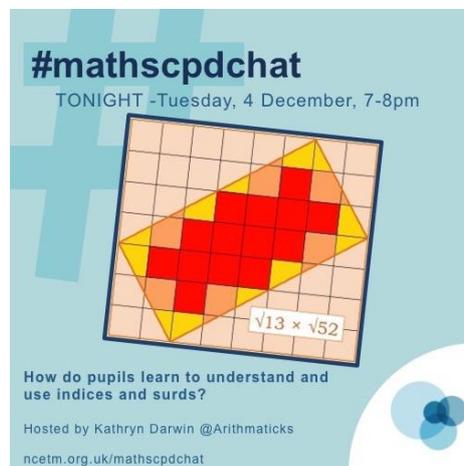


#mathscpdchat 4 December 2018

How do pupils learn to understand and use indices and surds?

Hosted by [@Arithmaticks](#)

This is a brief summary of the discussion – to see all the tweets, follow the hashtag #mathscpdchat in Twitter



Some of the areas where discussion focussed were:

- **'ways in'** to mastery of indices and surds, for example pupils creating images to represent simple positive integer powers, square numbers, estimating roots of non-square numbers, powers of ten ('Powers of 10' video), developing fluency with positive integer powers leading to $n^a \times n^b = n^{a+b}$;
- helping pupils to see that it **'makes sense' to represent \sqrt{n} as $n^{1/2}$** , for example comparing $n^{1/2} \times n^{1/2} = n^{1/2 + 1/2} = n^1 = n$ with $\sqrt{n} \times \sqrt{n} = n$, or asking 'if $n^a \times n^a = n^{a+a} = n^{2a} = n^1 = n$, what is a ?';
- using $n^a \times n^a \times n^a = n$, $n^a \times n^a \times n^a \times n^a = n$, and so on, to introduce **fractional powers in general**;

- extending sequences by using pattern-spotting in order to **introduce and 'justify' $n^0 = 1$ and negative powers**, for example using 'halving' or dividing by 2, to obtain ...
... .. $2^3(8), 2^2(4), 2^1(2), 2^0(1), 2^{-1}(1/2), 2^{-2}(1/4), 2^{-3}(1/8), \dots$;
- representing n^a where a is negative **as a decimal**, eg $10^{-2} = 1/10^2 = 1/100 = 0.01$, 'complications' that arise when the base generates a recurring or lengthy-digit decimal, such as $7^{-2} = 1/7^2 = 1/49 = 0.020408163\dots$;
- introducing **negative powers by linking to 'scientific notation' ($a \times 10^b$)** where b (the power of 10) is negative ... discussion about distinction between 'scientific form' and 'normalised scientific form' ('standard form') ... in 'standard form' $1 \leq a < 10$ in $a \times 10^b$, but in 'engineering form' b must be a multiple of 3 so $1 \leq a < 1000$;
- introducing **negative powers by pupils dividing** a base-raised-to-a-power by the same-base-raised-to-a-higher-power, for example $2^3 \div 2^5 = 2^{3-5} = 2^{-2}$ and $2^3 \div 2^5 = 8 \div 32 = 1/4$, so $2^{-2} = 1/4$;
- **representing n^a as a decimal when a is a fraction** ... leading to encountering non-recurring, non-terminating, decimals, and so to the idea of an irrational number, and to introducing the word 'surd' ... for example $2^{1/2} = \sqrt{2} = 1.412\dots$;
- **defining 'surd' for/with pupils** ... whether or not a surd is necessarily an irrational number ... 'surd' linked to 'absurd' through Latin derivation from 'surdus'/'deaf to reason' ... definition is often oversimplified in modern textbooks ... 'quadratic surds' for \sqrt{n} ... 'surdic expressions', eg ' $41 + \sqrt{6}$ ' ... 'radical' for any numerical expression under a n^{th} root (France) ... proof that $\sqrt{8}$ is irrational ... pupils 'pinning down' irrationality first so that they can then appreciate that, $\sqrt{0.25}$ (for example) is not a surd;
- using **prime factor decomposition to simplify surds**;
- where **surds first appear in schools' 'Schemes of Learning'** ... whether Y8 is too early for fractional indices ... worries about drive to address more advanced content earlier ... secure grasp of 'core basics' before KS4 ... 'conceptual exploration' when pupils are able to appreciate it ... 30 years ago did we teach more than we now do?
- pupils **coping with n^a where a is expressed as a decimal**, for example $32^{0.6} = 32^{3/5} = ({}^5\sqrt{32})^3 = 2^3 = 8$;
- **simplifying expressions involving 'nasty-decimal indices'**, for example $(n^{0.272727\dots})^{11} = n^{(0.272727\dots \times 11)} = n^{((27/99) \times 11)} = n^{((3/11) \times 11)} = n^3$;
- **Mrs Malley's fractional and negative indices dance** ... done with Y11!

An interesting 'conversation' of tweets, about the stage in their schooling at which pupils address indices (in some depth) and surds, followed from these two tweets by [Kathryn Darwin](#):

-  **Kathryn @Arithmaticks** · Dec 4
Where do surds first appear in your Scheme of Learning? They've popped up for us in Year 8 this year to 'extend and challenge' and to support KS4 problem solving... Is this too early? #MathsCPDChat
-  **Kathryn @Arithmaticks** · 17h
I was worried about this - I like the idea of the basics and seeing the link with indices but I was worried this was too early for fractional indices too....

including these two from [Mrs M](#) and [Jess Prior](#):

-  **Mrs M @MrsMandMaths** · 17h
Replying to @Arithmaticks
Year 10
- 
-
-  **Jess Prior @FortyNineCubed** · 17h
Replying to @Arithmaticks
Year 9 for us - this feels about right. I'm quite keen on delaying teaching topics as much as possible - if they've seen it all in year 8 then upper ks4 can develop a feeling of 'more of the same'. #mathscpdchat

these two from [Mr Blachford](#) and [Kathryn Darwin](#):

-  **Mr Blachford @MrBlachford** · 17h
The drive to do more advanced content earlier worries me. Some concepts take time to mature. I'd rather a secure grasp of core basics before ks4.
#MathsCPDChat
- 
-  **Kathryn @Arithmaticks** · 17h
I think we could cover some of it earlier, like $\sqrt{x} \times \sqrt{x} = x$ is something we take for granted I think but not obvious to students. It's hard to know where the line is, but sometimes I realise the students are 12/13 and I'm banging on about irrationality... #mathscpdchat

this from [Heather Scott](#):

-  **Heather Scott @MathsladyScott** · 18h
I spend a whole lesson on $\sqrt{9} \times \sqrt{9} = 9$... sometimes it takes more than 60 minutes for stdts to spot a pattern using different numbers - it fascinates me what is blocking their thinking on this 😞 #mathscpdchat

these two from [Mr Blachford](#) and [Jess Prior](#):

 **Mr Blachford** @MrBlachford · 17h
Thing is, I can 'show' kids lots of stuff. All I'll be achieving is procedural mimicry. I *could* teach basic calculus to a year 7 class if all I'm doing is showing the tricks. I prefer conceptual exploration when they're able to appreciate it
[#MathsCPDChat](#)

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 **Jess Prior** @FortyNineCubed · 17h
Yes - absolutely this! And when they are able and ready to appreciate it and meet something new and exciting - best teaching moments ever.
[#mathscpdchat](#)

these two from [Sharon Malley](#) and [Kathryn Darwin](#):

 **Sharon Malley** @mathsmumof2 · 17h
Replying to @Arithmatics
With top set y8 I introduced that the square roots of non-square numbers are irrational and we estimate their size by finding the two squares they are between
[#mathscpdchat](#)

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 **Kathryn** @Arithmatics · 17h
I think this is often a missed opportunity! So I assume this allows you to distinguish between irrational and a surd? [#mathscpdchat](#)

and this one from [Heather Scott](#):

 **Heather Scott** @MathsladyScott · 18h
I started teaching over 30 yrs ago - understand the spiral curriculum ... I tend to go as far as I think students are grasping and then revisit [#mathscpdchat](#) 30 yrs ago we taught more than we teach now - or maybe it's changed with new GCSE? 😊

(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

[Teaching surds – how Twitter came to the rescue!](#), which is an article by [Kathryn Darwin](#) in the NCETM Secondary Magazine 151 (published on 4 December 2018), shared by [Mary Pardoe](#)

[Powers of Ten](#), which is a film by Charles and Ray Eames that 'takes the viewer on an adventure in magnitudes' (by changing the 'scale of view' of a scene by both positive and negative powers of 10 ... in a most amazing way), shared by [Steve L](#)

[Engineer Symbols](#) which shows how to switch between ways of showing numbers on a particular Casio calculator, shared by [The Calculator Guide](#)

[Visualising Indices](#) which explores visual representations of indices and draws on creativity, shared by [Mary Pardoe](#)

[SUMAZE!](#) which is an enjoyable challenging online game from MEI/Integral; pupils using it are likely to develop fluency in using operations to combine numbers many of which are small positive whole numbers raised to a small positive integer power, shared by [Mary Pardoe](#)

[Index Issues](#) which is a page on Underground Mathematics; it addresses issues involved in working with indices (at Higher GCSE level and just beyond), shared by [Mary Pardoe](#)

[John and Betty's Journey into Complex Numbers](#) which is an unusual PowerPoint file; it reveals, slide-after-slide, an illustrated story in which the need for new (representations of) numbers arises naturally in an easy-to-understand context, shared by [Sharon Malley](#)