Planning to teach circles – Supporting document

This document, which accompanies the 'Planning to teach circles' video, gives guidance on planning lessons that allow students to explore and understand the properties of a circle, including calculation of circumference and area.

Part 1 – The big idea

What do students need to know?

The KS3 National Curriculum programme of study (Geometry and measures section) mentions:

- calculate and solve problems involving perimeters of 2-D shapes (including circles), areas of circles and composite shapes
- derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies.

There are some important implications in these programme of study statements.

- That students need to be able to ***derive*** the properties of a circle, not just memorise them. They need to be able to see why properties such as area of a circle = πr^2 are true.
- That the use of 'appropriate language' is important. We need to use appropriate language ourselves but also encourage students to use it while talking about circles. They may know about radius and diameter but not be familiar with terms like chord, arc, segment, sector, etc.
- The mention of 'technologies' implies some level of exploration and discovery. The use of dynamic geometry software is ideal for such explorations.
- Students need to understand the concepts of circumference and area, distinguish between them, understand what π is, and how it is related to these properties of a circle.
- Students need to apply these ideas to solve a range of standard and non-standard problems.

What things typically go wrong?

Common misconceptions or causes of confusion include:

- formulae for circumference and area are sometimes seen by students as very similar and they can confuse the two. This is why students need to appreciate how the formulae are derived and appreciate that π (a constant) × d (a length) is also a length and that πr (a length) × r (a length) gives an area
- students may see π as a rather abstract symbol and may associate it with algebra and so confuse it with a variable. Introducing the number π in a wide range of other situations where numbers are used in calculations will help students to become familiar with the idea that it is just another number.

Part 2 – Prerequisites

Expectations from KS2

The following statement from the Key Stage 2 National Curriculum programme of study outlines what we might expect students to have had experience of before they arrive at secondary school:

- illustrate and name parts of the circles, including radius, diameter and circumference, and know that the diameter is twice the radius.

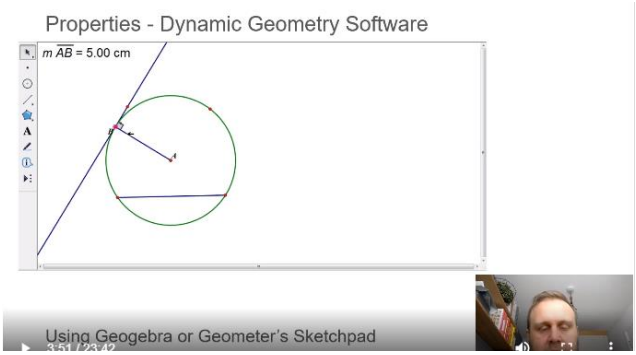
Notice that the mention of 'circumference' is only referring to knowing the 'name' of this feature, not its properties or how to calculate it. Introducing students to the rather surprising and potentially intriguing idea that whatever the size of the circle the ratio of the circumference to its diameter is the same will be a key learning experience to introduce at KS3.

A key related property often overlooked but equally surprising and intriguing is that whatever the initial size of the circle, if you scale up the diameter, the circumference will also scale up by the same factor.

Part 3 – Key teaching aspects

Key message

Exploring a circle using dynamic geometry software is a powerful way of teaching key terms, features and properties (you will see Matt model this in his video).

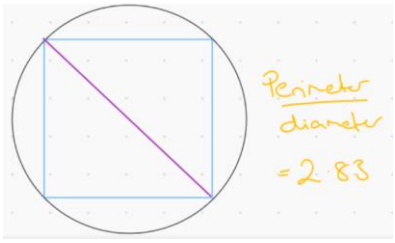


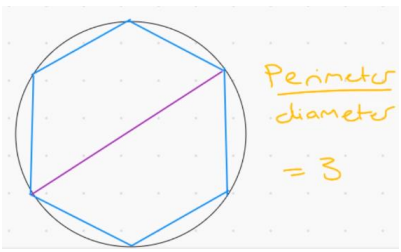
It will be important for students to be given some practical experience of:

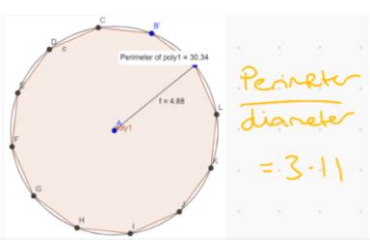
a) thinking about the problem "How do you measure the length of a curve?" and coming up with their own suggestions
b) testing out these ideas by measuring the circumference of different objects

and then having their attention drawn to the idea of a constant relationship.

Looking at the ratio of the perimeter to the 'diameter' of regular polygons (i.e. shapes whose vertices lie on a circle) can also provide an insight into this constant relationship $\pi = \frac{C}{d}$ and the corresponding formulae $C = \pi d$



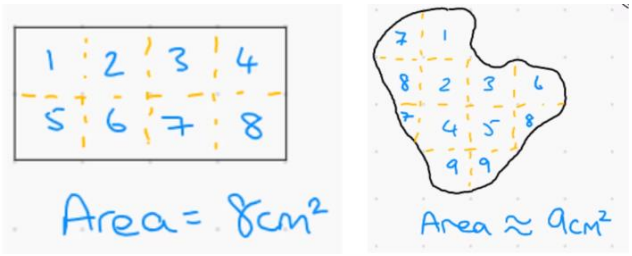




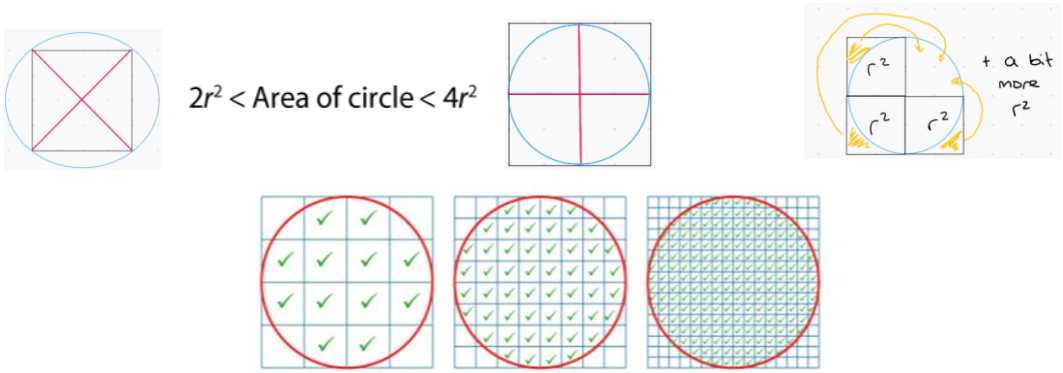
When offering problems to practise which require calculating the circumference, it will be important to make good use of variation in order to focus students' attention on key aspects such as:

- when the diameter or radius is multiplied by a certain amount, the circumference will also be multiplied by that same amount
- finding the circumference from the diameter and finding the diameter from the circumference are related calculations
- arc lengths are just proportions of the circumference; the proportion being determined by the angle subtended by the arc at the centre of the circle.

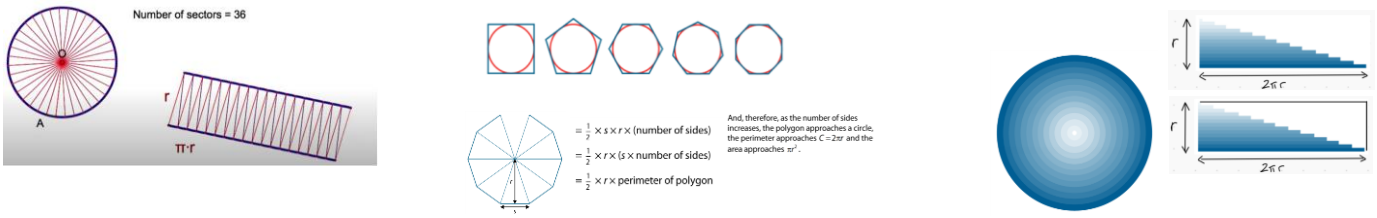
For the area of a circle, students need experiences which support them in thinking about the inherent difficulty in finding the area of a shape that cannot be easily divided up into square units:



They also need to experience certain representations which help give them a sense of how big the area of a circle of radius ***r*** might be:



This will then lead to the introduction of more analytical approaches which will serve to establish the exact relationship, $A = \pi r^2$:



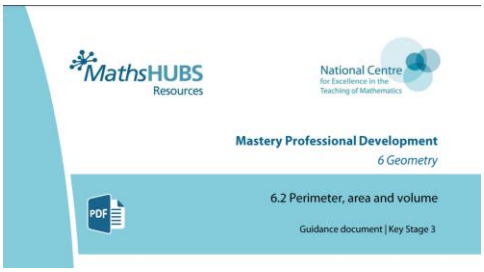
Part 4 – Why this is important

How will this support future learning?

The ideas in this topic helps students develop a deeper understanding of the properties of circles and support connections with learning in the following areas:

- calculate arc lengths, angles, and areas of sectors of circles
- identify and apply circle definitions and properties, including centre, radius, chord, diameter, circumference, tangent, arc, sector, and segment
- {**apply and prove the standard circle theorems concerning angles, radii, tangents, and chords, and use them to prove related results**}
- {**recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point**}.

Useful links

	NCETM Secondary Mastery Assessment materials www.ncetm.org.uk/classroom-resources/assessment-materials-secondary/ An activity related to finding the circumference can be found on page 37 of these materials.
	NCETM Mastery Professional Development Materials (6.2 Perimeter, area, and volume) Pages 9 and 10 and pages 18-25 www.ncetm.org.uk/media/1qabpyac/ncetm_ks3_cc_6_2.pdf
	Variation Theory: Circumference and area of a circle https://variationtheory.com/2018/03/20/circumference-of-a-circle/ https://variationtheory.com/2018/04/12/area-of-a-circle/
	Median Don Steward Mathematics Teaching 10 – 16 Circumference - https://donsteward.blogspot.com/search?q=circumference Area - https://donsteward.blogspot.com/search?q=area+of+circle