MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

3 Multiplicative reasoning

3.2 Trigonometry

Guidance document | Key Stage 3

Making connections

The NCETM has identified a set of six 'mathematical themes' within Key Stage 3 mathematics that bring together a group of 'core concepts'.

The third of these themes is *Multiplicative reasoning*, which covers the following interconnected core concepts:

3.1 Understanding multiplicative relationships

3.2 **Trigonometry**

This guidance document breaks down core concept *3.2 Trigonometry* into two statements of knowledge, skills and understanding:

3.2.1 Understand the trigonometric functions

3.2.2 Use trigonometry to solve problems in a range of contexts

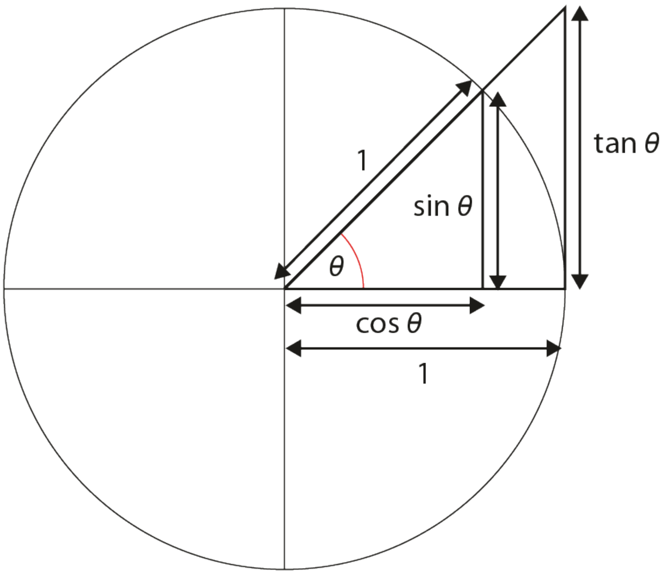
Then, for each of these statements of knowledge, skills and understanding we offer a set of key ideas to help guide teacher planning.

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Overview

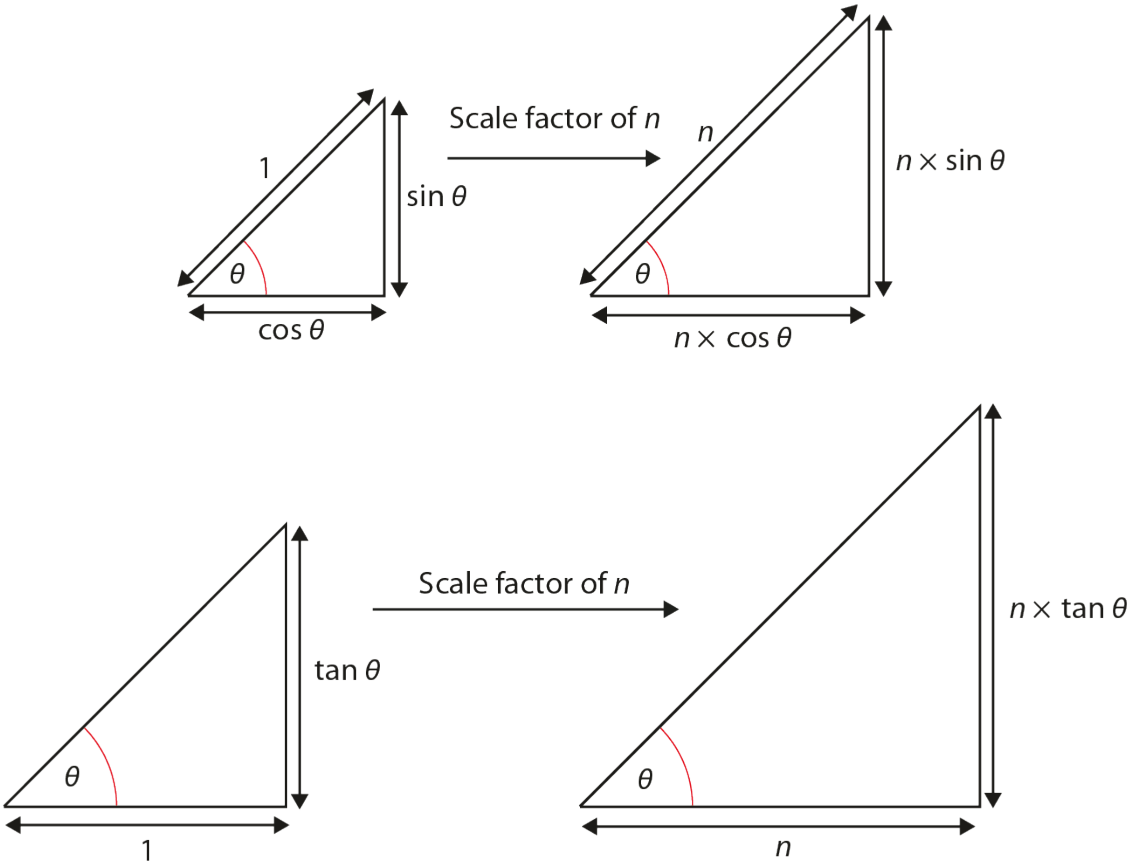
At Key Stage 2, students solved problems involving similar shapes, where the scale factor was known or could be found. At Key Stage 3, this work on similarity and scale factors is linked to the trigonometric functions and the fundamental ratios of $\sin\theta = \frac{opp}{hyp}$, $\cos\theta = \frac{adj}{hyp}$ and $\tan\theta = \frac{opp}{adj}$. The intention is that trigonometry is connected to previous learning and not perceived as a stand-alone topic.

In the key ideas 3.2.1.1 and 3.2.1.2, the connection is made explicit, beginning with where the trigonometric functions – sine, cosine and tangent (often called 'circular functions') – come from:

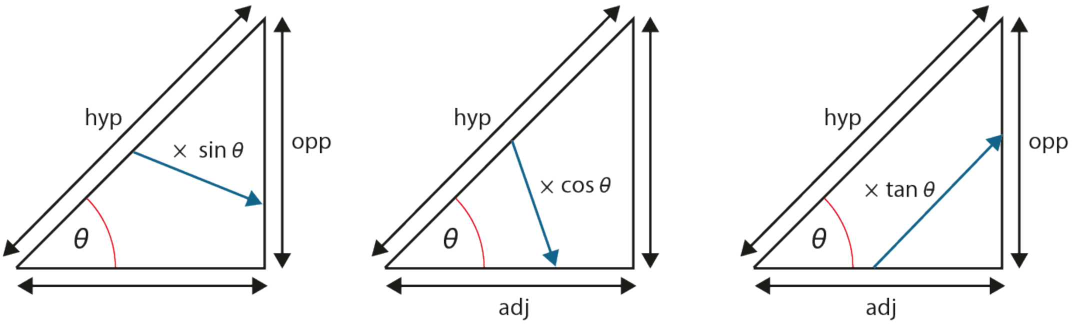


By using existing knowledge and understanding of similarity, scale factor and multiplicative relationships, an awareness is built of how the length of sides and size of angles in right-angled triangles can be calculated and, hence, provide solutions to a wide range of practical problems.

This sense of all right-angled triangles being a scaling of one of the two 'unit' right-angled triangles within the unit circle emphasises the multiplicative relationship *between* triangles:



Another important awareness is the multiplicative relationship (or ratio) *within* each right-angled triangle:



These trigonometric ratios are explored in key idea 3.2.1.3.

It is important for students to develop a secure understanding of trigonometry in right-angled triangles in 2D figures to support further study in Key Stage 4, such as:

- recognising, sketching and interpreting graphs of trigonometric functions
- applying the ratios to find angles and lengths in 3D figures
- knowing and applying the sine rule and cosine rule to find unknown lengths and angles in non-right-angled triangles.

Prior learning

Before beginning to teach *Trigonometry* at Key Stage 3, students should already have a secure understanding of the following from previous study:

Key stage	Learning outcome
Upper Key Stage 2	• Solve problems involving similar shapes where the scale factor is known or can be found
Key Stage 3	• 6.1.2 Understand and use similarity and congruence **Please note:** The numerical code refers to a statement of knowledge, skills and understanding in the NCETM breakdown of Key Stage 3 mathematics.

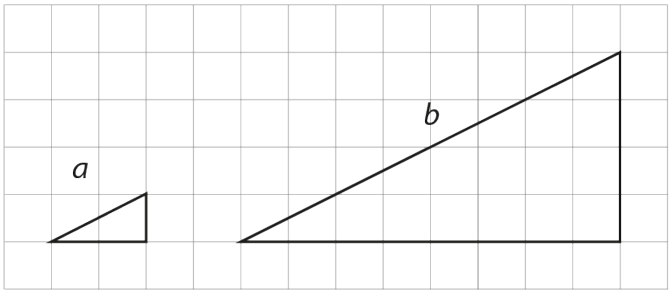
You may find it useful to speak to your partner schools to see how the above has been covered and the language used.

You can find further details regarding prior learning in the following segments of the NCETM primary mastery professional development materials[1]:

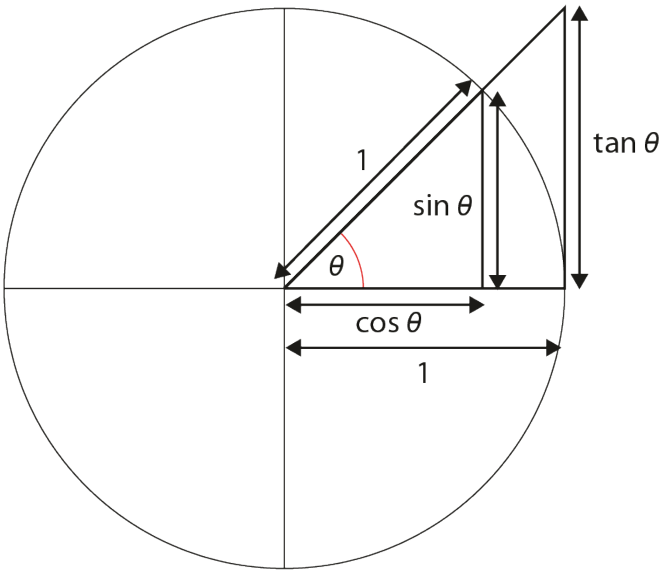
- Year 4: 2.17 Structures: using measures and comparison to understand scaling
- Year 6: 2.27 Scale factors, ratio and proportional reasoning
- Year 6: 2.30 Multiplicative contexts: area and perimeter 2

Checking prior learning

The following activity from a Standards & Testing Agency's past mathematics paper[2] offers a useful idea for assessment, which you can use in your classes to check whether prior learning is secure:

Reference	Activity
2017 Key Stage 2 Mathematics Paper 2: reasoning Question 22	*Here are two similar right-angled triangles.*  *Write the ratio of side a to side b.* Source: *Standards & Testing Agency* Public sector information licensed under the Open Government Licence v3.0

Key vocabulary

Term	Definition
adjacent	In trigonometry, one of the shorter two sides in a right-angled triangle. The side adjacent or next to a given angle.
hypotenuse	In trigonometry, the longest side of a right-angled triangle. The side opposite the right-angle.
opposite	In trigonometry, one of the shorter two sides in a right-angled triangle. The side opposite a given angle.
trigonometric functions (sine, cosine, tangent)	Functions of angles. The main trigonometric functions are cosine, sine and tangent. Other functions are reciprocals of these. Trigonometric functions (also called the 'circular functions') are functions of an angle. They relate the angles of a triangle to the lengths of its sides. The most familiar trigonometric functions are the sine, cosine and tangent in the context of the standard unit circle with radius 1 unit, where a triangle is formed by a ray originating at the origin and making some angle with the *x*-axis; the sine of the angle gives the length of the *y*-component (rise) of the triangle, the cosine gives the length of the *x*-component (run), and the tangent function gives the slope (*y*-component divided by the *x*-component).  Trigonometric functions are commonly defined as ratios of two sides of a right-angled triangle containing the angle. They can equivalently be defined as the lengths of various line segments from a unit circle. $\cos A = \frac{b}{c}$ $\sin A = \frac{a}{c}$ $\tan A = \frac{\sin A}{\cos A} = \frac{a}{b}$

Collaborative planning

Below we break down each of the two statements within *Trigonometry* into a set of key ideas to support more detailed discussion and planning within your department. You may choose to break them down differently depending on the needs of your students and timetabling; however, we hope that our suggestions help you and your colleagues to focus your teaching on the key points and avoid conflating too many ideas.

Please note: We make no suggestion that each key idea represents a lesson. Rather, the 'fine-grained' distinctions we offer are intended to help you think about the learning journey irrespective of the number of lessons taught. Not all key ideas are equal in length and the amount of classroom time required for them to be mastered will vary, but each is a noteworthy contribution to the statement of knowledge, skills and understanding with which it is associated.

The following letters draw attention to particular features:

D Suggested opportunities for **deepening** students' understanding through encouraging mathematical thinking.

L Examples of shared use of **language** that can help students to understand the structure of the mathematics. For example, sentences that all students might say together and be encouraged to use individually in their talk and their thinking to support their understanding (for example, *'The smaller the denominator, the bigger the fraction.'*).

R Suggestions for use of **representations** that support students in developing conceptual understanding as well as procedural fluency.

V Examples of the use of **variation** to draw students' attention to the important points and help them to see the mathematical structures and relationships.

PD Suggestions of questions and prompts that you can use to support a **professional development** session.

For selected key ideas, marked with an asterisk (*), we exemplify the common difficulties and misconceptions that students might have and include elements of what teaching for mastery may look like. We provide examples of possible student tasks and teaching approaches, together with suggestions and prompts to support professional development and collaborative planning. You can find these at the end of the set of key ideas.

Key ideas

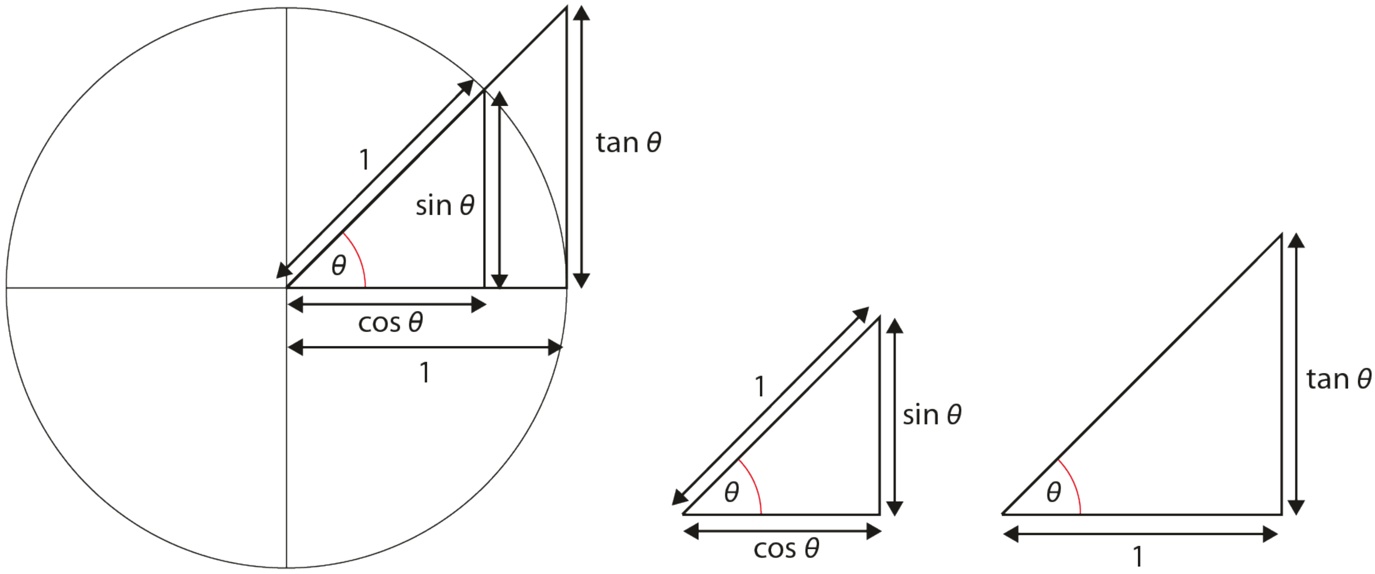
3.2.1 Understand the trigonometric functions

The trigonometric functions (sine, cosine and tangent) are introduced to students in an accessible and meaningful way: students observe the motion and position of a point moving around a unit circle. When coordinate axes are introduced, the circle is centred on the origin and students can make estimates of the *x*- and *y*-coordinates of the point as the radius going through that point rotates. This will give students what may be their first experience of a non-algebraic, non-linear function.

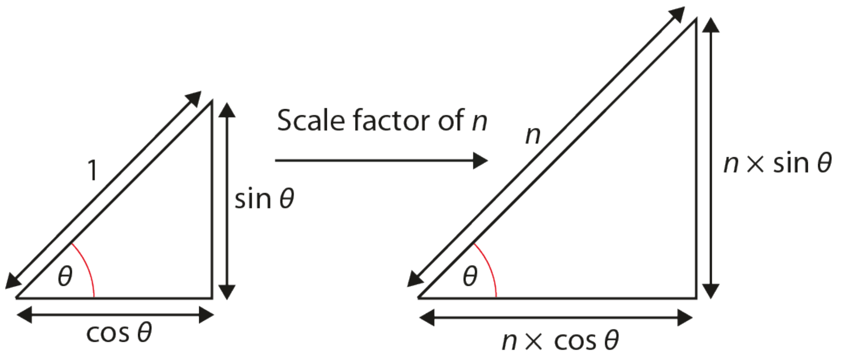
Once a good intuitive feel for these functions has been achieved, students' attention can be drawn to:

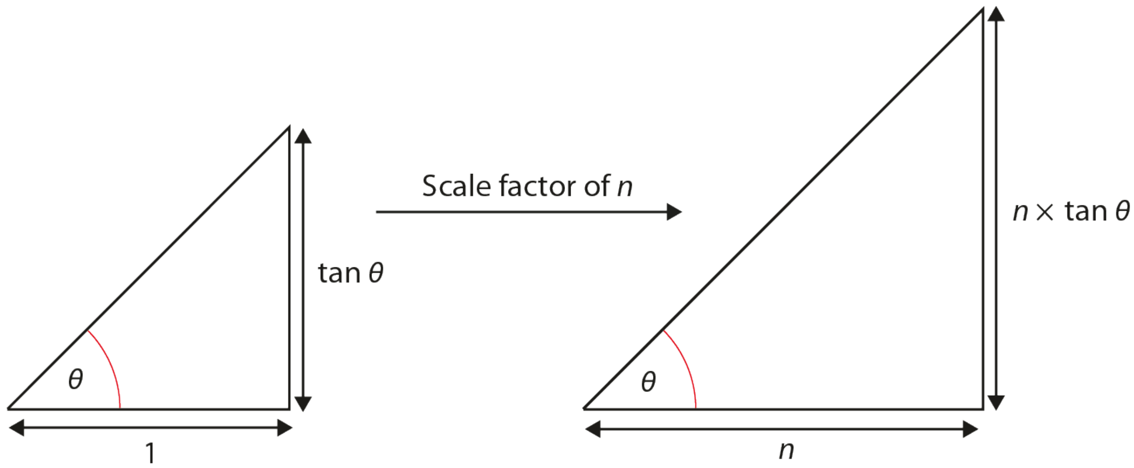
a) specific ratios within the two key right-angled triangles defined by the unit circle:

- one with a hypotenuse of length 1, where the opposite and adjacent sides of the triangle have lengths sin θ and cos θ, respectively
- another with adjacent side of length 1, where the opposite side has length tan θ.



b) the fact that, in similar triangles, the corresponding sides have all been scaled up by the same amount and the angle remains constant:





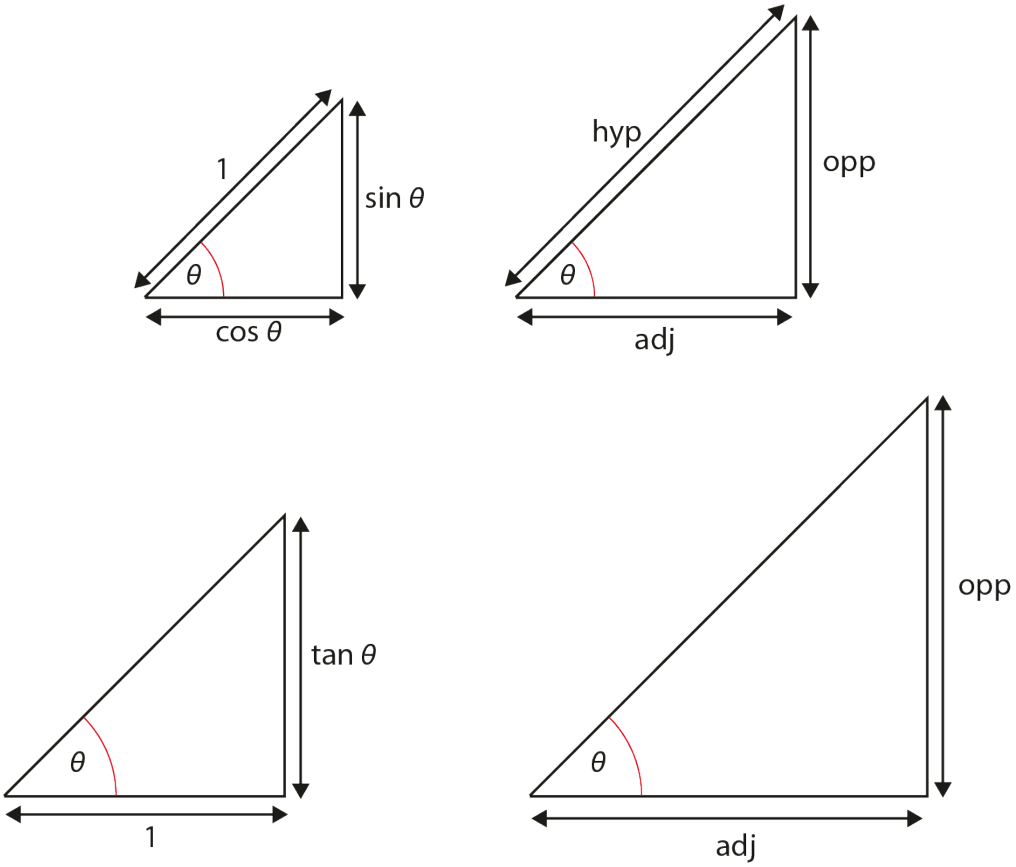
These give rise to the formulae:

length of opposite side = length of hypotenuse × sin θ ($o = h \times \sin\theta$)

length of adjacent side = length of hypotenuse × cos θ ($a = h \times \cos\theta$)

length of opposite side = length of adjacent side × tan θ ($o = a \times \tan\theta$)

The above is based on an understanding that there is a multiplicative link *between* two similar right-angled triangles. Another important awareness is that this implies a multiplicative link *within* each triangle. So, for example, in these pairs of similar triangles:



$$\frac{\sin\theta}{1} = \frac{opp}{hyp}\,;\ \frac{\cos\theta}{1} = \frac{adj}{hyp} \text{ and } \frac{\tan\theta}{1} = \frac{opp}{adj}$$

3.2.1.1* Understand that the trigonometric functions are derived from measurements within a unit circle

3.2.1.2 Recognise the right-angled triangle within a unit circle and use proportion to scale to similar triangles

3.2.1.3* Know how the sine, cosine and tangent ratios are derived from the sides of a right-angled triangle

3.2.2 Use trigonometry to solve problems in a range of contexts

A key awareness for students will be how the ability to find missing sides and angles in any right-angled triangle is extremely useful in so many practical situations (for example, finding: the height of inaccessible objects, the length of an object given the length of its shadow, and the direction in which to steer a boat across a river where there is a current).

It will be important to choose a wide range of standard and non-standard problems in order for students to be confident in modelling real-life situations mathematically, and in recognising what information is given, what information is required and which trigonometric relationship needs to be used to reach a solution.

As students practise their skills in 3.2.2.2 and 3.2.2.3, the opportunity arises to introduce a variety of contextual situations so students can appreciate that, once they strip away the context, the remaining mathematical model can be solved abstractly, which can then be interpreted to arrive at the contextual solution.

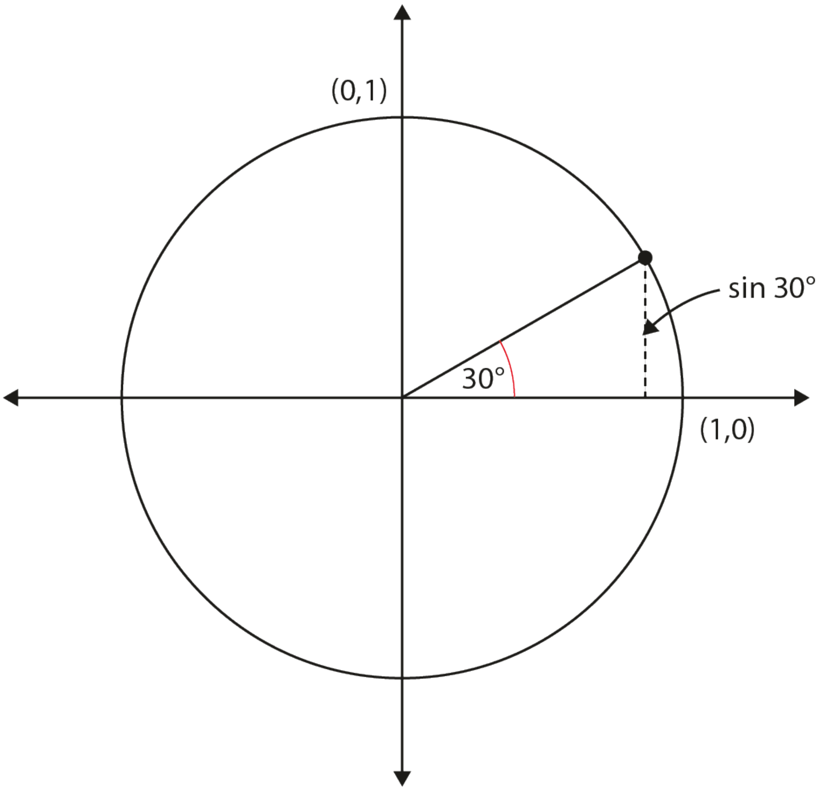
3.2.2.1 Choose appropriate trigonometric relationships to use to solve problems in right-angled triangles

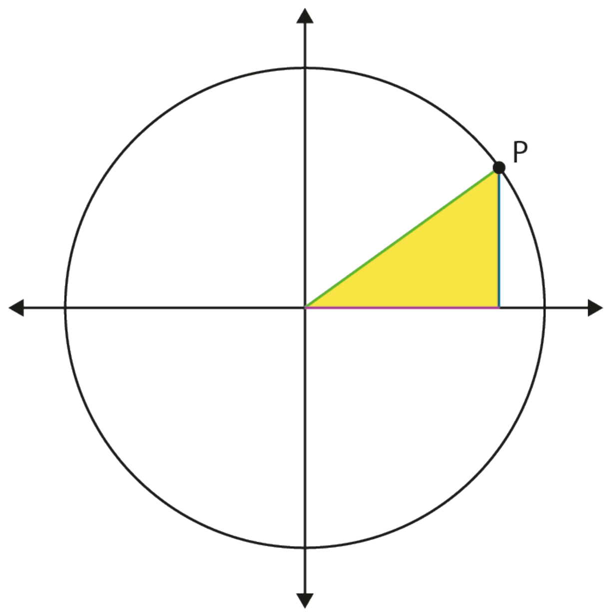
3.2.2.2 Use trigonometric ratios to find a missing side in a right-angled triangle

3.2.2.3 Use trigonometric ratios to find a missing angle in a right-angled triangle

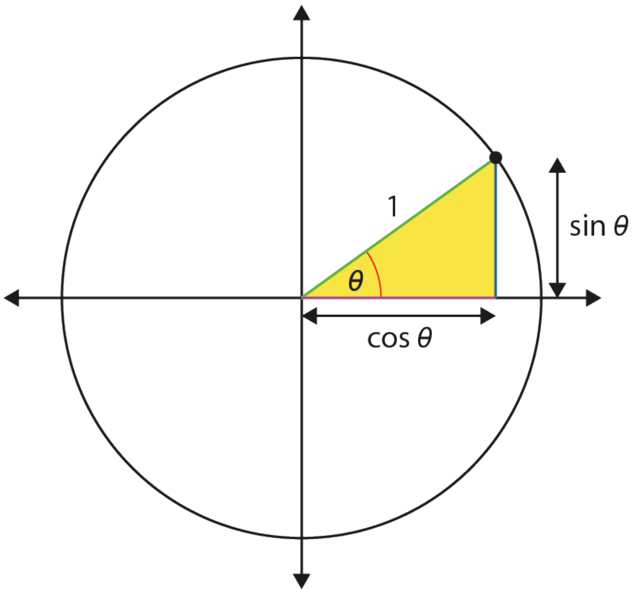
Exemplified key ideas

3.2.1.1 Understand that the trigonometric functions are derived from measurements within a unit circle

Common difficulties and misconceptions
Students may rely on mnemonics, such as SOHCAHTOA ($\sin\theta = \frac{opposite}{hypotenuse}$, $\cos\theta = \frac{adjacent}{hypotenuse}$, $\tan\theta = \frac{opposite}{adjacent}$), without understanding the underlying ideas, and so reduce the study of trigonometry to an entirely procedural application of these formulae. Students should understand the idea of the unit circle and the fact that, for example, the sine of an angle is the *y*-coordinate of the point where the radius has been rotated through that angle:  This should help students understand why sin 0° = 0 and sin 90° = 1, that the values in between do not follow a linear sequence and therefore, sin 30°, *not* sin 45°, is 0.5 and, later, why sin 30° = cos 60°, and so on. By introducing these trigonometric ratios (sine, cosine and tangent) through the accessible context of a point moving around a unit circle, students gain a coherent and connected view of these new ideas, which enables them to make sense of this area of mathematics.

What students need to understand	Guidance, discussion points and prompts
Identify a triangle within the unit circle and understand that the hypotenuse is the only side with a constant length. *Example 1:* *A line joins the centre of the circle to a point, P, on the circumference.*  *Imagine the point P moving anticlockwise around the circle. As it does so, think about the length of:* • *the green line (the radius)* • *the purple line (the x-coordinate of P)* • *the blue line (the y-coordinate of P).* *a) How does the length of the green line change?* *b) How does the length of the blue line change?* *c) How does the length of the purple line change?* *d) What type of triangle is formed by these three lines? Is it the same kind of triangle for all possible positions of P?*	**R** Students should be given time to imagine the point moving around the circle and to think about how the *x*- and *y*-coordinates of the point vary as the angle increases. Dynamic geometry software can helpfully be used in conjunction with mental imagery activities to develop a deep understanding of the structures and relationships involved. **PD** It could be argued that this image offers direct access to the topic of trigonometry because relatively little prior knowledge is required in order to be able to engage successfully with this part of the curriculum. To what extent do you agree with this? What does 'successful engagement with this part of the curriculum' look like to you?

Example 2:



The y-coordinate of the point as it moves around the circle is called the sine of the angle (sin θ). The x-coordinate is called cos θ.

Imagine the point P moving around the circle and the value of θ taking different values.

Find the value of:

a) sin 0°

b) cos 0°

c) sin 90°

d) cos 90°

Make an estimate for the value of:

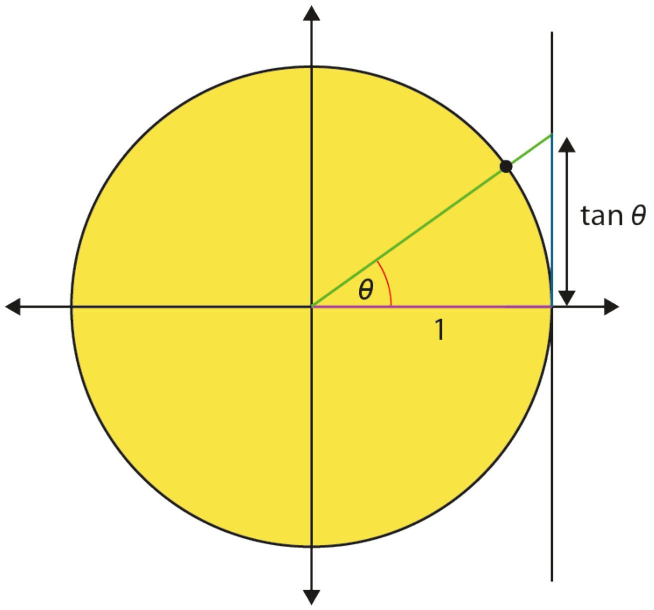
e) sin 45°

f) cos 45°

g) sin 30°

h) cos 30°

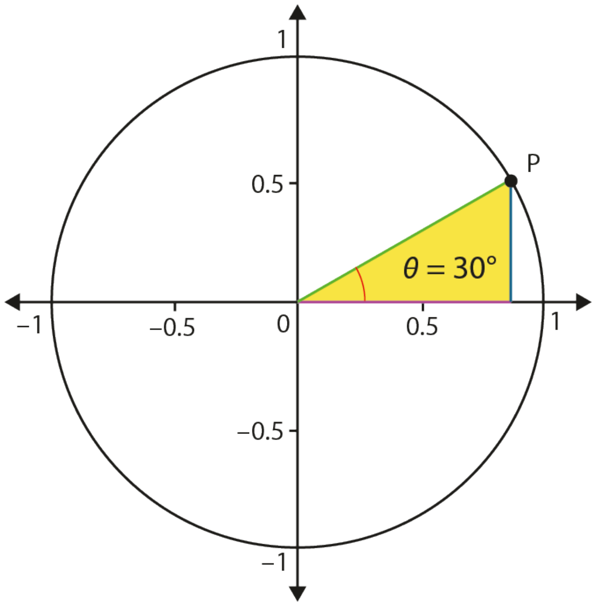
R In *Example 2*, the unit circle is used to give meaning to the trigonometric functions of sine, cosine and tangent, and the relationship between them. It does not offer a means to reach a solution to problems; rather, it offers a way for students to make sense of the mathematics involved and paves the way for more advanced study at Key Stages 4 and 5.

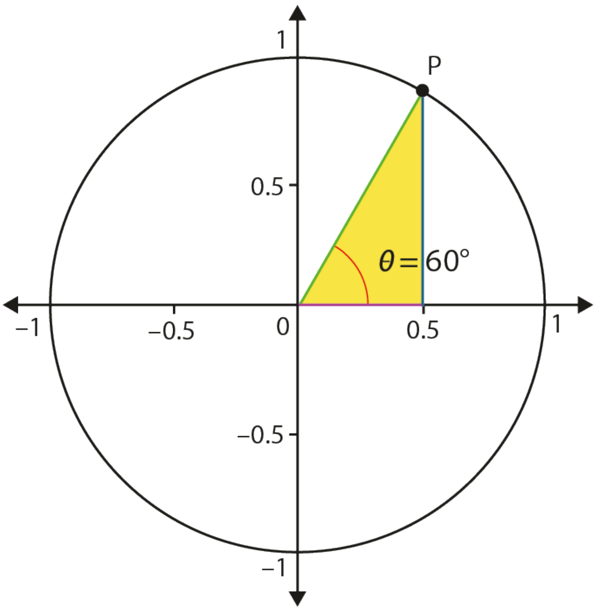
Example 3:  *The tangent of the angle is marked on the diagram above.* *Find the value of, or estimate, as appropriate:* *a) tan 0°* *b) tan 30°* *c) tan 45°* *d) tan 60°* *e) tan 90°*	The image in *Example 3* allows students to understand why the tangent function (unlike the sine and cosine functions) does not have a maximum or minimum value, but tends to infinity as θ tends to 90°.

Understand that the height and length of the base of the triangle do not change in a linear way as the point moves around the circle's edge.

Example 4:

The point P has been moved around the circle to create two different triangles. In one triangle, the green line makes an angle of 30° with the base; in the other triangle, it makes an angle of 60°.





a) *How long is the green line in each picture?*

b) *Estimate the length of the blue line in each picture.*

R In *Example 4*, the dimensions of the circle have been included for the first time.

Students' experience of functions often leads them to assume that relationships are proportional. The use of the unit circle as a representation of a trigonometric function provides an image of why doubling the angle in a triangle does not double the height. This may not be obvious to students, so encouraging them to explain why this is so, followed by teacher-led discussion, should help them understand rather than just accept this as a given fact.

It is important also to draw students' attention to the fact that there is a maximum height that can be reached. No matter how great the angle, the height of the triangle within the unit circle cannot exceed 1.

You could also ask students to estimate the height of the triangle when the angle is 45° (half-way between the two axes) to offer a further example of the non-linear relationship between the angle and the height.

If you are using a dynamic version of this image, then you might also like to show the height at, for example, 40° and ask students to then predict the height at 20° or at 80°. The use of dynamic software also allows for greater exploration of the height as the point moves out of the first quadrant.

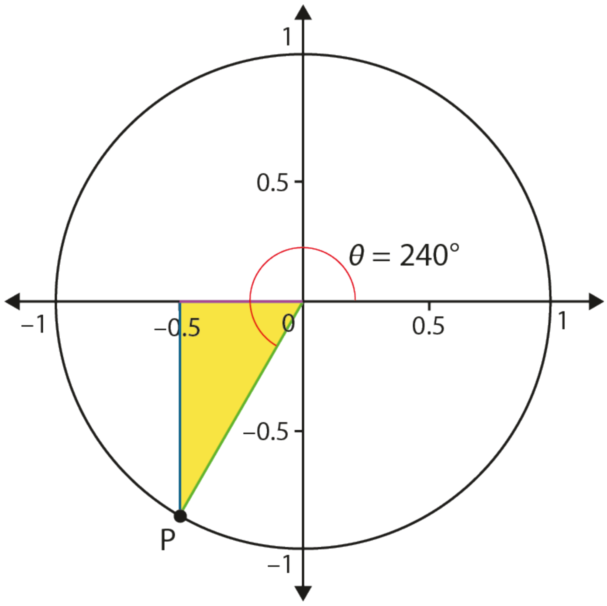
D The use of the unit circle allows students to experience using trigonometric functions with angles greater than 90°. In part e), students are asked to make sense of an angle of 240°. If they do not notice that the triangle is congruent to one of those above, then their attention should be drawn to this.

It should be noted that the intention here is to further explore the unit circle as a representation, not to calculate or formalise the relationships.

c) *Explain why, when the point has rotated 45° around the circle, it is not half-way up (i.e. at the point (0, $\frac{1}{2}$).*

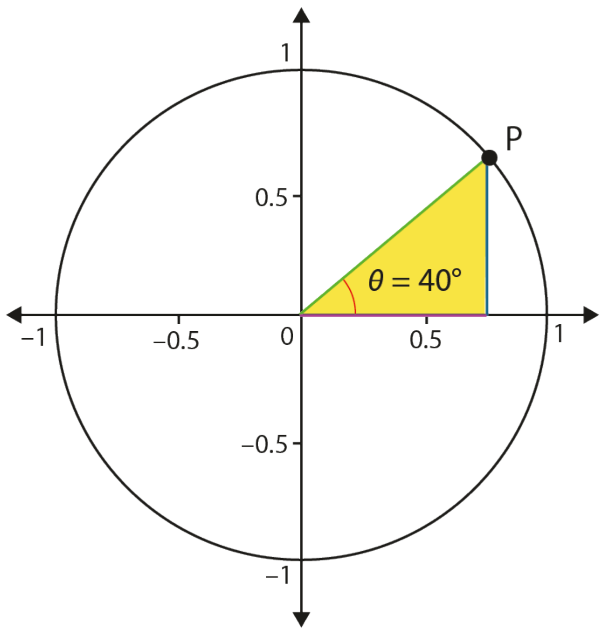
d) *Explain why doubling the angle at the centre of the circle does not double the height of the triangle.*

e) *How would you estimate and describe the 'height' of the triangle below?*

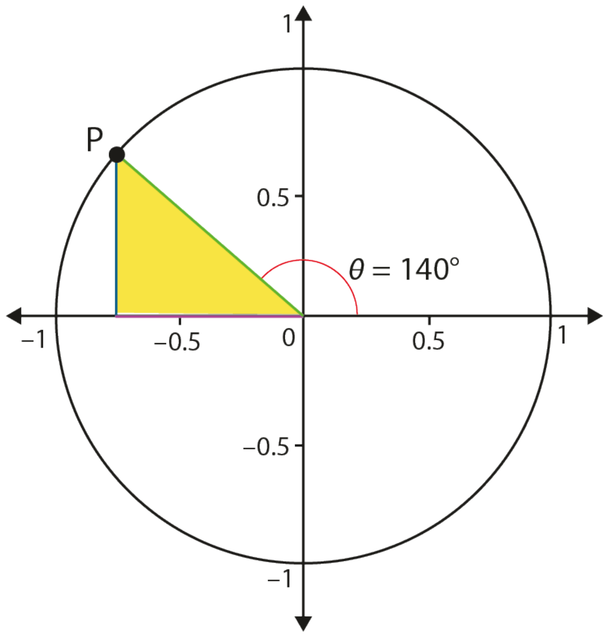


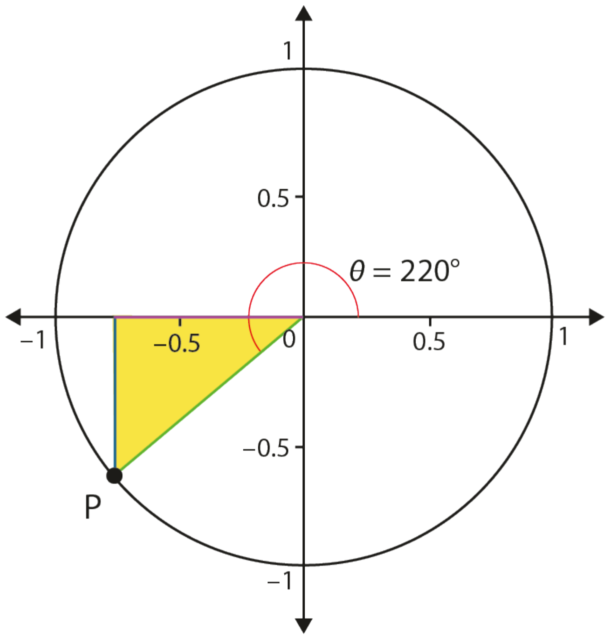
Example 5:

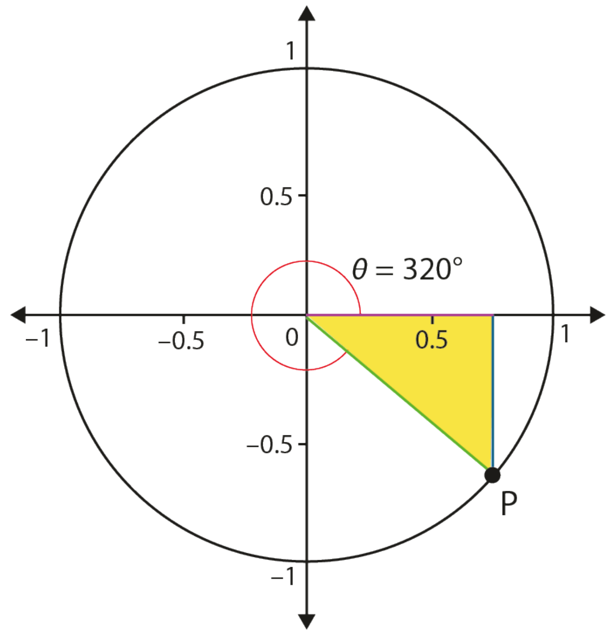
What is the same and what is different about these triangles? Explain how you know.



D In *Example 5*, students are given an opportunity to explore the unit circle further in order to make sense of some relationships. Encourage students to fully explain their reasoning.


1
P
0.5
θ = 140°
−1
−0.5
0
0.5
1
−0.5
−1


1
0.5
θ = 220°
−1
−0.5
0
0.5
1
−0.5
P
−1

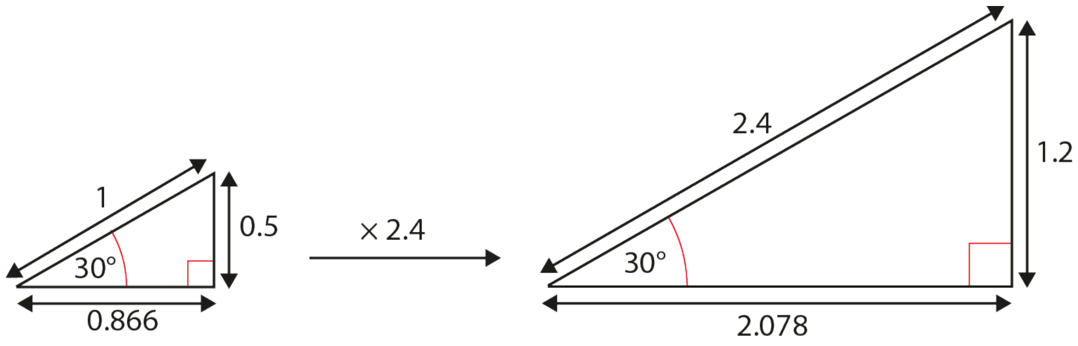
1 0.5 θ = 320° −1 −0.5 0 0.5 1 −0.5 P −1	

3.2.1.3 Know how the sine, cosine and tangent ratios are derived from the sides of a right-angled triangle

Common difficulties and misconceptions

Students may understand the relationships between similar triangles but struggle to see the same relationships within each triangle.

For example, in these two similar triangles, not only is there a relationship *between* the two triangles (i.e. a scale factor of 2.4), but also the ratios *within* each triangle are equal (i.e. $\frac{0.5}{1} = \frac{1.2}{2.4} = 0.5$ and $\frac{0.866}{1} = \frac{2.078}{2.4} = 0.866$).



All lengths correct to 3 decimal places.

This understanding will support students in seeing that, for any given angle, certain ratios in a right-angled triangle remain constant, and the value of these ratios corresponds to the values of sine, cosine and tangent of that particular angle. This will enable students to derive the trigonometric ratios:

$$\sin\theta = \frac{opp}{hyp}, \cos\theta = \frac{adj}{hyp} \text{ and } \tan\theta = \frac{opp}{adj}$$

The use of ratios supports efficient and fluent calculation. Students should eventually relinquish the use of the unit circle when the connection between the trigonometric functions (as defined on the circle) and these ratios is secure.

However, students may find it difficult to remember the various ratios and struggle to know which one to apply in which situation. In this case, it will be important to support them by continuing to emphasise the 'unit circle' diagram and to identify the right-angled triangle within the circle that relates to the problem to be solved.

What students need to understand	Guidance, discussion points and prompts

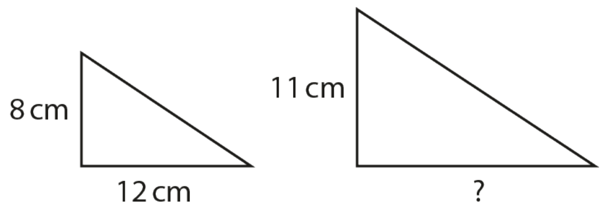
What students need to understand

Understand that the multiplicative relationship between sides of a right-angled triangle can be used to find missing sides in similar triangles.

Example 1:

Consider the shadow created by a vertical stick on a horizontal beach and how this could be represented as a right-angled triangle.

a) *If an 8 cm stick casts a 12 cm shadow, how long a shadow would an 11 cm stick cast at the same time of day?*



b) *At a different time of day, a 6 cm stick casts a shadow 5 cm long. How long would the shadow of a 7 cm stick be at this time of day?*

c) *Later, the 6 cm stick casts a 13 cm shadow. What length of stick would cast a 20 cm shadow at the same time of day?*

Guidance, discussion points and prompts

You could introduce the idea of a shadow of a person or a stick and ask students how the shadow changes at different times of the day. Use of dynamic geometry software can help illustrate this.

You could also include the scenario where the stick and shadow are the same value and ask students what that tells them about the angle of the sun at that point: *'At this same time of day, would all shadows be equal to the height of the objects that make them?'*

Through discussion of *Example 1*, the relationship between the sticks and the shadows can be explored.

Focusing on the relationship between the stick and its shadow is not the only way the answer can be deduced. The enlargement scale factor between the two similar triangles can be used. In this example, values have been chosen so that no one strategy is easier at this stage.

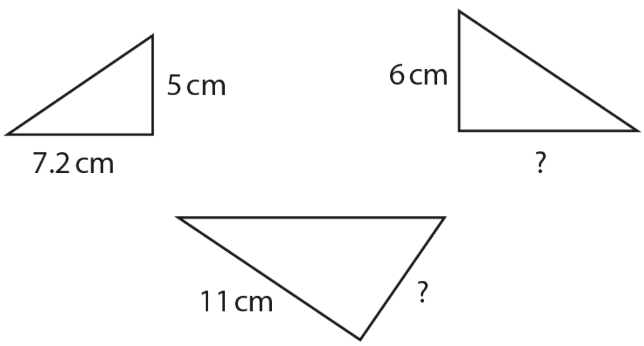
D You may wish to point out alternative methods, but attention should at least be drawn to the relationship between pairs of sides in the same triangle.

What students need to understand

Understand that corresponding sides are in proportion in similar triangles.

Example 2:

Here are three similar triangles.



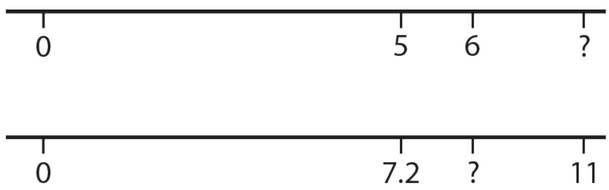
a) *What is the multiplier between the pair of labelled values for each triangle?*

b) *Find the missing values.*

Guidance, discussion points and prompts

Encourage students to use strategies they have developed in other aspects of the curriculum that have focused on multiplicative relationships.

R For example, they could use a double number line (also known as *'stacked number lines'*):



This will help students to focus on the common relationship between two corresponding sides and identify the inverse relationship, where appropriate.

D Students' attention can be drawn to the fact that, if the relationship between the short

	and medium sides in each triangle is identified as $\times\frac{7.2}{5}$, which is equivalent to $\times$ 1.44, the inverse relationship can be expressed as $\div$ 1.44 or, alternatively, $\times$ 0.694 (to 3 significant figures).

Understand the multipliers defined as sine, cosine and tangent refer to the relationships between each pair of sides within a right-angled triangle.

Example 3:

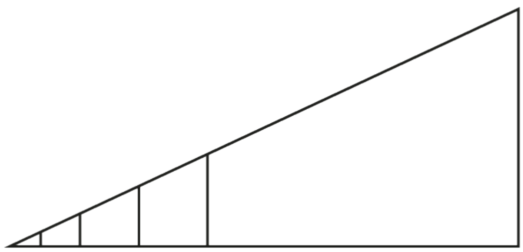
Using the relevant values in the table below (or using your calculator), find the missing side lengths.

Angle	Sine	Cosine	Tangent
10	0.1736	0.9848	0.1763
15	0.2588	0.9659	0.2679
20	0.3420	0.9397	0.3640
25	0.4226	0.9063	0.4663
30	0.5000	0.8660	0.5774
35	0.5736	0.8192	0.7002
40	0.6428	0.7660	0.8391
45	0.7071	0.7071	1
50	0.7660	0.6428	1.1918
55	0.8192	0.5736	1.4281
60	0.8660	0.5000	1.7321
65	0.9063	0.4226	2.1445
70	0.9397	0.3420	2.7475
75	0.9659	0.2588	3.7321
80	0.9848	0.1736	5.6713
85	0.9962	0.0872	11.4301
90	1	0	∞

V The numbers in *Example 3* have been chosen to initially allow students to use their image of the unit circle to find the answer to part a).

In subsequent parts, the numbers have been chosen to allow for some scaling (both up and down), and in order for students to notice the constant ratio between the opposite and hypotenuse in each case.

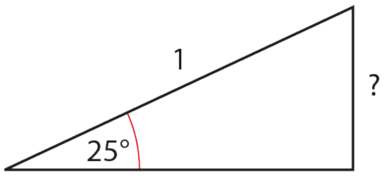
Discussion following this activity could usefully include asking, '*What would happen if these triangles were all drawn to scale and all "nested" together with the 25° angle positioned over each other, like this?*'



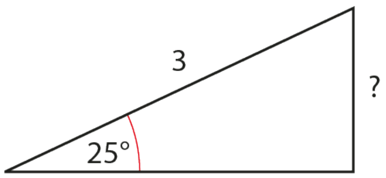
Not to scale

This could lead to analysing the connection between the opposite and the hypotenuse in each case and drawing students' attention to the fact that they are all equal to 0.4226 (sin 25°).

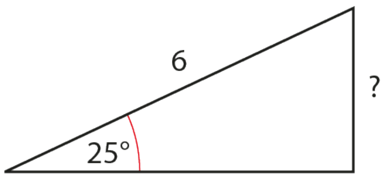
a)



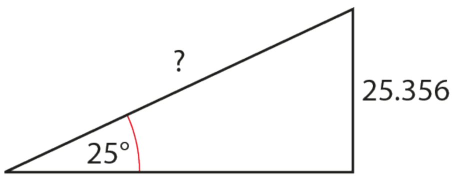
b)



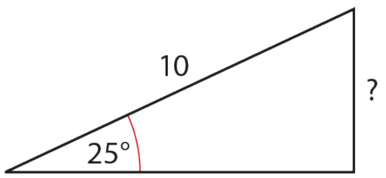
c)



d)



e)

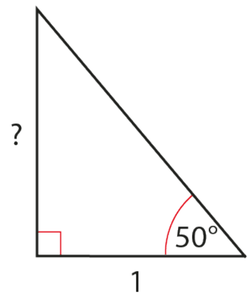


Not to scale

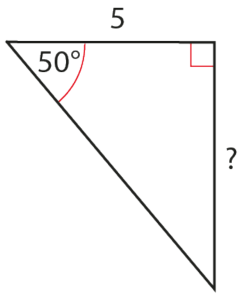
Example 4:

Using the relevant values in the table above (or using your calculator), find the missing side lengths.

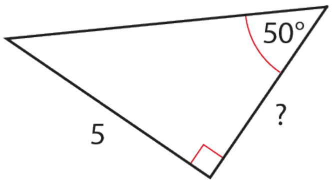
a)



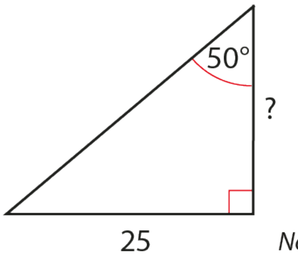
b)



c)



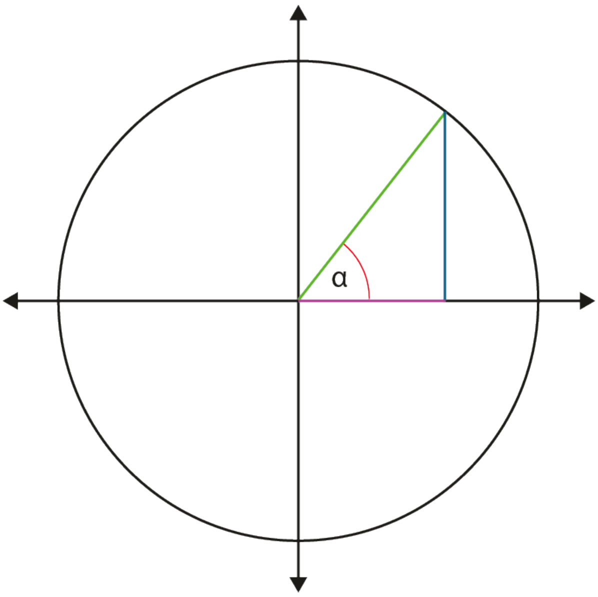
d)



Not to scale

V In *Example 4*, the lengths have been chosen to prompt discussion about the use of scaling (from the 'unit' triangle) and about the ratio between the opposite and the adjacent sides in each triangle being 1.1918 (tan 50°).

Example 5:



a) *If the adjacent (purple) side has a length of 5 cm and the angle a is 70°:*

  (i) what is the value of tan 70°?

  (ii) what is the length of the opposite (blue) side?

b) *If the hypotenuse (green side) has a length of 8 cm and the angle a is 60°:*

  (i) what is the value of sine 60?

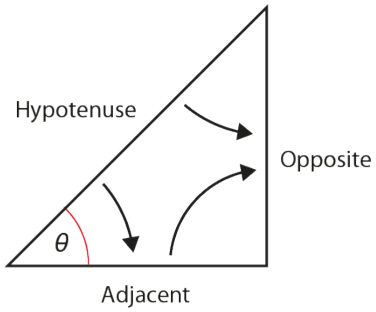
  (ii) what is the length of the opposite (blue) side?

  (iii) what is the value of cosine 60?

  (iv) what is the length of the adjacent (purple) side?

c) *If the hypotenuse is 1 cm and the angle a is 45°, what are the lengths of the opposite and adjacent sides?*

Students should understand the convention for naming the sides of a right-angled triangle in a standard triangle orientation and with a given angle, so that they can define sine, cosine and tangent as multipliers between pairs of sides.



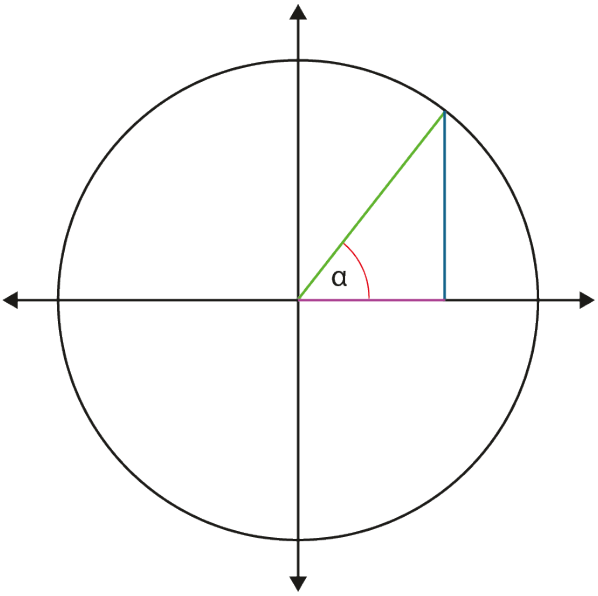
Adjacent	$\frac{\text{Opp}}{\text{Adj}}$ → $\tan\theta$	Opposite
Hypotenuse	$\frac{\text{Opp}}{\text{Hyp}}$ → $\sin\theta$	Opposite
Hypotenuse	$\frac{\text{Adj}}{\text{Hyp}}$ → $\cos\theta$	Adjacent

Attention needs to be drawn to the importance of the angle in each case, because the multiplier changes as the angle changes.

Note that a standard orientation of the triangle is used to emphasise the link with the unit circle at this stage.

Use the values of sine, cosine and tangent on a calculator to find unknown angles where the lengths of a right-angled triangle are known.

Example 6:



What are the values of a if the length of the blue line in this unit circle diagram is:

a) 0.5?

b) 0.38?

c) 0.9?

d) 1.2?

What are the values of a if the length of the purple line is:

e) $\frac{3}{4}$*?*

f) 0.7777?

g) −0.4?

You may wish to ask students to tackle *Example 6* by first using the table below (from *Example 3*). They can answer part a) and estimate part b) as being between 20 and 25 degrees.

Angle	Sine	Cosine	Tangent
10	0.1736	0.9848	0.1763
15	0.2588	0.9659	0.2679
20	0.3420	0.9397	0.3640
25	0.4226	0.9063	0.4663
30	0.5000	0.8660	0.5774
35	0.5736	0.8192	0.7002
40	0.6428	0.7660	0.8391
45	0.7071	0.7071	1
50	0.7660	0.6428	1.1918
55	0.8192	0.5736	1.4281
60	0.8660	0.5000	1.7321
65	0.9063	0.4226	2.1445
70	0.9397	0.3420	2.7475
75	0.9659	0.2588	3.7321
80	0.9848	0.1736	5.6713
85	0.9962	0.0872	11.4301
90	1	0	∞

However, at some stage, students will need to be shown how they can retrieve this information from a calculator and not only find an angle from a trigonometric value, but also the trigonometric value of any given angle.

Part d) has been chosen to remind students that the value of sine and cosine does not exist beyond 1.

In part e), there is the opportunity for you to point out that values do not have to be decimals; values given in a fractional format are just as acceptable.

	D You will need to consider whether introducing a negative number at this stage is appropriate for your students. This goes beyond the expectation at Key Stage 3, but may prove useful to link to the work on the unit circle in the previous key idea. Students should realise that the actual answer cannot be found as there are different angles for which −0.4 can be the value of cosine α, including a negative value of α. This understanding can be further deepened by inspecting the graphs of $y = \sin\theta$ and $y = \cos\theta$ to remind students that the values are repeated cyclically and there is a pattern they can predict. **PD** Students often struggle to understand the inputs and outputs of a calculator when using trigonometric ratios. To what extent might using tables of values be helpful as an intermediate step?

Weblinks

1 NCETM primary mastery professional development materials
https://www.ncetm.org.uk/resources/50639

2 Standards & Testing Agency past mathematics papers
https://www.gov.uk/government/collections/national-curriculum-assessments-practice-materials#key-stage-2-past-papers