MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.25 Addition and subtraction: money

Teacher guide | Year 4

Teaching point 1:

One penny is one hundredth of a pound; conventions for expressing quantities of money are based on expressing numbers with tenths and hundredths.

Teaching point 2:

Equivalent calculation strategies for addition can be used to efficiently add commonly-used prices.

Teaching point 3:

The 'working forwards'/'finding the difference' strategy for subtraction is an efficient way to calculate the change due when paying in whole pounds or notes.

Teaching point 4:

Column methods can be used to add and subtract quantities of money.

Teaching point 5:

Finding change when purchasing several items uses the part–part–(part–)whole structure.

Overview of learning

In this segment children will:

- link the fact that there are 100 pennies in a pound with their understanding of hundredths to express quantities of money using decimal notation
- compare and convert between quantities of money expressed in both units of pounds (decimal notation) and in units of pennies
- apply known mental strategies to calculations with money
- apply column methods to calculations with money, where appropriate
- identify the most efficient strategy to use for a given calculation with money
- use the bar model to understand and represent the structure of problems in which more than one cost is subtracted from a quantity of money.

In this segment children will move beyond using coins; they will learn to represent quantities of money in decimal notation and work with quantities independent of coin denominations. £1, 10 p and 1 p coins are used initially as place-value counters, supporting children's understanding of the decimal notation. However, it is important to move away from this fairly quickly and to use language carefully so that children don't become dependent on the coin representations, and also so that they don't assume that quantities must be made up of particular coin denominations (or, indeed, of coins at all). The conventions for reading money differ from those of reading other decimal fractions, for example, £1.42 is said as *'one pound forty-two'* (or *'one pound and forty-two pence'*) rather than as *'one-point-four-two pounds'*; this is addressed in *Teaching point 1*, and the correct use of language should be emphasised throughout the segment.

Once children are confident using decimal notation for money and converting quantities between pounds and pence, they will build on their understanding of tenths and hundredths, and known calculation strategies. The segment focuses on common calculations for money, such as calculating:

- the cost of several items
- change due from whole pounds
- the difference between two quantities of money.

Children have already learnt to use equivalent calculation strategies for addition (segment *1.19 Securing mental strategies: calculation up to 999*). Here these strategies are applied to adding several quantities of money (e.g. *£1.99 + £2.99 = £2 + £3 – 2 p*). There is emphasis on using the 'finding the difference' strategy, supported by number lines, for calculating the amount of change due from whole pounds, which is covered before the application of column methods, since column subtraction is generally not an efficient choice in this context, due to the need for exchange across many columns. Note that later (segment *1.29 Using equivalence and the compensation property to calculate*), children will learn how to use equivalent calculation strategies for subtraction; by that point, they could use this approach to transform calculations for easier application of the column method, for example, transforming *£50 – £34.49* into £49.99 – £34.48 to avoid the need for exchange.

At the end of the segment, the context of money is used to formally introduce the idea that two or more quantities can be subtracted by first adding together the subtrahends, then subtracting the resulting total from the minuend (e.g. *£2 – 55p – 72p = £2 – £1.27*). Bar models are used to reveal the structure of these problems and to support children in deciding which calculations they need to perform.

Throughout the segment, teachers should emphasise use of known facts; for example, in the 'finding the difference' strategy, the number line should only be used to identify the 'jumps' needed to get from the cost to the amount paid, with known facts being used to calculate those 'jumps' (rather than counting forward on the number line). Teachers should also ensure that children always seek to use the most efficient calculation strategy. Exemplar practice is provided within each teaching point, but it is recommended that, at the end of the segment, children are presented with a mixture of problem types until they can confidently identify an efficient approach for each given problem.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

One penny is one hundredth of a pound; conventions for expressing quantities of money are based on expressing numbers with tenths and hundredths.

Steps in learning

	Guidance	Representations
1:1	In preparation for using decimal notation, look at the relationship between 1 p, 10 p and £1, using the coins. Alongside exchanging, for example, a £1 coin for ten 10 p coins (and vice versa), encourage children to use the generalised statements: • ***'Ten groups of ten pence is equal to one pound, so ten pence is one tenth of a pound.'*** • ***'One hundred groups of one penny is equal to one pound, so one penny is one hundredth of a pound.'*** • ***'Ten groups of one penny is equal to ten pence, so one penny is one tenth of ten pence.'***	
1:2	Start to introduce decimal notation for money quantities, beginning with whole pounds. Explain that, unlike other measures, the unit (here the '£' sign) is represented *before* the digits (e.g. £3 or £3.00), but that we say/read *'three pounds'* not *'pounds three'*. Note that, in preparation for the next steps, children can practise writing whole-pound quantities with the decimal point (e.g. £3.00), but the meaning of the notation will only become clear in steps *1:3–1:4*. Children could practise using the notation by rolling a dice, taking the resulting number of £1 coins, then recording the quantity symbolically and saying the amount.	 £3 or £3.00 *'Three pounds'*

1:3

Once children are confident recording whole pounds, progress to the representation of multiples of ten pence greater than one pound (e.g. £2.40). Show an amount composed of pound coins and ten-pence coins, and record the quantities on a place-value chart, as well as without the place-value chart as shown opposite. Although the quantity could be made up of different coin combinations, using one pound and ten-pence coins, for now, helps children to understand the value of each column. Link to previous work on tenths and hundredths to show that the decimal point separates the whole pounds from the additional pennies (here, multiples of ten pence), for example:

- *'"2.40" represents two and four additional tenths, or two and forty additional hundredths.'*
- *'"£2.40" represents two whole pounds and four additional groups of ten pence, or two whole pounds and forty additional pennies.'*
- *'The "4" represents four additional groups of ten pence. Ten pence is one tenth of a pound, so the four groups of ten are shown in the tenths place.'*

Explain that, when recording money, we always show two decimal places, even if the value in the hundredths/one-penny place is zero*. Then model the correct language for saying the amount:

- Emphasise that the word 'pound' is said instead of 'point', so *'two-**point**-four-zero'* becomes *'two **pounds** forty'* or *'two **pounds** and forty pence'*.
- Explain that we always say the amount after the decimal point in pence, even if it is a multiple of ten pence; four groups of ten pence is equal to forty pence, so we say *'two*

£1 (or 100 p)	10 p	1 p
2	4	0

£	2 •	4	0

£2.40

'Two pounds forty'
or
'Two pounds and forty pence'

<table>
<tr>
<td></td>
<td>
pounds forty', or 'two pounds and forty pence.'

Start to use the generalised statement: 'The number to the left of the decimal point represents the number of whole pounds. The number to the right of the decimal point represents the number of additional pennies.'

*Note that when/if children use a calculator to calculate with money, they need to remember to add the zero, since the calculator will not display the final zero.
</td>
<td></td>
</tr>
<tr>
<td>1:4</td>
<td>
Now progress to record quantities with additional pennies (with a non-zero digit in each place-value position).

In a similar way to the link between multiples of ten pence and the tenths place, support children in making the link between additional pennies and the hundredths place, using their knowledge that one penny is one hundredth of one pound.

As before, model the correct language for saying amounts such as £1.32 and encourage children to describe the value of each digit, for example:

'The "1" represents one pound.'
'The "3" represents three groups of ten pence, or thirty pence.'
'The "2" represents two pence.'

Note that it is important that children don't begin to think that the digit in the tenths place always represents ten pence coins; step 1:8 suggests practice questions that will challenge this thinking.

Continue to use the generalised statement introduced in step 1:3.
</td>
<td>

<table>
<tr><th>£1 (or 100 p)</th><th>10 p</th><th>1 p</th></tr>
<tr><td>1</td><td>3</td><td>2</td></tr>
</table>

£
<table>
<tr><td>1</td><td>• 3</td><td>2</td></tr>
</table>

£1.32

'One pound thirty-two'

or

'One pound and thirty-two pence'
</td>
</tr>
</table>

1:5

Now, explore quantities with zero in the tenths (10 p) place. Emphasise that if there is less than ten pence in addition to the whole pounds, there will be a zero in the first space after the decimal point. Compare two values such as £3.50 and £3.05, and make sure children are able to explain the value of the digit '5', and to reason why the zero is needed, in each case. Convention dictates the inclusion of the zero in £3.50, whilst the zero in £3.05 is vital as it shows that the '5' has a value of 5 p and not 50 p.

Explain that when there is less than ten pence in addition to the whole pounds (i.e. a zero in the tenths place) we usually say, for example, '*three pounds and five pence*' rather than '*three pounds five*'.

£1 (or 100 p)	10 p	1 p
3	5	0

£			
£	3	• 5	0

£3.50

'Three pounds fifty'

or

'Three pounds and fifty pence'

£1 (or 100 p)	10 p	1 p
3	0	5

£	3	• 0	5

£3.05

'Three pounds and five pence'

1:6

Give children practice until they can confidently move between any pair of the following representations:

- quantity in decimal notation, e.g. £4.25
- name of the quantity, for example, *'four pounds twenty-five'* or *'four pounds and twenty-five pence'*

	• amount represented in pound, ten-pence, and one-penny coins. Include quantities greater than £10, as well as quantities with a zero in either the tenths or hundredths place. You can initially provide children with place-value charts for support, but these should then be removed to ensure that children can confidently record and interpret decimal quantities of money without them. Note that children may incorrectly write *'four pounds and twenty-five pence'* as £4.25 p. Address this error; we can write £4.25 or 425 p (see next step) but cannot combine the two.	
1:7	Children also need to be able to apply their knowledge that 100 pence is equivalent to £1 to convert between amounts represented in pence and in decimal notation (e.g. *685 p = £6.85*). Work through some examples together, initially physically exchanging coins if necessary, before giving children some practice. Make sure you include conversion in both 'directions'. There are a range of different approaches to converting quantities, including: • unitising in hundredths (learned in segment *1.24 Composition and calculation: hundredths and thousandths*), for example, 6.85 is 685 hundredths, so £6.85 is equal to 685 p • partitioning into pounds and pence, e.g., £6.85 = £6 + 85p = 600 p + 85 p = 685 p Try to avoid children working procedurally (counting two digits from the right and putting in the decimal	

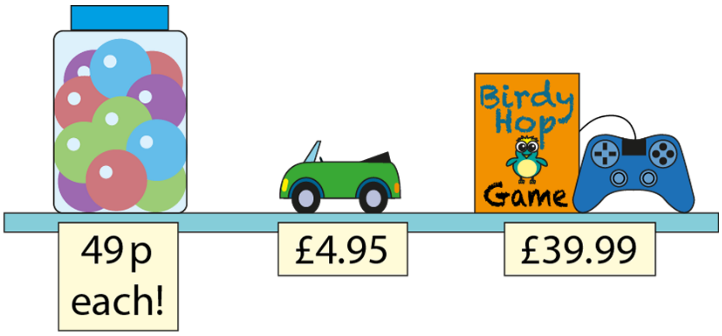
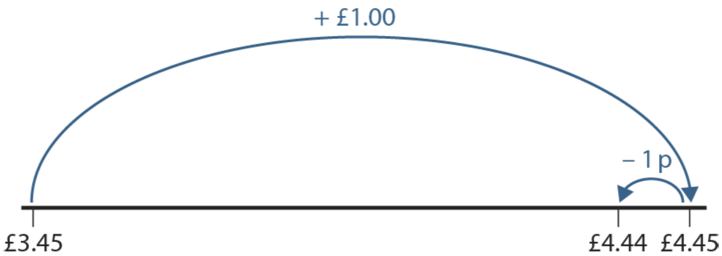
	point) without understanding the conversion between the units. Again, include quantities greater than £10, as well as quantities with zero in either the tenths or hundredths place (referring to the decimal representation), for example: • 2345 p = £23.45 • 6009 p = £60.09 • 1040 p = £10.40	
1:8	To complete this teaching point, provide practice, including word problems and real-life examples. Include examples where different coin denominations are used to represent a quantity, so that children do not think that, for example, the 25 pence in £4.25 *must* be made up of two ten-pence coins and five one-penny coins. This is important to ensure that children see that quantities are independent of the coins used to represent them. Similarly, include examples that are not linked to coins at all (such as quantities in the context of prices, balances or other payment methods; for example, children may be familiar with the idea of a certain amount of 'credit' on a pre-paid phone, and this has nothing to do with coins). Example problems: • *'Rebecca has 428 loyalty points to spend in a book shop. Each point is worth one penny. Write down how much she has to spend in decimal notation.'* • *'A toy car costs £3.33. Describe the value of each "3".'* • *'Fahari is paying for some toys. The shopkeeper says "That's twelve pounds and eight pence, please." Write the cost in decimal notation.'*	• *'Put the following prices in order from the cheapest to the most expensive.'* £2.99 150 p £39.95 99 p 395 p £1.05 • *'How much money is here? Write your answer in decimal notation.'* 

		• *'Felicity wants to buy this mug.'*  £2.75 *'What is the smallest number of coins she can use?'* ☐ £1 coins ☐ 50 p coins ☐ 20 p coins ☐ 10 p coins ☐ 5 p coins ☐ 2 p coins ☐ 1 p coins Felicity needs ___ coins.

Teaching point 2:

Equivalent calculation strategies for addition can be used to efficiently add commonly-used prices.

Steps in learning

	Guidance	**Representations**
2:1	As an introduction to this teaching point, show some common prices of items and ask children what they notice. They should realise that many prices are just under a whole number of pounds, a whole ten, or a whole hundred (pence or pounds). Use knowledge of rounding to find each price nearest to the whole pound/ten pounds/hundred as appropriate. Then discuss why prices might be set like this. Focus mainly on prices in decimal notation such as £4.95 or £34.99.	 
2:2	Remind children that they already know how to efficiently add and subtract near multiples of 10 or 100 using equivalent calculation strategies (segment *1.19 Securing mental strategies: calculation up to 999*), for example: • 52 + 29 = 52 + 30 – 1 = 82 – 1 = 81 ('adjusting' strategy) • 45 + 29 = 44 + 30 = 74 ('redistributing' strategy). Both strategies can be applied, where appropriate, to calculations with money, but now explore, in detail, how the adjusting strategy can be used to efficiently add common prices in decimal notation. Use the same representations as in segment *1.19* (number lines, initially, then progressing to use of linked equation	Adjusting one addend by one penny: £3.45 + 99 p • Number line  • Linked equations **£3.45 + 99 p = $\boxed{£4.44}$** £3.45 + $\boxed{£1.00}$ = $\boxed{£4.45}$ – 1 p

<table>
<tr>
<td></td>
<td>
pairs) to support children in making links to previous learning.

Begin with adjusting one addend by one penny, for example:

$$\begin{aligned} £3.45 + 99\text{ p} &= £3.45 + £1.00 - 1\text{ p} \\ &= £4.45 - 1\text{ p} \\ &= £4.44 \end{aligned}$$
Encourage children to describe the steps, using the following stem sentences:

'First we add: ___ plus ___ is equal to ___…'
'…then we adjust: ___ minus ___ is equal to ___. '

Ask children 'Why do we need to adjust?', encouraging them to answer 'Because we have added too much.'

Note that in the example shown, the quantity greater than £1 is shown in units of pounds (£3.45), while those less than £1 are shown in units of pennies (99 p). Children should be able to confidently work with both units, doing the necessary conversions in their heads. Throughout this, and later, teaching points, include a range of quantities and notations, for example:

55 p + 99 p
(99 p plus a quantity less than £1)
£0.55 + £0.99
(and in decimal notation)
£3.45 + 99 p
(99 p plus a quantity greater than £1; mixed notation)
£3.45 + £0.99
(and in decimal notation)
£3.45 + £1.99
(both quantities greater than £1 in decimal notation)

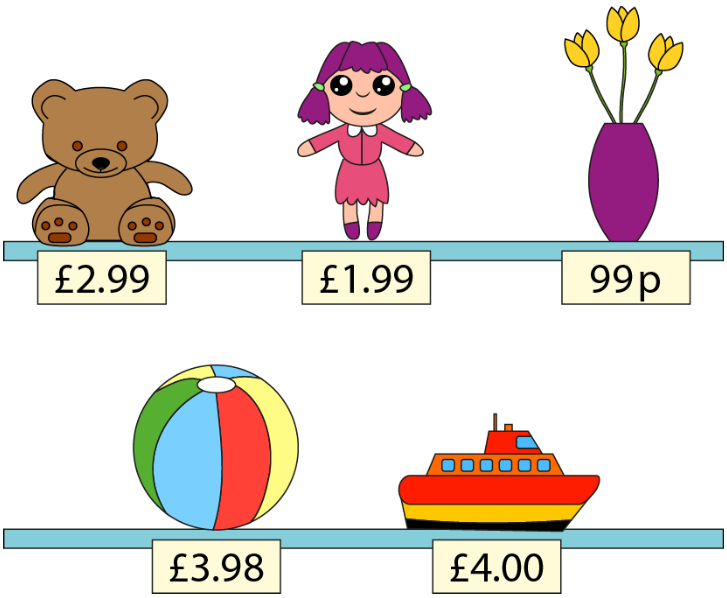
As discussed in Teaching point 1, ensure that children don't combine both notations, for example:

£3.45 + 99 p = £4.44 p ✗
</td>
<td>

'First we add: three pounds forty-five plus one pound is equal to four pounds forty-five…'
'…then we adjust: four pounds forty-five minus one penny is equal to four pounds forty-four. '

</td>
</tr>
</table>

	£3.45 + 99 p = £4.44 ✓ or £3.45 + 99 p = 444 p ✓	
2:3	Now extend to examples for which both addends are adjusted by one penny (e.g. 99 p + 99 p, £1.99 + £1.99 (near doubles) and £1.99 + £2.99). You can continue to scaffold the problems with linked equations, but gradually remove the level of scaffolding (as shown opposite). Ensure that children do not record, for example, 99 p + 99 p = £1 + £1 ✗ If any children are still making this sort of error, continue to reiterate the meaning of the equals symbol. You can also include examples with more than two addends.	Adjusting both addends by one penny – linked equations: **£1.99** + **£1.99** = $\boxed{£3.98}$ – 2 p £2 + $\boxed{£2}$ = $\boxed{£4}$ **£3.99** + **£1.99** = $\boxed{£5.98}$ – 2 p £4 + £2 = £6
2:4	You can use varied practice to highlight the patterns and to extend to adjusting by more than one penny, as shown opposite. Draw out the patterns by asking children: • *'What do you notice about these calculations?'* • *'What stays the same and what changes?'* Encourage children to reason about what each answer will be based on the previous calculation in the set.	Missing number problems: *'Fill in the missing numbers.'* £4.86 + 99p = $\boxed{}$ £4.86 + £1.99 = $\boxed{}$ £4.86 + $\boxed{}$ = £7.85 $\boxed{}$ + £3.99 = £8.85 $\boxed{}$ = £1.99 + 99p $\boxed{}$ = £1.99 + 98p £3.97 = $\boxed{}$ + £1.98 £4.97 = £2.99 + $\boxed{}$

<table>
<tr>
<td></td>
<td></td>
<td>☐ = £2.99 + £2.98
☐ = £2.99 + £2.97
☐ = £2.98 + £2.98</td>
</tr>
<tr>
<td>2:5</td>
<td>Once children are confident using the strategies in steps 2:2 and 2:4, provide them with some contextual practice, such as the examples shown opposite.</td>
<td>

'Choose two items and find the total cost.'
'Choose another two items and find the total cost.'
'And another…'
'Choose three items and find the total cost.'
'What costs about £1?'
'Which two items, together, cost about £4?'

Dòng nǎo jīn:
'Jim looks at the items for sale and chooses some to buy with his pocket money.
Which items did he buy if he spent:

£4.98
£4.97
£6.98?

Could he spend £5.98?'</td>
</tr>
</table>

Teaching point 3:

The 'working forwards'/'finding the difference' strategy for subtraction is an efficient way to calculate the change due when paying in whole pounds or notes.

Steps in learning

	Guidance	Representations
3:1	Before using the 'working forwards'/'finding the difference' strategy (introduced in segment *1.19 Securing mental strategies: calculation up to 999*) to calculate change due, ensure that children are secure with this strategy and also that they understand that finding change is a subtraction calculation (the bar model can be used to draw attention to the latter). Also make sure that children can confidently apply their knowledge of finding the next whole number (for numbers with tenths or hundredths) to round prices up to the nearest multiple of ten pence and to the nearest pound. Throughout this teaching point, emphasise the use of known facts, including complements to five, ten and one hundred. Begin by looking at whole-pound complements (e.g. *£5 – £3*, *£10 – £7*). Then progress to complements that are multiples of ten pence (e.g. *£5.00 – £3.40*). Using a number line with the cost and amount paid shown, first ask children to identify the next whole pound (here, £4.00), before finding the value of the 'jumps' (the bar model is still used here to show the structure of the problem). Using an example related to one of the whole-pound calculations, as shown here, will help children to see, for example, that if the cost is *more* than £3, the change from £5 will be *less* than £2. Children make a similar common	See the representations below.

Whole-pound complements:

'I buy something that costs three pounds. How much change will I get from a five-pound note?'

Amount paid: £5	
Cost of item: £3	**Change: £2**

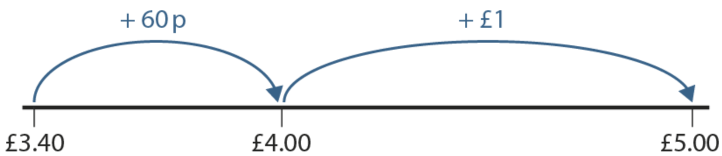
£5	
£3	**£2**

£3 + $\boxed{£2}$ = £5 £5 – £3 = $\boxed{£2}$

Complements that are multiples of ten pence:

'What if I spend three pounds and forty pence? How much change will I get from five pounds now?'

£5.00	
£3.40	?



£1 + 60 p = £1.60

£5.00	
£3.40	£1.60

<table>
<tr>
<td></td>
<td>error here to that made when finding two-digit complements to 100 (segment 1.17 Composition and calculation: 100 and bridging 100), giving an extra ten each time, e.g.:

62 + 48 = 100 ✗

Here, they commonly take the pound and pence values of the cost and mistakenly complement each, respectively, to the number of pounds in the change, and to one pound, i.e.:

£3.40 + ☐ = £5.00

40 p + 60 p = £1.00

£3 + £2 = £5

so

£3.40 + £2.60 = £5.00 ✗

Emphasise that when we spent £3, the change from £5 was £2, and now that we have spent more than £3, we will get less change.

Work through a range of examples, finding change from £5 or £10 notes, and other whole-pound quantities, until children have mastered this step. You may find it helpful to address the common error ‘head on’ by asking children spot-the-mistake or true/false questions.</td>
<td>Addressing the common error:

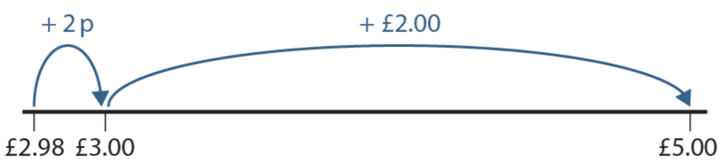
• ‘Mark each calculation with a tick or a cross.’

<table>
<tr><th></th><th>✓ or ✗</th></tr>
<tr><td>£5.00 – £2.30 = £3.70</td><td></td></tr>
<tr><td>£10.00 – £1.70 = £8.30</td><td></td></tr>
<tr><td>£3.90 + £1.10 = £5.00</td><td></td></tr>
<tr><td>£7.60 + £3.40 = £10.00</td><td></td></tr>
</table>

• True or false?
‘I buy a cup of coffee that costs £2.20. I pay with a £10 note and get £8.80 change.’</td>
</tr>
<tr>
<td>3:2</td>
<td>Now move on to costs that have a non-zero pence value (e.g. £5.00 – £3.98). Again, the example builds on the one used in the previous step; there the cost was £3.40, but now the cost is greater, so less change is given.

Note that here we have removed the scaffold of the bar model, which was previously used to represent the structure of the problem.

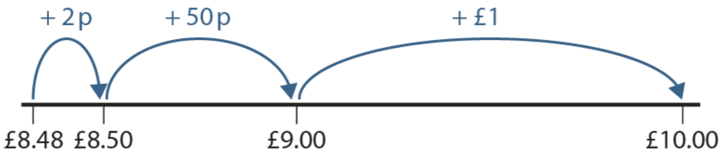
Also work through some examples where the price is more than 10 p away from the next whole pound, including</td>
<td>• ‘I had £5.00 and I spent £2.98 on football cards. How much money did I have left?’



£2.00 + 2 p = £2.02</td>
</tr>
</table>

an extra 'jump' on the number line (e.g. £10 – £8.48).

- *'I had £10 and I spent £8.48 on books. How much money did I have left?'*



£1.00 + 50p + 2p = £1.52

3:3

Complete this teaching point by providing varied practice, as shown opposite and below:

- *'Stephanie bought a sticker book for £3.20. She paid with a £5 note. How much change did she get?'*
- *'Nigel has £10 of credit on his phone. If he spends £3.50 on a game, how much credit will he have left?'*
- *'Eloise has £5 to spend in the joke shop. She wants to buy a pot of slime for 99 p and a water pistol for £3.50. How much change will she get from her £5?'*
- *'When Eloise gets to the checkout, she finds that the water pistol is in the sale and only costs £3. Will she have enough to buy another pot of slime?'*

- *'Find the amount of change I would get from a five-pound note if I spent the following amounts.'*

£5.00 – 99 p = $\boxed{}$

£5.00 – £1.99 = $\boxed{}$

£5.00 – £2.99 = $\boxed{}$

£5.00 – £2.98 = $\boxed{}$

£5.00 – $\boxed{}$ = £1.01

'What would my change be in each case if I paid with a ten-pound note instead of a five-pound note?'

- *'Find the change I would get from ten pounds if I spent the following amounts.'*

Cost of item	Amount paid	Change
£8	£10	£2
£7.99	£10	
£7.49	£10	
£7	£10	
£6.49	£10	

Teaching point 4:

Column methods can be used to add and subtract quantities of money.

Steps in learning

<table>
<tr><th></th><th>Guidance</th><th>Representations</th></tr>
<tr>
<td>4:1</td>
<td>When calculating with money, when whole coin and note values are not involved, or equivalent calculations are not appropriate, column methods can be the most efficient way to calculate. Children should already be confident in laying out column calculations for numbers with hundredths and with adding the quantities in columns from right to left, including regrouping for addition and exchanging for subtraction.

Begin this teaching point by demonstrating laying out column-addition problems in the context of money and completing the calculations. In this step, keep the totals within ten pounds.

As in segment 1.24 Composition and calculation: hundredths and thousandths, pay particular attention to the decimal point and encourage children to articulate the value of each digit as they lay out the calculation. You can initially represent each problem with a part–part–whole representation (bar model or cherry diagram) to reveal what operation is required (this is a useful practice when children encounter word problems with different structures).

To begin with, you can create the addends from one-pound, ten-pence and one-penny coins, aligning them to show ‘like values’ added together. However, move away from this fairly quickly since, by now, children should be confident with column methods and should not become dependent on a</td>
<td>Column addition:

‘I buy two magazines. One costs £1.37 and the other costs £2.45. How much do I spend altogether?’

<table>
<tr><td colspan="2">?</td></tr>
<tr><td>£1.37</td><td>£2.45</td></tr>
</table>

$$\begin{array}{cccccc} & £ & 1 & . & 3 & 7 \\ + & £ & 2 & . & 4 & 5 \\ \hline & £ & 3 & . & 8 & 2 \\ \hline & & & & 1 & \end{array}$$

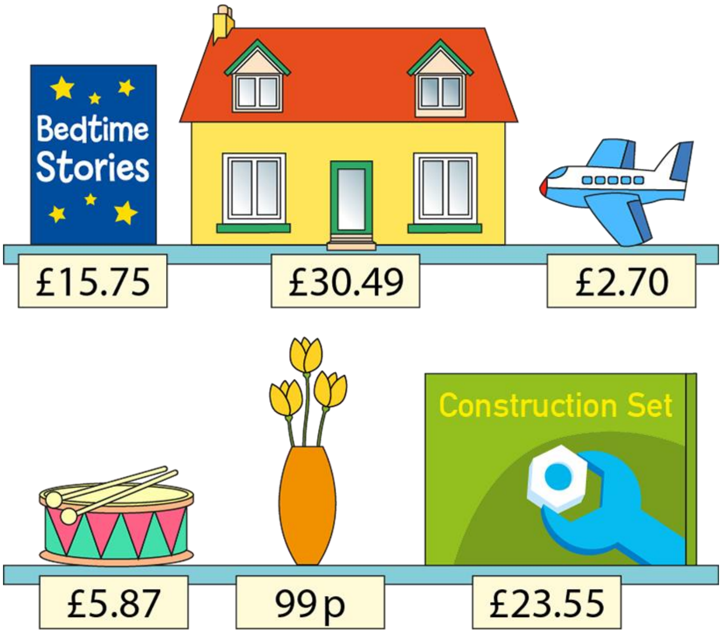
Example practice problems:

‘Complete the following calculations using column addition.’

£4.25 + £2.53
£3.73 + £5.19
£2.64 + £2.83
£5.86 + £3.47</td>
</tr>
</table>

<table>
<tr><td></td><td>link to coin values. This also helps to avoid reinforcing the belief that money values are always linked to real-life coin compositions. Similarly, when regrouping is required, you can initially use similar generalised sentences to those used in the hundredths segment:
• 'Ten pennies is equal to ten pence.'
• 'Ten groups of ten pence is equal to one pound.'
After working through a few examples, give children some practice laying out and completing calculations for themselves, including examples with regrouping of either or both ten pennies into ten pence and ten groups of ten pence into one pound.</td><td></td></tr>
<tr><td>4:2</td><td>Once the children are confident with laying out and completing the calculations described in the previous step, extend to examples with:
• addends greater than ten pounds
• addends with different numbers of digits
• addends with some digits equal to zero
• more than two addends
• regrouping to the left of the decimal point, as well as regrouping in the left-most digit.
Throughout, encourage children to describe and reason about the value of the digits.</td><td>• 'I want to buy two games, one for £24.55 and the other for £17.82. How much will I spend altogether?'
$$\begin{array}{rcccccc} & £ & 2 & 4 & . & 5 & 5 \\ + & £ & 1 & 7 & . & 8 & 2 \\ \hline & £ & 4 & 2 & . & 3 & 7 \\ \hline & & 1 & 1 & & & \end{array}$$
• 'I want to buy a book that costs £5.60 and a game that costs £27.05. How much will I spend altogether?'
$$\begin{array}{rcccccc} & £ & & 5 & . & 6 & 0 \\ + & £ & 2 & 7 & . & 0 & 5 \\ \hline & £ & 3 & 2 & . & 6 & 5 \\ \hline & & 1 & & & & \end{array}$$</td></tr>
</table>

		• *'I bought the following items. How much did I spend altogether?'*
		  • *chair: £50.35*
		  • *desk: £83.25*
		  • *lamp: £25.10*
		$\begin{array}{cccccccc} & £ & & 5 & 0 & . & 3 & 5 \\ & £ & & 8 & 3 & . & 2 & 5 \\ + & £ & & 2 & 5 & . & 1 & 0 \\ \hline & £ & 1 & 5 & 8 & . & 7 & 0 \\ \hline & & 1 & & & & 1 & \end{array}$

4:3	Give children practice for the different calculation types described in the previous step. Include some quantities for which column addition is not the most efficient strategy and encourage children to identify when this might be the case; for example, to find the cost of the construction set and the flowers (both shown opposite) it would be better to use the equivalent calculation: £23.55 + £1 – 1 p	 • *'Choose two items and find the total cost.'* • *'Choose another two items and find the total cost.'* • *'And another…'* • *'Choose three items and find the total cost.'*
4:4	Now follow a similar progression to steps *4:1–4:3* for subtraction (one subtrahend only): • Begin with two-addend problems, within ten pounds, with no exchange. • Then include examples with exchange of either or both one pound for ten groups of ten pence,	

and ten pence for ten pennies. Here you can 'reverse' the stem sentences from step *4:1*, i.e.:

- ***'One pound is equal to ten groups of ten pence.'***
- ***'Ten pence is equal to ten pennies.'***

- Extend to:
  - quantities greater than ten pounds
  - subtrahends with fewer digits than the minuend (e.g. *£12.50 – £7.08*); encourage children to reason about the 'empty' columns and describe how they are subtracting zero in those cases
  - examples with some digits equal to zero (including exchange through a zero)
  - exchange to the left of the decimal point.

Throughout, continue to encourage children to describe and reason about the value of the digits.

Note that contextual examples for subtraction in this teaching point should be appropriate for calculation using column methods; examples include questions about differences, such as saving. In *Teaching point 3*, we discussed that questions on calculating change for common prices from whole pounds are often efficiently solved using 'finding the difference' mental strategies; for example, column subtraction is not an efficient choice when finding the change from ten pounds, when spending £7.99.

Note that a common error children make when subtracting from zero in one of the columns is to 'ignore' the zero; they do not spot the need to exchange and, instead, copy the non-zero digit into their answer. For example, in the following subtraction, children write '4' in the pennies column

Column subtraction:

- *'I have saved £6.53 and my brother has saved £4.38. How much more money have I saved than my brother?'*

£6.53	
£4.38	?

$$\begin{array}{cccccc} & £ & 6 & . & \cancel{5}^{4} & {}^{1}3 \\ - & £ & 4 & . & 3 & 8 \\ \hline & £ & 2 & . & 1 & 5 \\ \hline \end{array}$$

- *'The game I want to buy costs £29.50. I have saved £18.94. How much more do I need to save before I can buy the game?'*

$$\begin{array}{ccccccc} & £ & 2 & \cancel{9}^{8} & . & \cancel{5}^{{}^{1}4} & {}^{1}0 \\ - & £ & 1 & 8 & . & 9 & 4 \\ \hline & £ & 1 & 0 & . & 5 & 6 \\ \hline \end{array}$$

(In this example, a common error is for children to ignore the zero in the £29.50 and not spot the need to exchange before subtracting the 4 p of the £18.94.)

- *'I need £12.50 to enter a swimming competition, but I only have £7.08. How much more money do I need?'*

$$\begin{array}{ccccccc} & £ & \cancel{1}^{0} & {}^{1}2 & . & \cancel{5}^{4} & {}^{1}0 \\ - & £ & & 7 & . & 0 & 8 \\ \hline & £ & 0 & 5 & . & 4 & 2 \\ \hline \end{array}$$

	of their answer instead of exchanging one ten pence for ten pennies in the minuend: $\begin{array}{rcccccc} & £ & 2 & \cancel{9}^{8} & . & {}^{1}5 & 0 \\ - & £ & 1 & 8 & . & 9 & 4 \\ \hline & £ & 1 & 0 & . & 6 & 4 \\ \hline \end{array}$ ✗	
4:5	Give children practice for all the calculation types discussed in the previous step. As for addition (step *4:3*), include some quantities for which column subtraction is not the most efficient strategy and encourage children to identify when this might be the case. Also include examples where possible errors might arise, such as those with a zero in the tenths or hundredths column of the minuend.	*'Complete the following calculations.'* £13.05 – £2.10 £27.40 – £12.75 £17.45 – £8.90 £34.65 – £12.05 £15.00 – £12.99 £50.70 – £8.95
4:6	Complete this teaching point by providing children with a range of contextual problems, including both addition and subtraction, for example: • *'Alicia bought a t-shirt for £8.50 and some shorts for £15.66. How much did she spend altogether?'* • *'Red trainers cost £12.75 and blue ones cost £27.40. How much more do the blue ones cost than the red ones?'* As with steps *4:3* and *4:5*, continue to include some examples for which column methods may not be the most appropriate, for example: • *'Giant gobstoppers cost £2.99 each. Ahmad bought two. What was the total cost?'* • *'Deborah bought a football shirt for £17.99. She paid with a £20 note. How much change did she get?'*	

Teaching point 5:

Finding change when purchasing several items uses the part–part–(part–)whole structure.

Steps in learning

5:1

Guidance

Many problems that children encounter, especially in the context of money, will involve subtraction calculations with more than one subtrahend. This teaching point explores the two ways in which these problems can be solved:

- 'repeated subtraction', i.e. subtracting one subtrahend, then the next and so on
- 'adding first', i.e. adding together all of the subtrahends, then subtracting the resulting total from the minuend.

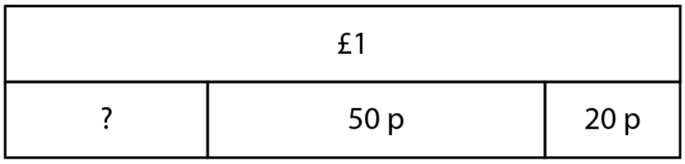
As well as exploring these strategies in general, it is also important for children to be aware that, unlike addition problems with several addends, we can't use a single-column calculation to subtract multiple subtrahends.

Throughout this teaching point, you will also need to consider carefully the context of the problems you use. The wording of the problem may not always indicate the most effective strategy and children will need to consider the numbers involved when choosing their approach.

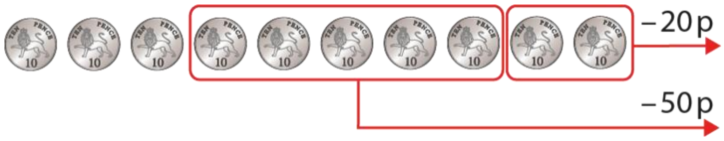
Begin by demonstrating that the 'repeated subtraction' and 'adding first' strategies both result in the same answer. To draw attention to the structures, use simple quantities. You can either use a single context to illustrate both strategies (as opposite) or fit the contexts to the strategies (i.e. money spent on two separate occasions to illustrate 'repeated subtraction' to contrast with two items bought in the same transaction to

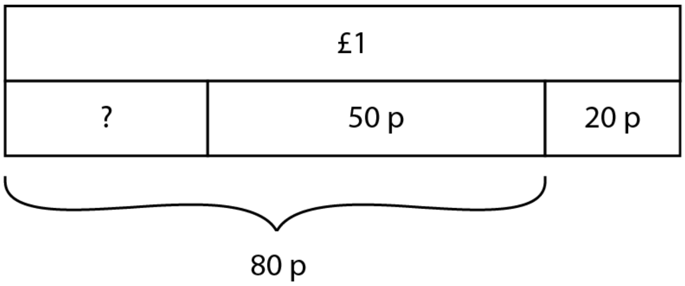
Representations

'Tim had £1.00. He bought a pencil for 20 p and a pen for 50 p. How much money does he have left?'



Repeated subtraction:





£1.00 – 20 p = 80 p

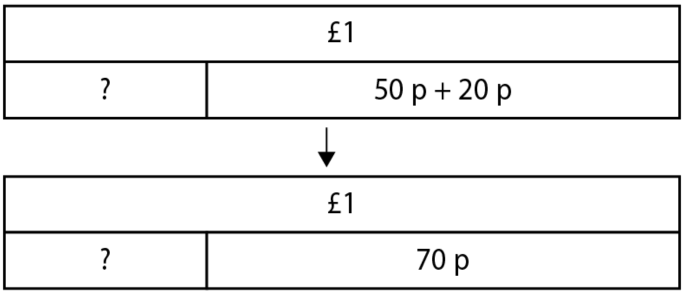
80 p – 50 p = 30 p

or

£1.00 – 20 p – 50 p = 30 p

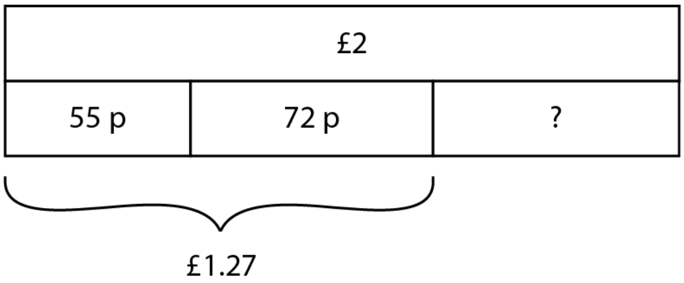
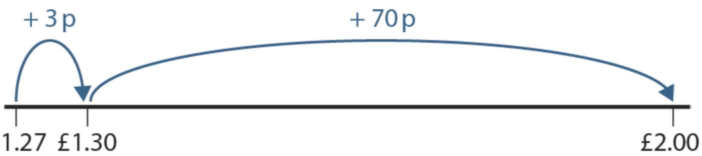
<table>
<tr>
<td></td>
<td>illustrate ‘adding first’). For the ‘adding first’ strategy, you can link to Teaching point 4, where children learnt to calculate the combined cost of several items.

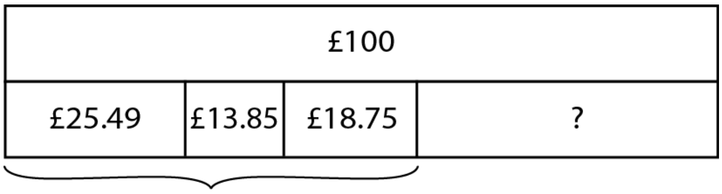
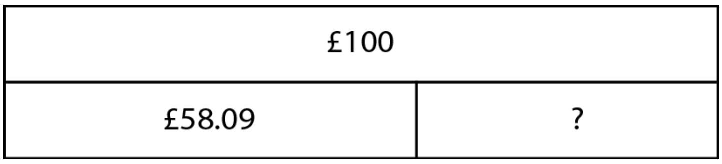
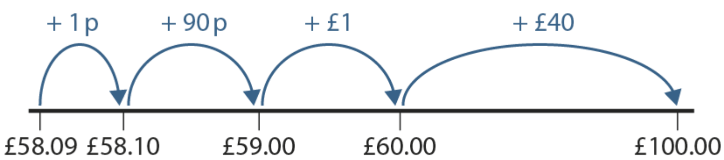
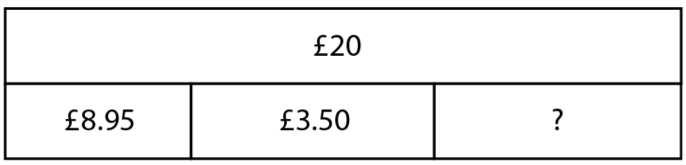
Work through several examples to consolidate understanding of the two strategies. To begin with, you could use coins alongside the bar models, to help demonstrate that the two strategies are ‘equivalent’, as illustrated opposite, but you should progress quite quickly to using just bar models or equations.</td>
<td>Adding first:


20 p + 50 p = 70 p
£1.00 – 70 p = 30 p</td>
</tr>
<tr>
<td>5:2</td>
<td>Now explore some examples involving less simple arithmetic, including some that require written methods. Continue to use bar models to illustrate what the questions are asking and to support children in identifying the calculations they need to perform.

For each example, encourage children to see that we could subtract one cost after another from the amount paid, but that it is often inefficient. If necessary, work through one of the examples in this way to demonstrate that it is less efficient and that a separate column calculation must be written for each subtraction.

For the final step, remind children that often, when subtracting to find change, the ‘finding the difference’/‘working forward’ strategy is usually the most efficient approach, because it avoids the need for more complicated exchanging if using the column method. Note that, later, in segment 1.29 Using equivalence and the compensation property to calculate, children will learn how to use equivalent calculation strategies for subtraction; by that point, they could instead perform the final step of</td>
<td>Example 1:
‘An apple costs 55 p and a drink costs 72 p. How much change will I get from £2.00 if I buy both?’

• Step 1

• Step 2

Change = 73 p</td>
</tr>
</table>

	subtraction problems like these using the column algorithm after first subtracting one from both the minuend and subtrahend, thus avoiding the need for exchange (e.g., £100 – £58.09 = £99.99 – £58.08). Also encourage children to check their answers by adding the 'parts' back together to ensure that the 'whole' is still the same.	Example 2: *'The class has raised £100 to spend on a party. They spend £25.49 on pizzas, £13.85 on drinks and £18.75 on decorations. How much do they have left to spend on the entertainment?'* • Step 1  $\begin{array}{rr} & £\ 2\ 5\ .\ 4\ 9 \\ & £\ 1\ 3\ .\ 8\ 5 \\ + & £\ 1\ 8\ .\ 7\ 5 \\ \hline & \mathbf{£\ 5\ 8\ .\ 0\ 9} \\ \hline & 1\ \ 2\ \ \ 1\ \ \ \end{array}$ • Step 2   £40 + £1 + 90 p + 1 p = £41 + 91 p = £41.91 The class has £41.91 left to spend on entertainment.
5:3	Spend some time looking at word problems as a class, encouraging children to draw a bar model to represent each problem and to decide whether they can easily use repeated subtraction or whether they should add the subtrahends first. The same arithmetic could be present in more than one problem, but the wording/context of the problem could	 Repeated subtraction: *'Joe gets £20 for his birthday on Monday and spends £8.95 on a game. At the weekend, he spends £3.50 on an*

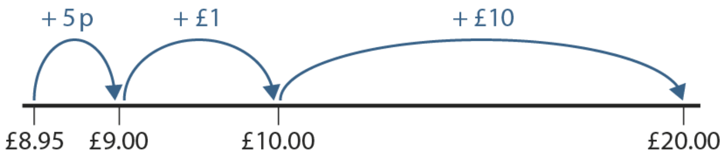
lead children to select one strategy over another, without thinking about which is the most efficient, for example:

- *'Joe gets £20 for his birthday on Monday and spends £8.95 on a game. At the weekend, he spends £3.50 on an ice-cream sundae. How much money does Joe have left?'*
  (Joe spends the money on two separate occasions, which may lead children to subtract twice, rather than adding first.)
- *'Joe buys a game for £8.95 and some football cards for £3.50. He pays with a £20 note. How much money does Joe have left?'*
  (Joe buys two items at the same time, which is more likely to lead children to add first, rather than subtract twice.)

You could ask children to perform the calculations both ways and discuss which they prefer.

ice-cream sundae. How much money does Joe have left?'

- Step 1



$$\begin{aligned} £20 - £8.95 &= £10 + £1 + 5\text{ p} \\ &= £11.05 \end{aligned}$$

- Step 2

$$\begin{array}{rcccccc} & £ & \cancel{1}^{0} & \cancel{1}^{10} & . & {}^{1}0 & 5 \\ - & £ & & 3 & . & 5 & 0 \\ \hline & £ & 0 & 7 & . & 5 & 5 \\ \hline \end{array}$$

Joe has £7.55 left.

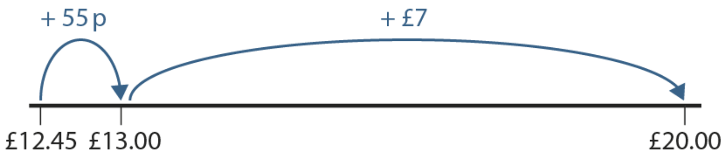
Adding first:

'Joe buys a game for £8.95 and some football cards for £3.50. He pays with a £20 note. How much money does Joe have left?'

- Step 1

$$\begin{array}{rcccccc} & £ & & 8 & . & 9 & 5 \\ + & £ & & 3 & . & 5 & 0 \\ \hline & £ & 1 & 2 & . & 4 & 5 \\ \hline & & 1 & 1 & & & \end{array}$$

- Step 2



$$\begin{aligned} £20 - £12.45 &= £7 + 55\text{ p} \\ &= £7.55 \end{aligned}$$

Joe has £7.55 left.