachers to develop insight into their students' thinking<sup>76</sup>.

Doerr and Lesh suggest that multimedia case studies that explicitly include lesson plans and interviews with the teacher before and after each lesson<sup>77</sup> can be particularly effective. The work of Olivero<sup>78</sup> with video papers also supports this idea. The distinguishing feature about videopapers is that they incorporate in one document videos, text, slides and interactive links between different elements. Research suggests<sup>79</sup>, that the experience of creating and reading a videopaper differs from the experience of reading a text or watching a video, and what emerges is the fact that both video and text are needed in order to make meaning<sup>80</sup>.

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75 - Balacheff and Sutherland 1994; Christou, et al. 2004; Goldenberg 1988; Hennessy 1999; Hoyles and Noss 2003; Laborde 2002; Noss, et al. 1997; Ruthven, et al. 2004; Sutherland, et al. 2005

76 - Ginsburg and Seo 1999; Lesh and Doerr 2003; Schorr and Lesh 2003

77 - Doerr, et al. 2000

78 - Olivero, et al. 2004

79 - Nemirovsky, et al. 2005

80 - Jewitt 2005

## 6. Important characteristics of ‘effective’ professional development programmes: synthesising the literature

### 6.1 Introduction

The majority of the literature reports on professional development initiatives which are viewed to a certain extent as being successful<sup>81</sup>. In many cases, the authors of these reports conclude with recommendations for programmes of professional development. Other literature is broader, drawing together ‘lists’ of characteristics of ‘effective’ professional development.<sup>82</sup>

There is also literature on barriers teachers encounter that prevent them from taking full advantage of the professional development in which they are engaged, and again it is common for the authors of this literature to conclude with recommendations, pointing out pitfalls to avoid.

This section draws together this literature.

The literature recognises that change is gradual<sup>83</sup> and that integrating new ideas and practices into ongoing and established practices of teachers is not simple or straightforward<sup>84</sup>. Eraut argues this point in detail in his discussion of the transfer of knowledge between educational and workplace settings as discussed more fully in Section 3.

Therefore it is generally agreed that sustained, longer term professional development programmes that unfold over time are more effective than individual workshops and seminars, which are one-time events<sup>85</sup>. There is no suggestion that such workshops should **not** take place, but it is generally agreed that they are unlikely to lead to long-term sustained professional learning.

In the literature, various frameworks are used to unpick the complexity of factors contributing to the programmes of professional development. For example, programmes can be described in terms of the ‘domain of influence’, ‘capacity for professional autonomy and transformative practice’ and ‘sphere of action’<sup>86</sup>, in terms of ‘structural features’ and ‘core features’<sup>87</sup> or in terms of ‘delivery mechanism’, ‘context’ and ‘duration’<sup>88</sup>. We choose to adopt a similar framework to the latter of these<sup>89</sup>, which takes into account these categories but also provides a mechanism for looking at the context of teacher learning. Our framework comprises the following elements:

- the context, or setting of the professional development
- the content of the programme and
- the processes involved

It is generally agreed that sustained, longer term professional development programmes that unfold over time are more effective than individual workshops and seminars, which are one-time events.

81 - In the majority of cases, the teachers participating in the CPD are volunteers and are willing to learn and committed to the CPD. There is a suggestion (see, for example, Borko 2004) that we need to understand better ‘typical’ CPD for ‘typical’ teachers, that goes largely unreported.

82 - Abdal-Haqq 1996; Clarke and Clarke 2005; Cordingley, et al. 2004; Desimone, et al. 2003; Guskey 2003; Harwell 2003; Hopkins and Reynolds 2001; Loucks-Horsley, et al. 2003; Schmoker 2004

83 - Clarke and Hollingsworth 2002; Guskey 2002

84 - Clarke and Clarke 2005; Etchberger and Shaw 1992; Putnam and Borko 2000; Wilson and Berne 1999

85 - ACME 2002; Harwell 2003; Smith Senger 1999; Wilson and Berne 1999.

86 - Fraser, et al. 2007

87 - Garet, et al. 2001

88 - Lawless and Pellegrino 2007

89 - This is a popular framework used by, for example, Harwell 2003. See also <http://www.nsd.org/standards/index.cfm>

## 6.2 Context: schools, classrooms, on-site and off-site

All professional development takes place within a setting which is culturally and temporally situated. One aspect of the cultural context is the views and beliefs of those involved in the professional development initiative<sup>90</sup>. This includes both the participating teachers' views and beliefs and those of their leadership teams. There is clear evidence in the literature that school leaders should provide ongoing support for teachers<sup>91</sup> but that teachers also need to be committed to professional learning and change.<sup>92</sup> This sort of support includes both practical support (such as providing time and funding<sup>93</sup>) and support such as active encouragement.

As Harwell discusses, teachers and leaders should have a shared sense of a need for change, and it is particularly important for teachers to take ownership of their own learning<sup>94</sup>. By implication, professional development programmes imposed on teachers are unlikely to succeed<sup>95</sup>.

Interestingly Cobb, Yackel and colleagues (who have been involved in a 10 year programme of supporting teachers' learning which involves the establishment of a professional teaching community) argue that in their early work they did not adequately take into account the institutional context, and this sometimes led to conflicts between the teachers with whom they were collaborating and the governors and the senior management. They argue that it is important to conduct ongoing analyses of the institutional contexts in which teachers work, which can guide collaboration with principals and administrators.

In a recent paper, Cobb and Smith<sup>96</sup> develop this argument, suggesting that large scale reforms seldom produce lasting changes unless schools make deliberate plans to develop support for teachers.

A further key recommendation emerging from the literature is that professional development programmes should be grounded in classroom practice, allowing teachers to experiment with new ideas and reflect on their experiences of doing so<sup>97</sup>. As Fullan and Stiegelbauer<sup>98</sup> point out, professional development programmes that do not build connections between what teachers learn and their classroom practices are not likely to have lasting effects.

This does not necessarily imply that programmes should take place on site or in the classroom: it can be helpful for teachers to work away from the familiar classroom environment and to participate in programmes that take place in multiple settings<sup>99</sup>. As discussed in Section 3, within the classroom teachers routinise their activity and they may find it easier to reflect on their routines and tacit knowledge in settings that are outside the school or college.

Programmes of professional development should include commitment to the enterprise by both institutions and teachers.

*'... in addition to developing new approaches for supporting students' and teachers' learning, reformers also need to view themselves as institution changing agents.'* (Cobb and Smith, 2008, p 2)

*'... the design of professional development should follow similar principles to those used in classroom activities and, when it does, changes to teachers' beliefs and practices become possible. Teachers learn through discussion and reflection just as students.'* (Swan, 2006, p 19)

90 - Fullan and Stiegelbauer 1991; Loucks-Horsley 1995

91 - Millett and Johnson 2000; Rogers, et al. 2007b

92 - Britt, et al. 2001

93 - Both lack of time and funding are cited in the literature as obstacles to taking up professional development opportunities Bobis 2004; Goodall, et al. 2005; Long 2004.

94 - Clarke and Clarke 2005; Krainer and Goffree 1998.

95 - Bishop and Denleg 2006

96 - Cobb and Smith 2008

97 - Abdal-Haqq 1996; ACME 2002; Loucks-Horsley, et al. 2003

98 - Fullan and Stiegelbauer 1991

99 - Putnam and Borko 2000

As discussed already change in practice is likely to be linked inextricably to change in teachers' knowledge and beliefs<sup>100</sup>, and the literature recommends that professional development programmes provide environments which encourage teachers to explore and change both their knowledge and beliefs and their classroom practice. The argument is that networks, communities of practice and group discussions are social spaces that provide environments in which teachers feel safe to explore their practice<sup>101</sup> and hence allow teachers the space for 'informed dissent'<sup>102</sup>. For this reason, and others, such as peer support<sup>103</sup> and reducing teacher isolation<sup>104</sup>, the literature calls strongly for collaborations between teachers. However it is also emphasised that it is important for such collaborative networks to support disagreement and critique.<sup>105</sup>

However, the constraints of cost, time and space limit teachers' ability to participate in face-to-face networks or communities of learning. Because of this it is claimed that online networking may be able to offer rich, extensive and meaningful opportunities for professional development and collaborative work between teachers<sup>106</sup>.

The literature cites a number of reasons to back up this claim; foremost is the practicality of 'anytime anyplace' networking<sup>107</sup>. Further with the increasing sophistication of 'social networking' tools, particularly in terms of building content (in the sense of Web 2.0 technologies)<sup>108</sup> communities could be set up and grow in new and unforeseen ways. In this way, ICT is used to develop a creative community rather than simply a communication channel.<sup>109</sup> We see the NCETM portal as having the potential to support such creative communities.

What is important here is to recognise the potential of online networking and to work out ways in which it can be set up and supported. Networking technologies are in their infancy, and research about how they have been used in professional development is scanty (and not always positive<sup>110</sup>). The research has tended to focus more on the potential and limitations of the various networking tools (mostly discussion boards), and recommends that discussion should be moderated and focused on concrete problems<sup>111</sup>.

It is important to create 'communities' where disagreement and critique are welcomed as teachers refine teaching practices and develop ideas. However, this is not always easy to achieve.

Online communities for teachers of mathematics have the potential to overcome the constraints of time and space but their development needs support

The NCETM portal is used to create and develop creative communities of teachers of mathematics, allowing both 'anytime anyplace' networking as well as collective building of knowledge.

100 - Clarke and Clarke 2005; Harwell 2003; Lerman 2001; Swan 2006

101 - Fullan and Stiegelbauer 1991; Little 2002; Loucks-Horsley 1995

102 - Little 2002

103 - Adsit 2004a

104 - Schlager and Fusco 2003

105 - Cohen and Hill 2000 p 3

106 - Adsit 2004b; McMahon 1997; Sprague 2006; Stephens and Hartmann 2002

107 - Dexter and Greenbow 2002

108 - Wellman, et al. 1996 Hargreaves 2003

109 - Hargreaves 2003

110 - Schlager and Fusco 2003

111 - Dexter and Greenbow 2002; Ginsburg, et al. 2004.

### 6.3 Content: teacher knowledge, learning and mathematics

We have already discussed the issue of ‘knowledge for mathematics teaching’ in Section 4, and we pointed out that there is consensus in the literature that professional development should address all areas of teacher knowledge<sup>112</sup>, as well as the relationship between different aspects of teacher knowledge.

In this respect the focus of professional development should attend to both issues of subject matter knowledge and issues of teaching and learning as they come together in classroom practice<sup>113</sup>. Importantly, professional development should stimulate intellectual thinking. As Little suggests, in her list of ‘design principles’ for professional development, ‘professional development offers meaningful intellectual engagement with ideas, with materials ...’<sup>114</sup>. Similarly, Goodall et al point out that professional development should promote ‘inquiry, creativity and innovation’<sup>115</sup>.

Research also suggests that professional development programmes tend to be more successful when they are designed to take into account the current knowledge and understanding of the participants and treat them as active, adult, professionals<sup>116</sup>. What seems to be particularly important is that professional development should meet the needs of teachers.

The literature reports tensions between innovations and the status-quo. For example, teachers are sometimes encouraged by their professional development programmes to implement new approaches, which do not sit easily with the schools or departments in which they work.

Related to this point is the suggestion that teacher educators are frequently not in touch with the reality of day-to-day classroom and school life and that more successful professional development programmes take this reality into account<sup>117</sup>. These realities include practical issues such as high stakes assessment regimes<sup>118</sup>, curriculum materials used in particular schools and access to technology or tools, but also the routines teachers adopt in order to cope with the ‘hot action’ of the classroom (as discussed in Section 3).

As far as the mathematical knowledge of teachers is concerned, the Smith report<sup>119</sup> suggested that many teachers of mathematics need to extend their knowledge regarding specific areas of the curriculum, such as statistics, data handling, modelling and applications. Examples in the literature of CPD which addresses specific areas of mathematics include focusing on fractions<sup>120</sup>, statistics<sup>121</sup>, proportional reasoning<sup>122</sup> linear and exponential growth<sup>123</sup> and algebra<sup>124</sup> and geometry<sup>125</sup>.

The content of professional development programmes should stimulate intellectual thinking and should attend not only to the teaching and learning of mathematics, but also to the domain of mathematics.

Professional development programmes should meet the needs of teachers and also take into account the realities of day to day teaching.

112 - ACME 2006

113 - Cohen and Hill 2000

114 - Little 1993 p 138

115 - Goodall, et al. 2005 p 25

116 - Abdal-Haqq 1996; Little 2002; Putnam and Borko 2000

117 - Brown and McIntyre 1993

118 - Long 2004

119 - Smith 2004

120 - Baturu 2004; Saxe, et al. 2001

121 - McClain 2003; Watson 1998

122 - Ben Chaim, 2002

123 - Alagic and Palenz 2006

124 - Cedillo and Santillan 2005; Hough, et al. 2007; Swan 2006

125 - Fiorentini, et al. 2005; Gal and Linchevski 2005; Leikin 2003; Swafford, et al. 1997

A particular point arising in the literature is the mathematical knowledge of primary school teachers, who are frequently not specialists in mathematics and may lack confidence in their own mathematical ability. The recently published Williams Report in the UK emphasises the point that most primary school teachers and early years practitioners lack sufficient in-depth mathematics subject knowledge and calls for measures (including CPD) to address this problem. However, the literature suggests that many of these teachers need support in overcoming the ‘emotional baggage’<sup>126</sup> associated with learning mathematical knowledge.

## 6.4 Process

Running through many of the recommendations relating to the processes of professional development is the notion that teacher learning should also be based on sound educational practice. One aspect of ‘sound educational practice’ is the modelling of approaches which reflect the sorts of approaches which teachers might adopt in the classroom<sup>127</sup>. Putnam and Borko<sup>128</sup> explicitly recommend that teacher educators should treat teachers as they expect teachers to treat students.

The recommendations also strongly suggest that processes of implementing changes in the classroom should be coupled with processes of reflection on the teachers’ experiences both as learners and as teachers, in discussions and writing<sup>129</sup>. This notion of the duality of making changes (sometimes referred to as experimenting) and reflecting includes Cordingley’s recommendations<sup>130</sup> for classroom coaching and encouraging, extending and structuring professional reflection and dialogue.

There is a small, but growing, body of literature reporting on how video is used in the reflective activities, both in face to face and online discussions<sup>131</sup>. For example, Galvis and Nemirovsky describe how video is used to encourage teachers to reflect on their own practice.

Some of the literature extends this idea of experimenting and reflecting to suggest that professional development should involve a process of inquiry and prepare teachers to employ the techniques and perspectives of inquiry in the classroom. Jaworski<sup>132</sup> discusses the notion of inquiry within professional development in detail, suggesting that inquiry can be seen as a ‘tool’ and as a ‘way of being’ and makes the strong case for communities of inquiry. She describes these communities as:

‘two or more people [work] together to embody an inquiry approach to learning and teaching, developing inquiry as a way of being within the community – acknowledge and address issues and tensions within their practice. Such acknowledging and addressing (usually) does not remove the issues and tensions, but allows a critical appraisal of what is needed, and possible with respect to social forces and constraints’.  
(p 206).

Processes involved in CPD should be based on sound educational practices and should involve the implementation of new approaches to teaching and learning in the classroom, together with reflective activity.

*‘The positive outcomes... were linked to CPD interventions which combined reflection with active experimentation.’  
(Cordingley, et al. 2005, p 18)*

126 - Bibby 1999; Hodgen and Askew 2007

127 - Clarke and Clarke 2005; Putnam and Borko 2000

128 - Putnam and Borko 2000

129 - Borasi, et al. 1999; Day 1999; Irwin and Britt 1999; Olson and Barrett 2004

130 - Cordingley, et al. 2005

131 - Galvis and Nemirovsky 2003; Kuntze and Reiss 2005; Lenfant, et al. 2003; Nemirovsky, et al. 2005; Olivero, et al. 2004; Pea 2005

132 - Jaworski 2006

133 - Doerr and Lesh 2003

134 - See for example, Chamberlin 2005

As discussed already much of a teacher's personal knowledge is tacit knowledge, not available to them in an explicit form. Doerr and Lesh<sup>133</sup> (2003) argue that teaching is a complex and ill-structured domain, and that knowing how to teach means knowing how to see and interpret such a domain:

'the nature of teacher knowledge is much more about how teachers interpret the complexity and the situated variability of the practical problems of the classroom' (p 127).

We suggest that this emphasis on seeing and interpreting relates to making tacit knowledge explicit, through reflection and inquiry, the importance of which is echoed in much of the literature on professional development for teachers of mathematics<sup>134</sup>.

At the same time as drawing attention to the importance of 'seeing and interpreting' practice as a key component of professional development programmes, Doerr and Lesh argue that there are two flawed and implicit assumptions within many professional development programmes. The first is that 'Learning to teach is a matter of learning how to do what effective teachers do' and the second is that 'There is some generally unspecified image of a good teacher that novices should strive to emulate in action' (p 125). They suggest that these two implicit assumptions can lead to a focus on the development of skills, procedures, guidelines and rules of thumb that can be used in the classroom. Such a focus can detract from a focus on reflection, interpretation and the unpacking of tacit knowledge.

However somewhat in contrast to this emphasis on reflection and inquiry, Claxton suggests that 'conscious deliberate reflection runs the risk of undermining skilled performance'<sup>135</sup> (p 36). Claxton argues for the importance of educating intuition, where intuition refers to a loose-knit family of 'ways of knowing'.

'The members of this family include the ability to function fluently and flexibly in complex domains without being able to describe or theorise one's expertise; to extract intricate patterns of information that are embedded in a range of seemingly disparate experiences ('implicit learning'); to make subtle and accurate judgments based on experience without accompanying justification; to detect and extract the significance of small, incidental details of a situation that others may overlook; to take time to mull over problems in order to arrive at more insightful or creative solutions; and to apply this perceptive, ruminative, inquisitive attitude to one's own perceptions and reactions – 'reflection' (Claxton, 2000, p 50).

What seems to be important is to develop a balance within professional development activities between intuition and reason. As Claxton argues 'A recognition of the intricate ways in which intuition and intellect, reason and reflection, experience and explanation, balance and complement each other challenges theorists of professional development to come up with a rather more sophisticated model of adult professional learning than we have at present [ ] Professional development involves a shifting, dynamic interplay of different ways of knowing, and models of specific situations need to be developed which take into account their rhythms and 'melodies' of learning'. (p 46).

Professional development programmes should take the intuition of teachers into account.

135 - Claxton 2000

## 7. Conclusions

### 7.1 Summary and recommendations

#### What do we mean by CPD?

CPD and teacher learning includes formal and informal opportunities, which may be serendipitous or planned. The ways in which these opportunities are woven together varies from teacher to teacher.

The literature emphasises the importance of informal learning in the workplace suggesting that such learning is an important aspect of CPD.

Planned CPD opportunities may be voluntary or mandatory and could involve training days within schools, joint team planning and teaching, peer observation, work shadowing, courses, conferences and networking. Participation in these events may or may not result in accreditation.

#### Models of CPD

The literature reports on a wide range of different models of CPD. These fall into three groups, relating to the way the CPD is organised:

- teachers within a school (or within several schools) jointly develop their own professional development. Examples include peer review, mentoring, coaching, study groups, lesson study and 'self-study'
- courses which concentrate on particular aspects of mathematics or mathematics for teaching. These might focus on, for example, specific mathematical topics, the integration of ICT into mathematics classrooms or cognitive acceleration. They may also be associated with the implementation and revision of the National Strategy
- teachers and researchers collaborating in classroom 'enquiry' with teachers choosing their own research questions, implementing an innovation in the classroom. The researcher and the teacher review and discuss the processes and results.

The majority of literature reviewed focused on a particular model of CPD. More research comparing a range of CPD approaches is needed.

We see the learning of teachers within the workplace as important, yet most research does not pay attention to this aspect of teacher learning.

In addition the voice of the teacher is under-represented in most research.

## The complexity of professional knowledge for teachers of mathematics

Teachers predominantly learn through practice, developing personal knowledge related to teaching and learning that is mostly tacit, uncodified and difficult to uncover. By contrast cultural knowledge includes the codified knowledge of the academic community and the know-how of the profession. Whereas CPD tends to focus on cultural knowledge, the literature suggests that very little of this is usable as it stands. It has to be transformed into a form that suits the user's practical context and purpose.

It is generally agreed that mathematics knowledge for teaching falls into three interrelated areas:

- knowledge about mathematics
- knowledge about students' mathematical conceptions
- knowledge about ways of teaching mathematics

We suggest that the challenge of CPD for teachers of mathematics is to work on all three aspects of mathematical knowledge for teaching, taking into account the complex relationship between cultural and personal knowledge.

Research needs to:

- pay more attention to the role of tacit and intuitive knowledge in professional development;
- take into account informal workplace learning.

CPD research should pay attention to all aspects of mathematics knowledge for teaching as well as the relationship between different aspects of mathematical knowledge for teaching.

## How do we understand change in teacher awareness, knowledge and practice?

Professional development is related to teacher change, which could occur in the interrelated areas of teachers' awareness, beliefs and knowledge and in the ways in which they work with students. These changes should, in turn, enhance the mathematical learning of students.

If we are investigating change, we need to be clear about what change counts as improvement, how sustained the change is and about how we understand that change.

CPD is frequently evaluated in terms of change in teacher beliefs, and in particular in terms of the reactions of teachers to their CPD experiences. It is more difficult to investigate change in teacher knowledge, teacher practice and student learning. It is more difficult still to investigate the interrelationships between these, particularly since changes in student learning are influenced by many factors other than changes in teacher knowledge and practice.

CPD activities in which all three areas of potential change are revealed are of value, and the suggestion is that teachers can produce resources that can be used to provide evidence of students' learning, which could be reusable. The value of such resources is that they can be used in multiple settings and shared with other teachers.

CPD programmes need to be clear about what change counts as improvement, how sustained the change is and about how we understand that change.

The NCETM portal can play an important role in supporting teachers to share resources that can be used to provide evidence of students' learning.

## What do we know about the characteristics of effective CPD?

The literature suggests that CPD works best when programmes of professional development:

- include commitment to the enterprise by both institutions and teachers
- encourage purposeful networking amongst teachers
- are grounded in classroom practice
- are based on sound educational practice
- build on what teachers already know, taking into account the voice of the teacher
- avoid adopting a 'deficit model' of teacher knowledge and practice
- focus on 'mathematics for teaching'
- centre around activities that reveal aspects of 1) teachers' awareness, beliefs, and knowledge 2) teachers' practice and 3) students' learning
- support reflection and inquiry by teachers on both their own learning and their own classroom practice
- are explicit about 'what change counts as improvement'
- support the development and evaluation of classroom-based activities.

## 7.2 Areas that are under-researched

In this section we draw attention to some important aspects of continuing professional development for teachers of mathematics about which we found little or no research.

The majority of the literature reports on specific professional development initiatives, which may be short or long-term but have a limited time span with respect to a teacher's lifetime of teaching. Research is needed to understand the multiple experiences which constitute a teacher's continuing professional development throughout their career as a teacher.

There is very little research on the professional development of teachers in Post 16 education, which includes those teaching young people on academic and vocational courses and those teaching basic numeracy to adults. Teachers in Post 16 education will play a key role in teaching mathematics to students who study for the new 14 – 19 diplomas in the UK<sup>136</sup>, and to contributing to the success of these diplomas. Teachers of adult learners face specific challenges, and, as Coben points out, are frequently not sufficiently well prepared to meet these challenges<sup>137</sup>.

There is very little research on the professional development of teaching assistants who work with teachers of mathematics in both the primary and secondary sectors. Teaching assistants sometimes take classes in the absence of a teacher and frequently work with small groups of students who have specific needs. However, many teaching assistants are not qualified to teach<sup>138</sup>, and this can be seen as a concern; as the recent Ofsted report in the UK<sup>139</sup> points out, there is 'scope to improve teaching assistants' mathematical interactions with pupils during group and individual work, so that they become more skilled in asking questions rather than telling pupils what to do' (p 21).

Very little research on professional development for teachers of mathematics addresses the question of the relationship between changes in teacher's classroom practice and enhanced student learning. While we recognise that it is difficult to untangle the complex web of factors that may influence the mathematical learning of students, we suggest that researchers need to think about ways in which to address this issue.

The issue of how difficult it is for teachers to transfer knowledge between different settings is seldom a focus of attention in the literature on professional development for teachers of mathematics. There is a tendency to assume that knowledge learned in professional development programmes is unproblematically transferred to the classroom.

It is well recognised that the ultimate aim of professional development for teachers of mathematics is to improve learning opportunities for students but in general, research does not attend to the voice of the students. However, we suggest that the voice of students could add a valuable perspective on changes in student learning.

There is very little in the literature that discusses the people who design, plan and deliver CPD, but we think it is crucially important that we know about, and understand more about, this group of people because of their influential position on the teaching of mathematics.<sup>140</sup>

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136 - See <http://yp.direct.gov.uk/diplomas/>

137 - Coben and Chanda 1997

138 - There are opportunities to gain qualifications; for example see <http://www.tda.gov.uk/support/qualificationsandtraining.aspx>

139 - Ofsted 2008

140 - Exceptions include Even 1999; Rogers, et al. 2007b

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