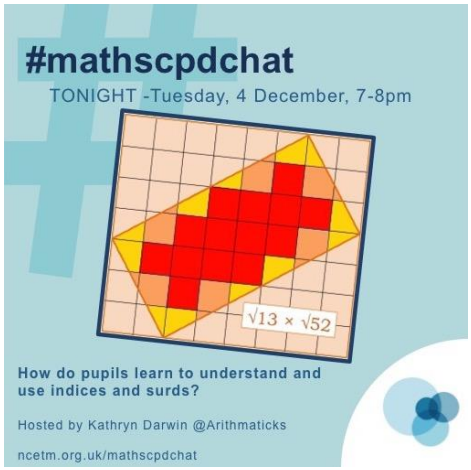
National Centre for Excellence in the Teaching of Mathematics

#mathscpdchat 4 December 2018

How do pupils learn to understand and use indices and surds?

Hosted by @Arithmaticks

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Some of the areas where discussion focussed were:

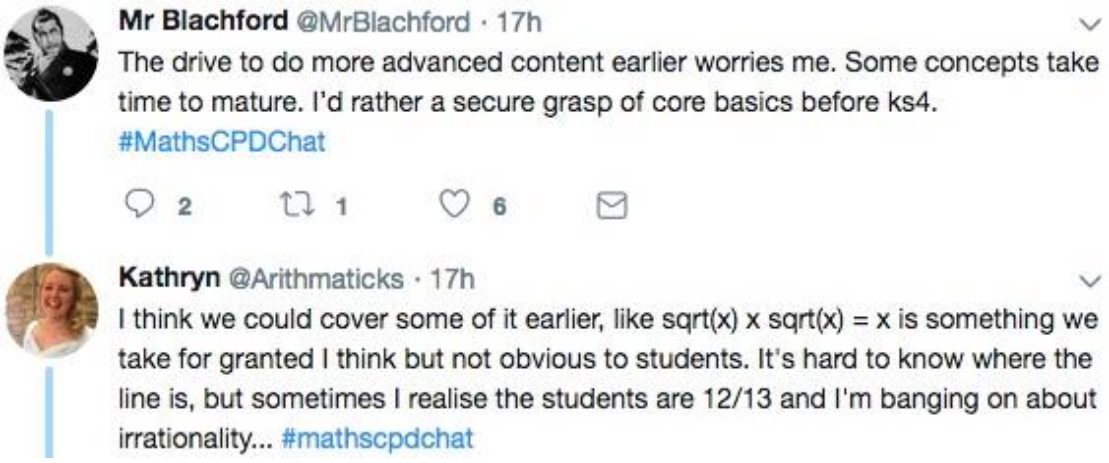
- **'ways in'** to mastery of indices and surds, for example pupils creating images to represent simple positive integer powers, square numbers, estimating roots of non-square numbers, powers of ten ('Powers of 10' video), developing fluency with positive integer powers leading to $n^a \times n^b = n^{a+b}$;
- helping pupils to see that it **'makes sense' to represent $\sqrt{n}$ as $n^{½}$**, for example comparing $n^{½} \times n^{½} = n^{½+½} = n^1 = n$ with $\sqrt{n} \times \sqrt{n} = n$, or asking 'if $n^a \times n^a = n^{a+a} = n^{2a} = n^1 = n$, what is a?';
- using $n^a \times n^a \times n^a = n$, $n^a \times n^a \times n^a \times n^a = n$, and so on, to introduce **fractional powers in general**;

- extending sequences by using pattern-spotting in order to **introduce and 'justify' $n^0 = 1$ and negative powers**, for example using 'halving' or dividing by 2, to obtain … … … … $2^3(8)$, $2^2(4)$, $2^1(2)$, $2^0(1)$, $2^{-1}(½)$, $2^{-2}(¼)$, $2^{-3}(⅛)$, …;
- representing n^a where a is negative **as a decimal**, eg $10^{-2} = 1/10^2 = 1/100 = 0.01$, 'complications' that arise when the base generates a recurring or lengthy-digit decimal, such as $7^{-2} = 1/7^2 = 1/49 = 0.020408163…$;
- introducing **negative powers by linking to 'scientific notation' ($a \times 10^b$)** where b (the power of 10) is negative … discussion about distinction between 'scientific form' and 'normalised scientific form' ('standard form') … in 'standard form' $1 \leq a < 10$ in $a \times 10^b$, but in 'engineering form' b must be a multiple of 3 so $1 \leq a < 1000$;
- introducing **negative powers by pupils dividing** a base-raised-to-a-power by the-same-base-raised-to-a-higher-power, for example $2^3 \div 2^5 = 2^{3-5} = 2^{-2}$ and $2^3 \div 2^5 = 8 \div 32 = ¼$, so $2^{-2} = ¼$;
- **representing n^a as a decimal when a is a fraction** … leading to encountering non-recurring, non-terminating, decimals, and so to the idea of an irrational number, and to introducing the word 'surd' … for example $2^{½} = \sqrt{2} = 1.412$ … ;
- **defining 'surd' for/with pupils** … whether or not a surd is necessarily an irrational number … 'surd' linked to 'absurd' through Latin derivation from 'surdus'/'deaf to reason' … definition is often oversimplified in modern textbooks … 'quadratic surds' for $\sqrt{n}$ … 'surdic expressions', eg '$41 + \sqrt{6}$' … 'radical' for any numerical expression under a n^{th} root (France) … proof that $\sqrt{8}$ is irrational … pupils 'pinning down' irrationality first so that they can then appreciate that, $\sqrt{0.25}$ (for example) is not a surd;
- using **prime factor decomposition to simplify surds**;
- where **surds first appear in schools' 'Schemes of Learning'** … whether Y8 is too early for fractional indices … worries about drive to address more advanced content earlier … secure grasp of 'core basics' before KS4 … 'conceptual exploration' when pupils are able to appreciate it … 30 years ago did we teach more than we now do?
- pupils **coping with n^a where a is expressed as a decimal**, for example $32^{0.6} = 32^{⅗} = (\sqrt[5]{32})^3 = 2^3 = 8$;
- **simplifying expressions involving 'nasty-decimal indices'**, for example $(n^{0.27272727…})^{11} = n^{(0.27272727… \times 11)} = n^{((27/99) \times 11)} = n^{((3/11) \times 11)} = n^3$;
- **Mrs Malley's** fractional and negative indices **dance** … done with Y11!

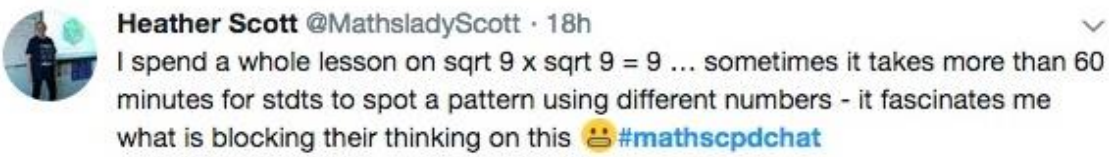
An interesting ‘conversation’ of tweets, about the stage in their schooling at which pupils address indices (in some depth) and surds, followed from these two tweets by Kathryn Darwin:

including these two from Mrs M and Jess Prior:

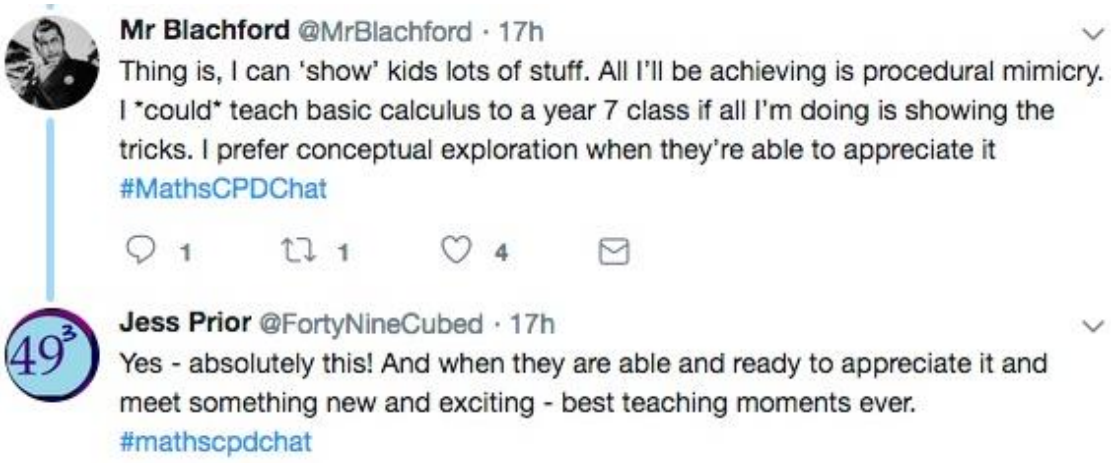
these two from Mr Blachford and Kathryn Darwin:



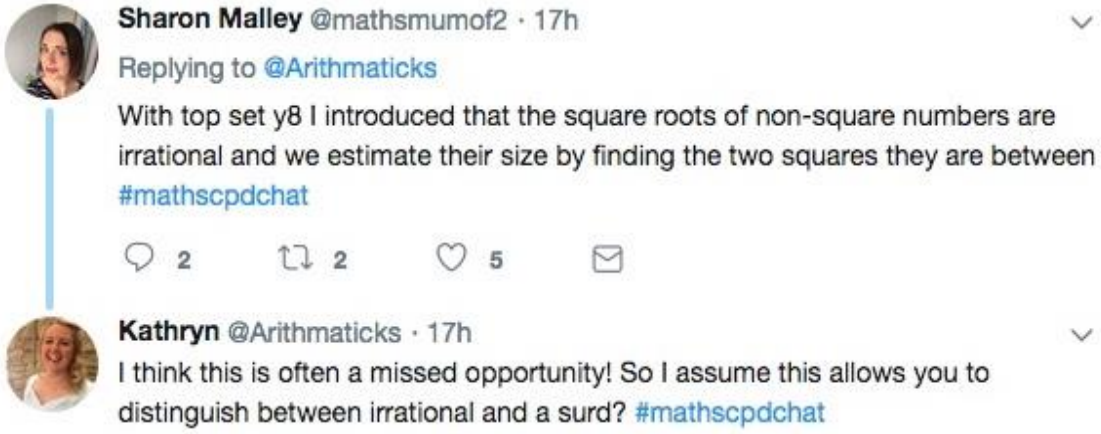
this from Heather Scott:



these two from Mr Blachford and Jess Prior:



these two from Sharon Malley and Kathryn Darwin:



and this one from Heather Scott:

(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Teaching surds – how Twitter came to the rescue!, which is an article by Kathryn Darwin in the NCETM Secondary Magazine 151 (published on 4 December 2018), shared by Mary Pardoe

Powers of Ten, which is a film by Charles and Ray Eames that 'takes the viewer on an adventure in magnitudes' (by changing the 'scale of view' of a scene by both positive and negative powers of 10 … in a most amazing way), shared by Steve L

Engineer Symbols which shows how to switch between ways of showing numbers on a particular Casio calculator, shared by The Calculator Guide

Visualising Indices which explores visual representations of indices and draws on creativity, shared by Mary Pardoe

SUMAZE! which is an enjoyable challenging online game from MEI/Integral; pupils using it are likely to develop fluency in using operations to combine numbers many of which are small positive whole numbers raised to a small positive integer power, shared by Mary Pardoe

Index Issues which is a page on Underground Mathematics; it addresses issues involved in working with indices (at Higher GCSE level and just beyond), shared by Mary Pardoe

John and Betty's Journey into Complex Numbers which is an unusual PowerPoint file; it reveals, slide-after-slide, an illustrated story in which the need for new (representations of) numbers arises naturally in an easy-to-understand context, shared by Sharon Malley