Multiplicative Reasoning

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

Question 7

How confident are you that you understand and can support children to explore different written algorithms for multiplication?

1 ☑ 2 ☐ 3 ☐ 4 ☐

How would you respond …?

a. Solve this calculation using an informal written method. You could use a pictorial representation.

One pack of three peppers costs 84p.
How much do 6 packs cost?

b. How can expanded algorithms be used to support children's understanding of regrouping in multiplication? Use this calculation to support your answer.

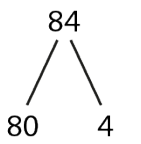
521×3

Responses

Note your responses to the questions here before you engage with the rest of this section:

Did you notice that…?

a.



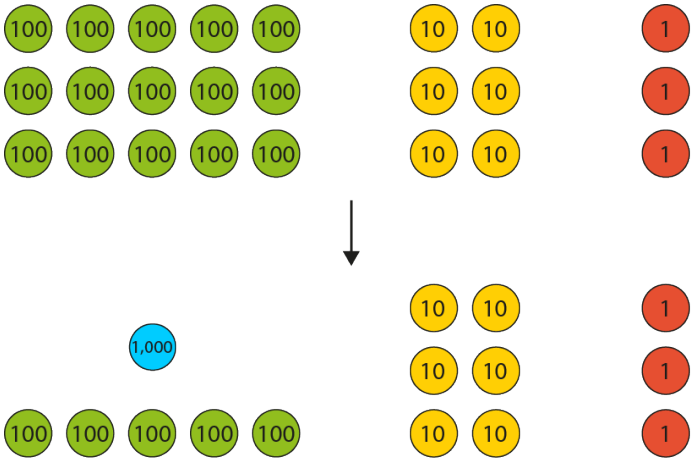
$84 \times 6 = 504$

$80 \times 6 = 480$

$4 \times 6 = 24$

$480 + 24 = 504$

b. Use place value counters to support working through the expanded layout. This will demonstrate where regrouping will be required.



	1,000s	100s	10s	1s
		5	2	1
×				3
				3
			6	0
	1	5	0	0
	1	5	6	3

3 × 1 ones = 3 ones

3 × 2 tens = 6 tens

3 × 5 hundreds = 15 hundreds

= 1 thousand + 5 hundreds

Informal and formal written methods

In this section, strategies for multiplication will be explored, focussing on the sequence when moving from informal written methods to a compact written algorithm for multiplication.

Where appropriate, children should use mental methods to multiply a two-digit number by a single-digit number. The short multiplication algorithm is applied to such calculations to develop children's understanding of how the algorithm works; they are then able to apply it to three-digit by single-digit multiplications.

Informal methods

When introducing this strategy, begin by reviewing the application of distributive law to multiplying a teen number by a single-digit number. Children should already be familiar with representations for distributive law, as they may have used this when deriving multiplication facts.

When writing the corresponding equations, draw attention to the factors being written in either order.

$13 \times 7 = 10 \times 7 + 3 \times 7$

$= 70 + 21$

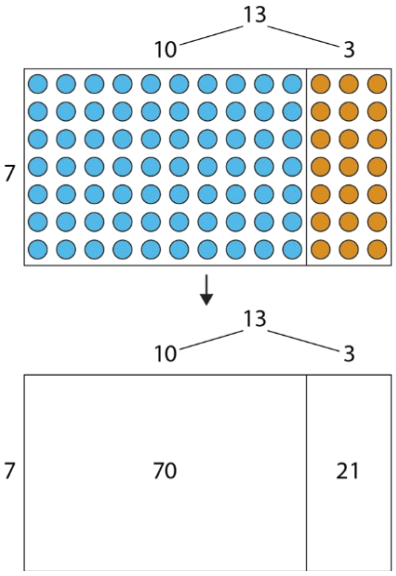
$= 91$

$7 \times 13 = 7 \times 10 + 7 \times 3$

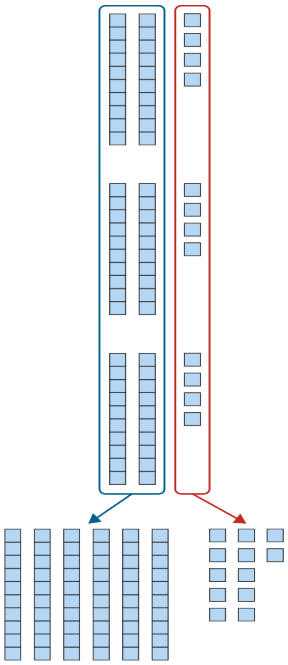
$= 70 + 21$

$= 91$

When children are confident with teen numbers, they can progress to multiplying non-teen numbers in the same way.



There are 3 rows, each with 24 chairs. How many chairs are there altogether?



A manipulative, such as Dienes, can be used to support children's understanding of the structure and help them to physically partition the number into tens and ones to multiply.

Partition the 24, describing it in terms of tens and ones and represent it using Dienes. Then, make two more copies of the representation of 234 because we are multiplying by three. Ask the children to bring the tens together and the ones together, recombining to represent the product.

Record equations and use unitising language as you work through the steps.

$24 = 20 + 4$	*'Twenty-four is equal to two tens and four ones.'* 24 = 2 tens + 4 ones
24	*'Two tens and four ones multiplied by three.'*
$24 \times 3 = 20 \times 3 + 4 \times 3$ $= 40 + 12$ $= 52$	*'Two tens and four ones multiplied by three is equal to two tens multiplied by three and four ones multiplied by three.'* 2 tens × 3 = 4 tens 4 ones × 3 = 12 ones

When working with larger numbers, children will need to use regrouping, which they should be familiar with from their work on addition. When children are confident of the regrouping structure, partitioning the tens and ones and multiplying the resulting parts by the single digit before recombining, the Dienes should be removed.

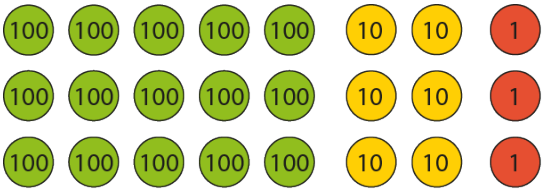
This strategy could be written informally.

$80 \times 6 = 480$ Or $84 \times 6 = 80 \times 6 + 4 \times 6$

$4 \times 6 = 24$ $= 480 + 24$

$480 + 24 = 504$ $= 504$

The same principles can be applied when working with three- or four-digit numbers. You may choose to use a different representation as the numbers get larger. For example, 521 × 3 could be represented using place value counters.



Short multiplication

Short multiplication is used as a consistent algorithm to record and apply the principles explored above. Children will be familiar with partitioning the product and describing the calculation with unitising language. This can be applied to the convention of the written algorithm, along with the need to work through the calculation from right to left.

Use expanded layout for a short time only, to help the children understand the algorithm.

Step 1 – write the factors:

	10s	1s
	3	4
×		2

Step 2 – multiply the single-digit number by the ones:

	10s	1s
	3	4
×		2
		8

2 × 4 ones = 8 ones

Step 3 – multiply the single-digit number by the tens:

	10s	1s
	3	4
×		2
		8
	6	0

2 × 4 ones = 8 ones

2 × 3 tens = 6 tens

Step 4 – add the partial products:

	10s	1s
	3	4
×		2
		8
	6	0
	6	8

2 × 4 ones = 8 ones

2 × 3 tens = 6 tens

Provide the children with opportunities to compare these methods.

Informal written method

$34 \times 2 = 30 \times 2 + 4 \times 2$

$= 60 + 8$

$= 68$

Expanded multiplication algorithm

	10s	1s
	3	4
×		2
		8
	6	0
	6	8

2 × 4 ones = 8 ones

2 × 3 tens = 6 tens

When children understand the process, a compact method can be used. Discuss where to place the number when regrouping is required, drawing upon their understanding of written addition algorithms.

Present an expanded layout alongside a compact one to allow comparisons and consolidation of the structure.

Multiplication algorithm – expanded layout

	100s	10s	1s
		3	8
×			4
		3	2
	1	2	0
	1	5	2

Multiplication algorithm – compact layout

$$\begin{array}{r} 3\ 8 \\ \times\quad 4 \\ \hline 1\ 5\ 2 \\ \hline 3 \end{array}$$

When working through examples, ensure precise language is used.

'First, write the largest factor: "367".'
'Then, write the smallest factor below, lining up the digits according to place value: "4".'
'Now multiply, starting with the ones: four times seven ones is equal to twenty-eight ones…'
'…and regroup: twenty-eight ones is equal to two tens and eight ones; write "8" in the ones column and "2" below the tens column.'
'Then, move to the tens: four times six tens is equal to twenty-four tens…'
'…and regroup: twenty-four tens is equal to two hundreds and four tens…'
'…and add the two tens from regrouping to give two hundreds and <u>six</u> tens: write "6" in the tens column and "2" below the hundreds column.'
'Then, move to the hundreds: four times three hundreds is equal to twelve hundreds…'
'…and regroup: twelve hundreds is equal to one thousand and two hundreds…'
'…and add the two hundreds from regrouping to give one thousand and <u>four</u> hundreds; write "1" in the thousands column and "4" in the hundreds column.'

$$\begin{array}{r} 3\ 6\ 7 \\ \times\quad\quad 4 \\ \hline 1\ 4\ 6\ 8 \\ \hline \quad 2\ \ 2 \end{array}$$

Common errors in this area may include:

- children do not use 0 as a place holder when multiplying by 10
- regrouped digits are not added to find the final product.

What to look for

Can a child:

- apply short multiplication in a range of contexts?
- explain the structure of long multiplication?
- use mental strategies when appropriate?

Links to supporting materials:

NCETM Primary Professional Development materials, Spine 2: Multiplication and Division:

- Topic 2.14: Multiplication: partitioning leading to short multiplication

Notes:

Key learning from support material and self-study:

What I will focus on developing in my classroom practice: