MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Fractions

3.2 Unit fractions: identifying, representing and comparing

Teacher guide | Year 3

Teaching point 1:

A whole can be divided into any number of equal parts.

Teaching point 2:

Fraction notation can be used to describe an equal part of the whole. One equal part of a whole is called a unit fraction. Each unit fraction has a name.

Teaching point 3:

Fractional notation can be applied to represent one part of a whole in different contexts.

Teaching point 4:

Equal parts do not need to look the same.

Teaching point 5:

Unit fractions can be compared and ordered by looking at the denominator. The greater the denominator, the smaller the fraction.

Teaching point 6:

If the size of a unit fraction is known, the size of the whole can be worked out by repeated addition of that unit fraction.

Overview of learning

In this segment children will:

- develop their understanding of the connections between wholes and equal parts
- be introduced to the word *'fraction'*, considering unit fractions only
- learn how to write a fraction using written notation (e.g. $\frac{1}{4}$) and how the denominator and numerator of a written fraction correspond to the parts within a whole
- learn the names of the fractions they have met so far (i.e. that $\frac{1}{4}$ is read as 'one-quarter')
- apply fractions in the contexts of area, linear and quantitative (sets of objects) models
- compare and order unit fractions of the same whole
- reason how to create the whole from knowledge of one part.

In segment *3.1 Preparing for fractions: the part–whole relationship*, children became experienced in spotting and describing parts of wholes, and practised identifying equal and unequal parts. However, they have not yet used the word 'fraction', seen any written fraction notation (e.g. $\frac{1}{4}$) or said any fraction names (e.g. 'one-quarter'). All of this is introduced here.

In this segment, children are taught to use the stem sentence they learnt in segment *3.1* to help them write fractions. They will be taught to write the division bar first, then the denominator and then then numerator. It is common for people to write a fraction 'from the top down' (writing the numerator, then the bar, then the denominator), but the 'bar first' method is very supportive to the children in linking the notation of the fraction to what it means, so it is worth writing fractions in this order. Check that the children also write them in this way.

The title of this segment refers to unit fractions. A unit fraction has a numerator of one (e.g. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{10}$ etc.) and these are the only types of fractions children will meet during this segment. Non-unit fractions (i.e. any fraction with a denominator other than one) will be introduced in segment *3.3 Non-unit fractions: identifying, representing and comparing*. Unit fractions are so-called because they represent the unit being worked in, defined by the denominator; it is not the 'one-ness' that is special in a unit fraction. For example, in the case of one-sixth, it is the 'sixth-ness', the unit of a sixth, that is important. Children are more familiar with working in units of whole numbers, in particular hundreds, tens and ones, and now they are working with a variety of different units which are all less than one, so this is a big step. The denominator represents the number of equal parts within the whole, and the numerator represents the number of these parts. The unit fraction is one of these parts.

The phrase 'unit fraction' is used in the teaching points throughout this section. However, it is not used extensively with the children. As they have not met any non-unit fractions yet, they will not fully understand how a unit fraction is distinct from any other fraction. It is more important at this stage to focus on notation and the names of the different unit fractions, which don't become regular until sixths. Children need to be confident with the concept of unit fractions because unit fractions are hugely significant in understanding fractions in general. When non-unit fractions are introduced in segment *3.3*, children will still be working with quantities of these unit fractions. Addition and subtraction of fractions also rely on unitising, so children need to be able to verbalise the units that they are working in, rather than just saying numbers, and link them to other units that they are familiar with. For example:

- 3 + 2 = 5
- 3 cats + 2 cats = 5 cats
- £3 + £2 = £5

- 3 kg + 2 kg = 5 kg
- 3 million + 2 million
- 3 sixths + 2 sixths = 5 sixths
- 3 eighths + 2 eighths = 5 eighths.

As in segment *3.1*, concepts are shown to children in a variety of contexts, including area, length and sets of objects. The focus in this segment, as in segment *3.1*, is on fractions as operators. Fractions as numbers will be introduced in segment *3.3*. It will become evident that all of the suggested representations for sets of objects are clearly partitioned into equal parts. For example, a set of 12 partitioned into three groups with four in each. This segment does not progress to the point of saying that one-third of twelve is four. Instead, the focus is on the idea that each of these groups is one-third of the whole, in the same way as for fractions of shapes and lines. This focus on the language of wholes and equal parts builds a strong foundation from which to progress to fractions of quantities in later segments; children who can confidently identify parts within sets, alongside parts of shapes and linear representations, will find it easier to find fractions of quantities later on.

In addition, the pouring of liquid or rice is used as a non-partitioned (continuous) model. Using 'stuff' like liquid or rice, which is not clearly partitioned into equal parts, requires the children to make approximate judgements about the size of the parts relative to the whole; it develops their *fraction sense* and requires them to move beyond counting parts within a whole. As described in segment *3.1*, *Overview of Learning*, it is the relative size of a part to a whole that is critical in determining the value of a fraction.

At the end of this segment, children will again practise creating a whole using their knowledge of one part, this time based on the fraction of the whole that the part forms. Throughout this fractions spine, children are encouraged to work both from whole to part and from part to whole, so that they become familiar with the inverse relationship between parts and wholes and how fractions relate to both multiplication and division.

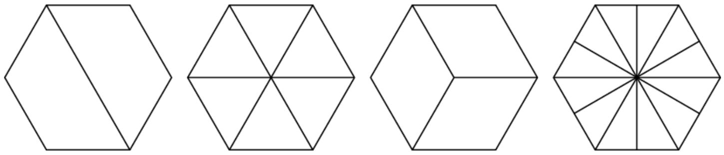
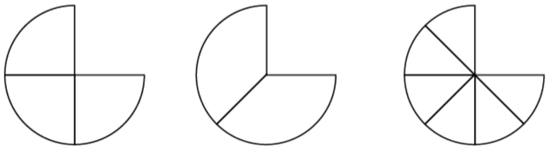
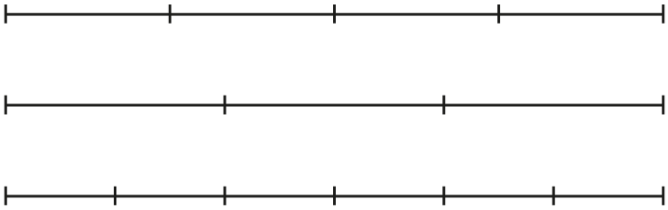
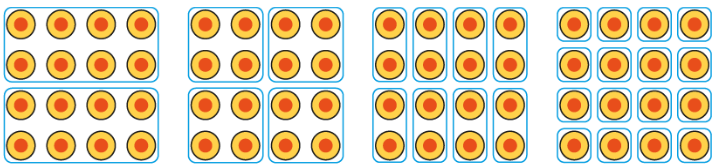
As with segment *3.1*, it may feel like this segment is progressing very slowly, in that by the end of it, children have only met unit fractions. However, while the range of fractions met is limited, the concepts that are met are not. As well as learning how to write and name fractions, in this segment the children also develop their ability to unitise and make judgements about proportional relationship between the part and the whole. Overall, this forms substantial new learning and a strong foundation from which to move forward.

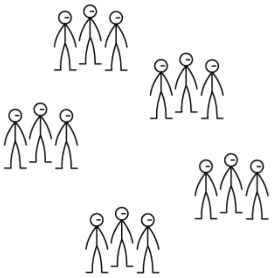
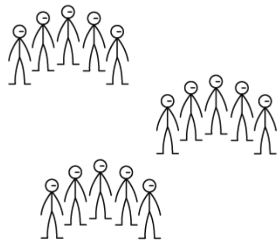
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

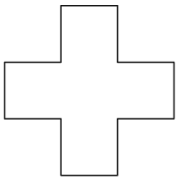
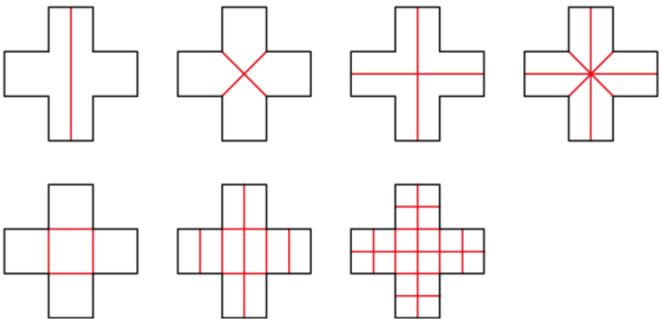
Teaching point 1:

A whole can be divided into any number of equal parts.

Steps in learning

	Guidance	Representations
1:1	In segment *3.1 Preparing for fractions: the part–whole relationship*, children identified wholes and parts of wholes through the context of area, shape, linear and measures, and cardinal and quantity value. They looked at how a whole can be divided into equal parts or unequal parts using the following stem sentences: • ***'If ___ is the whole, then ___ is part of the whole.'*** • ***'The whole has been divided into ___ equal/unequal parts.'*** Now we look at these models alongside each other, and explore how the same whole can be divided into different numbers of equal parts. Discuss the shapes, lines and sets in the images shown opposite. For each set of pictures, ask: • *'What is the same?'* • *'What is different?'* Children should realise that the whole is the same within each set of pictures, but each whole has been divided into a different number of parts; within each whole, the parts are equal. Use the stem sentences above to describe the pictures. Note that a 'whole' can be whatever we define as a whole. *Example 2* (opposite) can be used to check that the children are comfortable with wholes that do not look like a recognisable whole. In the first of these shapes, the whole is divided into three equal parts (but some children may see this as a circle divided into four equal parts). You may	Different numbers of equal parts: • *'What is the same?'* • *'What is different?'* • Area contexts Example 1:  Example 2:  • Linear context Example 3:  • Cardinal contexts Example 4: 

	want to give some time to discussion of this. In segment *3.1*, children were exposed to situations where the parts do not look the same, but the quantity within each part is equal. Remind them that it is possible to divide a class into equal parts because, while children are not all identical to each other, we can have the same number of children in each part. To reinforce this, define the whole as a number of people (rather than, for example, 'the class'), and identify the part as a smaller number of people, as in *Example 5* (opposite). Finding a fraction of an amount is the same as the partitive structure for division where we partition/share/distribute into equal parts (see *Spine 2: Multiplication and Division*).	Example 5:   • *'If fifteen children are the whole, then one team of three children is part of the whole.'* • *'The whole has been divided into five equal parts.'* • *'If fifteen children are the whole, then one team of five children is part of the whole.'* • *'The whole has been divided into three equal parts.'*

1:2	Give the children plenty of practice looking at shapes, lines and sets, and identifying how many equal parts each is divided into.
	
	To deepen understanding, use dòng nǎo jīn problems like the one opposite. You could give children a worksheet with several copies of the shape for them to divide into parts. Some of the possibilities are shown opposite.
		Dòng nǎo jīn:
		• *'Find how you can divide this shape into:'*
		  • *'two equal parts'*
		  • *'four equal parts'*
		  • *'five equal parts'*
		  • *'eight equal parts.'*
		'You might find more than one way for each.'
		• *'What other number of equal parts can you divide it into?'*
		
		Possible answers
		

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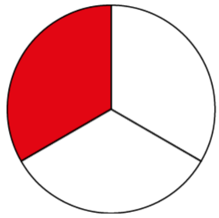
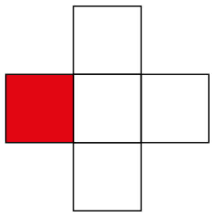
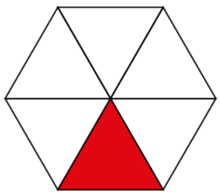
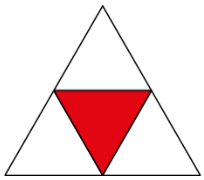
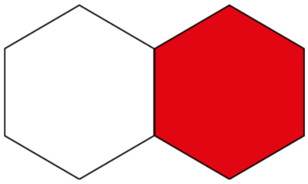
Teaching point 2:

Fraction notation can be used to describe an equal part of the whole. One equal part of a whole is called a unit fraction. Each unit fraction has a name.

Steps in learning

	Guidance	Representations
2:1	Having completed step *1:2*, children should be comfortable with the idea that a whole can be divided into any number of equal parts. Now look at a range of shapes, each with a different number of equal parts and with one of the equal parts shaded. Ask the children to describe the shaded part of each shape, using the stem sentences: • ***'The whole has been divided into ___ equal parts.'*** • ***'___ of the parts has been shaded.'*** These stem sentences are central to this spine, and will be used repeatedly to scaffold the children's learning. Take time during this teaching point to make sure that all children can use them confidently. It is important that children understand that we have a choice about what we take as our 'whole'. For example, the two hexagons joined together opposite can be seen as one whole.	 • *'The whole has been divided into three equal parts.'* • *'One of the parts has been shaded.'*  • *'The whole has been divided into five equal parts.'* • *'One of the parts has been shaded.'*  • *'The whole has been divided into six equal parts.'* • *'One of the parts has been shaded.'*  • *'The whole has been divided into four equal parts.'* • *'One of the parts has been shaded.'*



- *'The whole has been divided into two equal parts.'*
- *'One of the parts has been shaded.'*

2:2 Next, tell the children that we can record the relationship between the part and the whole symbolically using written notation. At this stage, do not say the fraction names ('one-third', 'one-fifth' etc.); these will be taught in the next step.

Show the first image with its stem sentences from step *2:1* again. Demonstrate how to write a fraction, saying the stem sentence and writing the corresponding part of the written fraction at the same time:

Say	Write
'The whole has been divided…'	The division bar: –
'…into 3 equal parts.'	The denominator: **3**
'One of the parts has been shaded.'	The numerator: **1**

Repeat this several times, saying the stem sentence and writing the fraction at the same time, so that children clearly see the links between the notation and the stem sentences.

Next, discuss the written form of the fraction. Explain that the division bar shows the division relationship between the whole and the part. Ask '*What does the "3" represent?*' and agree that it stands for the three equal parts that the whole has been divided into. Then ask '*What does the "1" represent?*' and establish that it stands for the one equal part that is shaded.

Now look at the other shapes from step *2:1*, one by one. Ask the children to write the fractions as you say the stem sentence as a class. If the children are working in pairs, they can take it in turns to write the fraction, checking that the order is being followed. Ensure throughout that the children are writing each part of the fraction as they say the relevant part of the stem sentences. When each fraction has been written, ask the children what each number represents.

Again, remind children that we have a choice about what we take as our 'whole', referring to the two hexagons joined together below.

Expand on this idea by returning to the hexagon. Taking the whole hexagon as the whole, one-sixth is shaded; taking a half of the hexagon as a whole, one-third is shaded.

You might normally write a fraction 'from the top down' (writing the numerator, then the bar, and then the denominator), but the 'bar first' method is very supportive to the children in linking the notation of the fraction to what it means, so it is worth writing fractions in this order whenever you write them in front of the children.

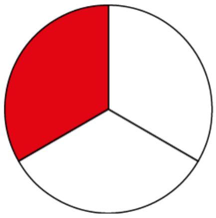
Finally, summarise for children what they have learnt during this step:

'There is a special name for the relationship between a whole and a part. It is called a fraction.'
Explain that they have seen images of fractions (refer back to the images you displayed) and they have written a fraction (refer to their written symbols/fraction notation).

Note: to write a fraction in Microsoft Word or PowerPoint:

- press Alt =
- type the fraction as (for example) 1/3
- press Enter

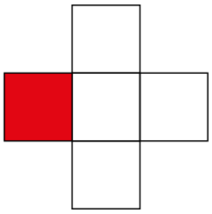
The fraction will automatically be presented in vertical form, e.g. $\frac{1}{3}$. Try to use this vertical format in any lesson resources you are preparing.



- *'The whole has been divided into three equal parts.'*
- *'One of the parts has been shaded.'*

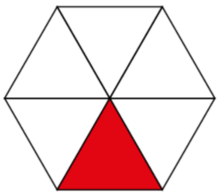
$$\frac{\square}{\square} \rightarrow \frac{\square}{3} \rightarrow \frac{1}{3}$$

- *'What does the "3" represent?'*
- *'What does the "1" represent?'*



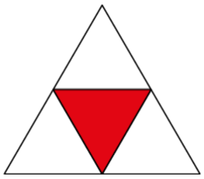
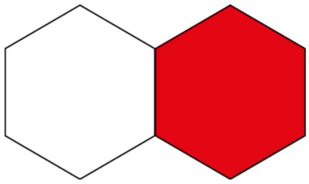
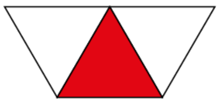
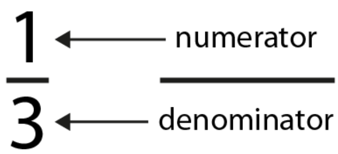
- *'The whole has been divided into five equal parts.'*
- *'One of the parts has been shaded.'*

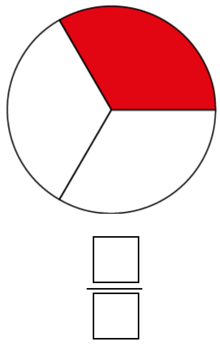
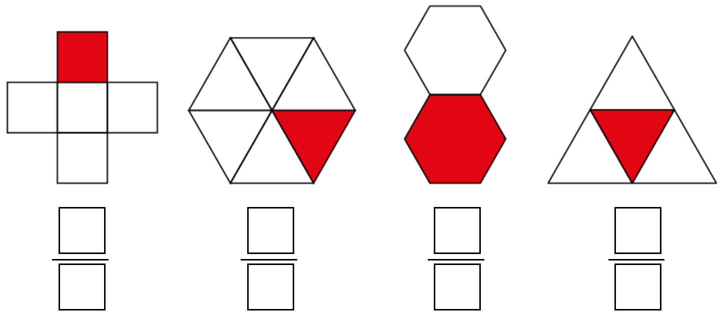
$$\frac{\square}{\square} \rightarrow \frac{\square}{5} \rightarrow \frac{1}{5}$$

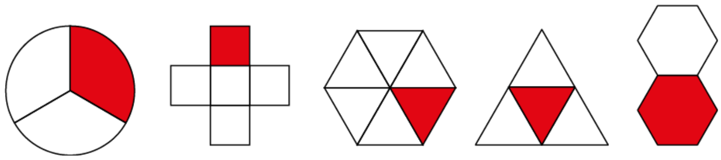
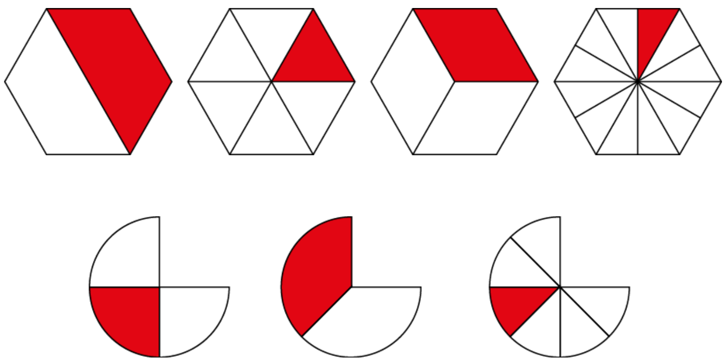


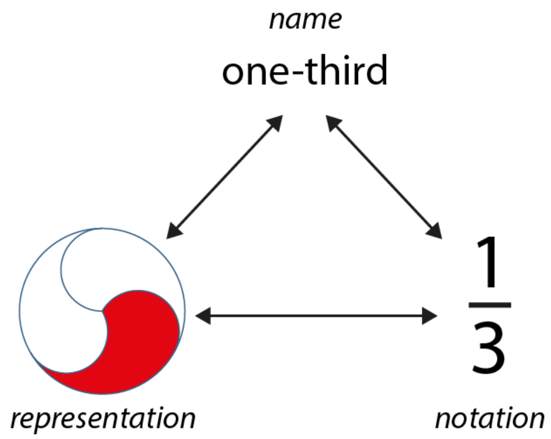
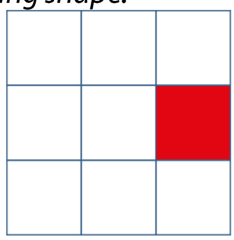
- *'The whole has been divided into six equal parts.'*
- *'One of the parts has been shaded.'*

$$\frac{\square}{\square} \rightarrow \frac{\square}{6} \rightarrow \frac{1}{6}$$

	 • *'The whole has been divided into four equal parts.'* • *'One of the parts has been shaded.'* $\frac{\square}{\square} \rightarrow \frac{\square}{4} \rightarrow \frac{1}{4}$  • *'The whole has been divided into two equal parts.'* • *'One of the parts has been shaded.'* $\frac{\square}{\square} \rightarrow \frac{\square}{2} \rightarrow \frac{1}{2}$  • *'The whole has been divided into three equal parts.'* • *'One of the parts has been shaded.'* $\frac{\square}{\square} \rightarrow \frac{\square}{3} \rightarrow \frac{1}{3}$	
2:3	Now that children can write fractions, introduce the vocabulary of *'denominator'* and *'numerator'*. Explain that the denominator is the number of equal parts in the whole. It is written below the division bar. (The word 'denominator' comes from the Latin for 'to name'. This will be useful later, as the denominator is the name of the unit we are working with.) The numerator is the number of parts of the whole (one for a unit fraction). It is written above the division bar. (The word 'numerator' comes from the Latin word for 'number'. This will be also	

	useful later as the numerator describes the number of parts we are working with.) Reiterate how to write a fraction, this time using the new vocabulary: *'To write a fraction, start by drawing the division bar. Next, write the denominator beneath to show the number of equal parts in the whole. Finally, write the numerator.'*	
2:4	Continue to reinforce the new vocabulary with a matching activity. Look at the first shape opposite. Remind children that the denominator tells us the number of equal parts. Ask *'What is the denominator?'* Encourage children the answer using the stem sentence: ***'The denominator is ___ because the whole is divided into ___ equal parts.'*** Then remind them the numerator tells us how many parts are shaded. Ask *'What is the numerator?'* Encourage children the answer using the generalised statement: '***The numerator is one because one part is shaded.'*** Give the children copies of images, such as those provided opposite, and ask them to fill in the denominators and numerators, using the stem sentences to justify their reasoning. Allow children to further explore this concept using a dòng nǎo jīn problem: *'Can you draw me a shape that can be represented by a fraction with a denominator of seven? Of eight?'*	 • *'The denominator is 3 because the whole is divided into three equal parts.'* • *'The numerator is 1 because one part is shaded.'* 
2:5	The next step is to name one equal part of the whole. Note that this is the first time in this spine that children will say any of these fraction names. Begin by showing the children the shapes from step *2:4*, now labelled with their fractional notation. Also show the written names for each fraction ('one-	

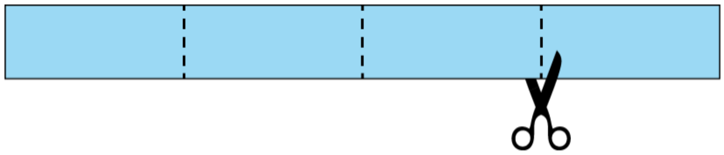
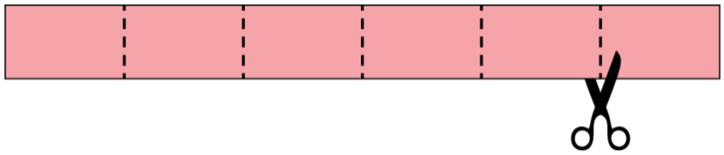
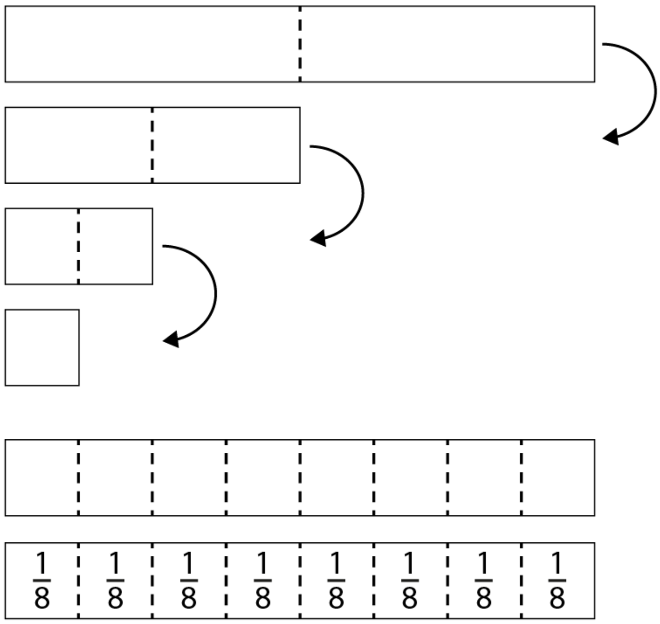
	quarter' etc.), in random order. Ask the children to reason about which name goes with which fraction. They should notice that 'six' occurs unaltered in 'one-sixth'. For 'one-third' and 'one-fifth' there are elements of the words 'three' and 'five' in the name. The words 'half' and 'quarter' bear no resemblance to the words 'two' and 'four', although children are of course likely to have used these names in everyday talk in relation to their ages, or to sharing food, for instance. Spend time practising the names of the fractions, recognising that it is the denominator which gives a fraction its name. If the children make mistakes, like calling one-quarter one-fourth, acknowledge that the shape is indeed divided into four equal parts, but explain that the name we give it is one-quarter rather than one-fourth. From sixths onwards, the naming follows a regular pattern, using the ordinal number of the denominator (*'**seven**th, **eigh**th, etc. …'*). Provide examples of these for children to practise.	Fraction names – fractions to one-sixth: *'Which name matches which shape? Are there any clues?'* one-quarter one-third one-half one-sixth one-fifth 
2:6	Children should now be able to name and write any unit fraction (don't use the phrase 'unit fraction' with them at this stage, as they have only encountered fractions with a numerator of one so far). Give them further practice by returning to the area, line and set images from *Teaching point 1*, now with one part shaded. Ask children to write the fraction that is shaded for each shape. Finally, it is worth making sure that children can confidently move between the name, written notation and visual representation for a fraction. Using a diagram with three interconnected ideas, such as the one opposite, give	Writing unit fractions: 

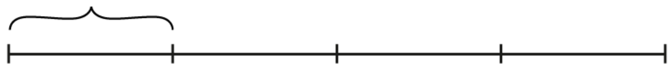
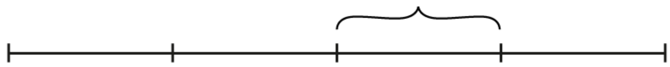
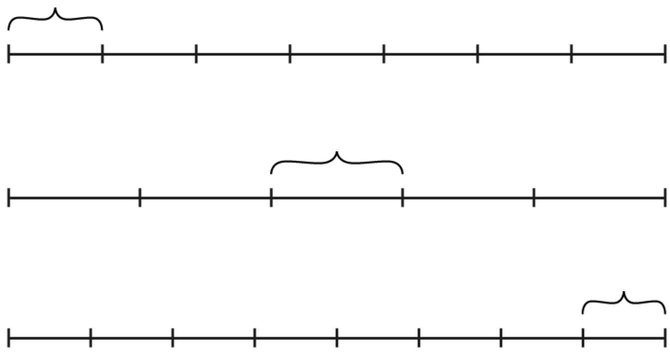
	them one of the elements and ask them to complete the other two. Provide further varied practice by presenting children with written notation, fraction names and visual representations and asking questions.	Moving between the name, written notation and visual representation for a fraction:  Sample questions: • *'How do we say $\frac{1}{7}$ in words?'* • *'How do we write "one-twelfth"?'* *'Look at the following shape.'*  • *'What fraction is shaded?'* • *'How do we say it in words?'* • *'How do we write it?'*

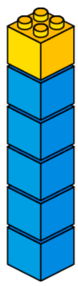
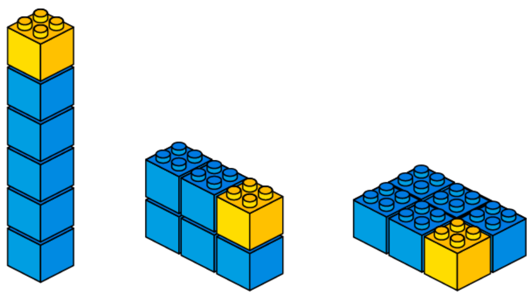
Teaching point 3:

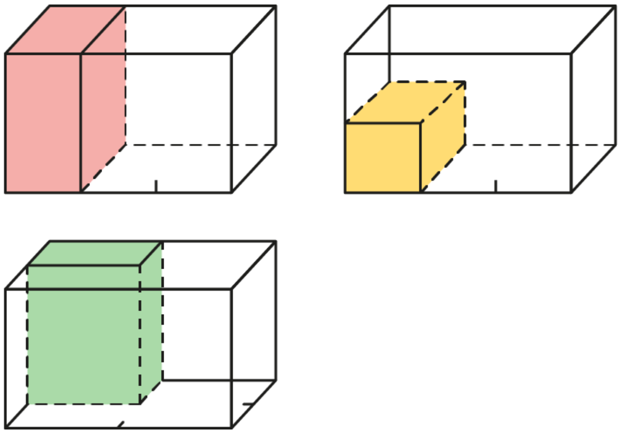
Fractional notation can be applied to represent one part of a whole in different contexts.

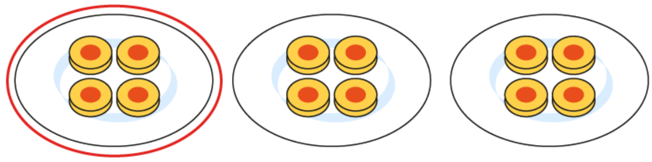
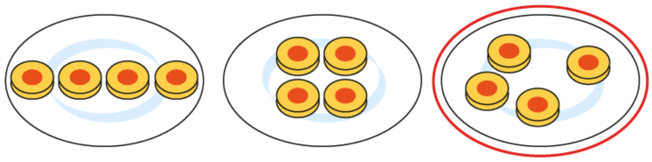
Steps in learning

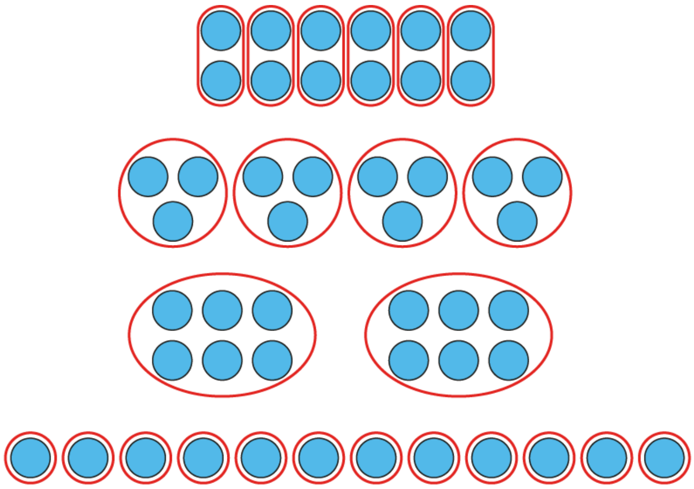
	Guidance	Representations
3:1	So far, children have been exposed to shaded equal parts of a whole in an area context. In this teaching point, they will apply fractional notation and the fraction names in the context of measures and cardinality (quantity values). Show the children the ribbons. Ask them to describe what is happening using the following stem sentences: • ***'The whole has been divided into __ equal parts. '*** • ***'Each equal part is one- __ of the whole.'*** • ***'__ of the whole ribbon has been cut off.'*** Next, ask children to create their own 'ribbon'. Give them a strip of paper each and ask them to fold it in half, then in half again and in half again. Now ask them to unfold it and label each equal part with its fraction. Ask the children to describe their strip using the stem sentences. • ***'The whole has been divided into ___ equal parts.'*** • ***'Each equal part is one- ___ of the whole.'*** Strictly speaking, these linear bars are another area model. They are valuable, however, for making links between an area model and a number line. Children will be introduced to the use of a number line with fractions in segment *3.3 Non-unit fractions: identifying, representing and comparing.*	Fractional notation and names – measures context (length): Example 1:  • *'The whole has been divided into four equal parts.'* • *'Each equal part is one-quarter of the whole.'* • *'One-quarter of the whole ribbon has been cut off.'* Example 2:  • *'The whole has been divided into six equal parts.'* • *'Each equal part is one-sixth of the whole.'* • *'One-sixth of the whole ribbon has been cut off.'* Example 3:  • *'The whole has been divided into eight equal parts.'* • *'Each equal part is one-eighth of the whole.'*

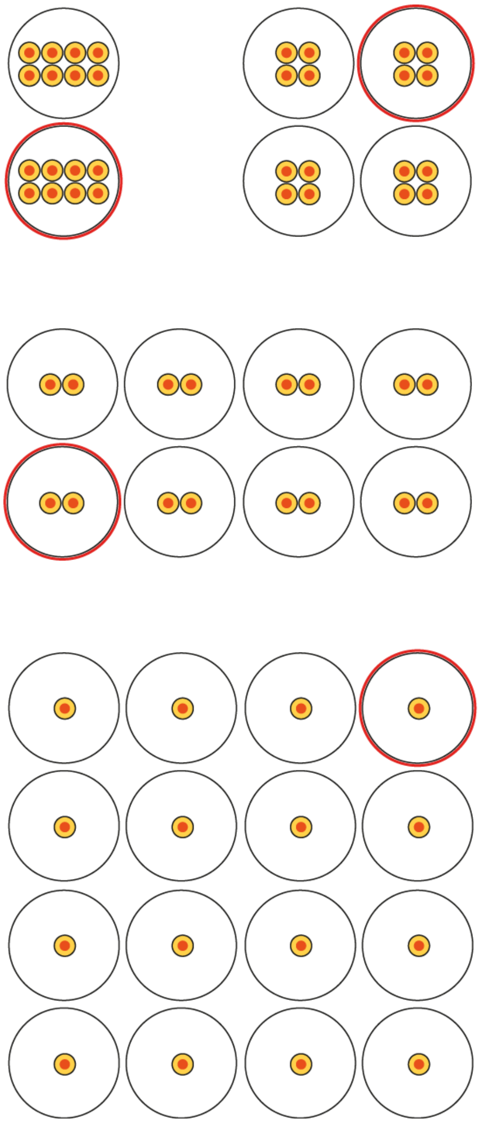
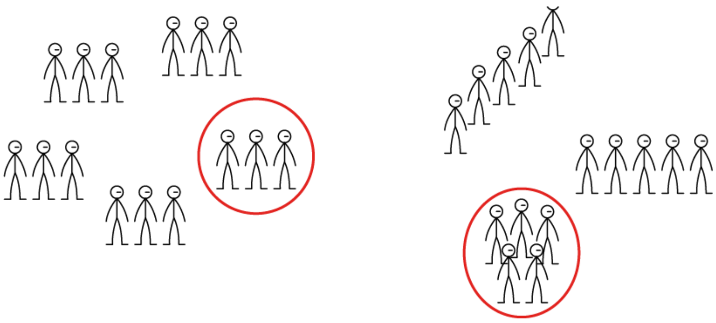
3:2	Now move from the linear bars to dividing a line into equal parts. Show children the first image opposite and ask them to describe what they see using the stem sentences:
	• ***'The whole has been divided into ___ equal parts.'***
	• ***'One of these parts is highlighted. This part is one-___ of the whole line.'***
	As children say the fraction name, label the section on the line with the fraction, writing the division bar first, then the denominator, and then the numerator, as before.
	Once you have identified that one-quarter of the first line is highlighted, look at the second line. Ask:
	• *'What is the same?'*
	• *'What is different?'*
	Children should realise that, as for the first line, one-quarter is highlighted but that it is a different quarter.
	Note that we are still using fractions as an operator here: one-quarter *of this line* has been highlighted. Children will meet fractions as numbers on a number line in segment *3.3 Non-unit fractions: identifying, representing and comparing*. However, as there are similarities between this image and a number line, children may ask whether this is three-quarters. If necessary, refer the children back to the stem sentences to clarify why the highlighted section is still one-quarter.
	It is important to emphasise the equal sections shown on the line, not the dividing marks. A common misconception is that, because there are three dividing marks, each mark represents one-third.
	Children usually find it easier to move from the image to the written notation,
		Fractional notation and names – measurement context (length):
		
		• *'The whole has been divided into four equal parts.'*
		• *'One of these parts is highlighted. This part is one-quarter of the whole line.'*
		
		'Now look at this line.'
		
		• *'What is the same?'*
		• *'What is different?'*
		  ◦ *'The whole has been divided into four equal parts.'*
		  ◦ *'One of these parts is highlighted. This part is one-quarter of the whole line.'*
		
		Emphasising sections:
		

	and hence to the fraction name. Writing the fraction as the stem sentence is verbalised (as the children did extensively in the previous teaching point) will help give them confidence in naming the fraction. In time, they should be able to move from the visual representation straight to the fraction name. Work through examples similar to those shown. Ensure you vary the orientation of the line and the length of the line.	
3:3	Now explore the concept with a 3D (volume) model. Present children with an image of coloured bricks. Use a story like this: • *'Rupinder made this tower using coloured bricks. How many bricks did she use?'* • *'Each brick is one equal part. What fraction of the tower is yellow?'* Looking at the bricks in two dimensions, the yellow brick appears to have a bigger area than the blue bricks because it is the top brick. It would be worthwhile allowing children to make a similar model using six equal-sized bricks. Show the children more 3D models made of six bricks. Ask: • *'What is the same?'* • *'What is different?'* Give the children some coloured bricks and ask them to make and describe models that show different unit fractions, such as one-quarter, one-eight, one-ninth and one-tenth. This draws on learning from segment *3.1 Preparing for fractions: the part–whole relationship*, where children had to work out what a whole was when given one equal part and told the number of parts in the whole. Make the	Fractional notation and names – measures context (volume):  • *'There are six bricks in the whole tower.'* • *'One brick is yellow, so one-sixth of the tower is yellow.'* *'Now look at these models.'*  • *'What is the same?'* • *'What is different?'* • *'All the models have the same fraction of yellow, but the yellow brick is in different positions.'*

	link back to that learning. For example, say *'If I know that one brick is one-quarter of the whole, I need four of those to make the whole (because my yellow brick is one of four equal parts)'*. Look out for children who take four bricks *in addition to* the yellow brick. If this happens, ask them how many equal parts they have once the bricks are put together and relate their model back to the stem sentences.	
3:4	Ask children to look at other 3D models and say what fraction of the whole is filled or highlighted. Also look at diagrams such as the ones shown opposite, where children need to be able to visualise how many of each shape will fit into the whole. The broken lines can be used as a guide.	Fractional notation and names – measures context (volume): 

3:5	The same approach of identifying parts and wholes can be used for identifying fractions of sets. Provide a picture like the one shown opposite. The biscuits are arranged in three equal groups, so the biscuits on one plate can described as a one-third of all the biscuits.
	You could equally say that the whole has been divided into twelve equal parts and one plate of biscuits is four-twelfths of the whole. If children suggest this, let them know that we will be looking at fractions with a numerator other than one (non-unit fractions) in the next segment – don't dwell on this here.
	At this stage you will notice the obvious link with fractions of quantities: we can see from the picture that one-third of twelve is four. Tempting as it may be to say this, resist the urge. At this early stage, it is better to continue to focus on the language of wholes and equal parts using the stem sentence: ***'The whole has been divided into ___ equal parts. One part is one- ___ of the whole.'***
	Children who can confidently identify parts within sets, alongside parts of shapes and linear representations, will find it easier to find fractions of quantities later on; this is covered in segment *3.6 Multiplying whole numbers and fractions.*
	Include some images in which the parts do not look the same. For example, show children the second image opposite and ask what they notice.
	• *'What is the same?'*
	• *'What is different?'*
	Make sure children understand that, even though the arrangement of biscuits is different, the whole is still divided into equal parts as there are the same number in each part.
		Fractional notation and names – cardinal context:
		
		• *'The whole has been divided into three equal parts.'*
		• *'One plate of biscuits is one-third of the whole.'*
		
		'Now look at these plates of biscuits.'
		
		• *'What is the same?'*
		• *'What is different?'*
		  • *'All the plates have the same number of biscuits.'*
		  • *'The whole has been divided into three equal parts.'*
		  • *'One plate of biscuits is one-third of the whole.'*
		  • *'The biscuits are arranged in different ways.'*

3:6	Explore this concept further by asking children to divide a whole into different groups for themselves. Give each child (or pair) 12 counters and remind them that they have already seen that a whole of 12 can be divided into three equal parts, with four in each part. Ask them to see what other equal parts they can divide the whole into. Some possible options are shown opposite. Ask them to describe their parts using the stem sentences: • ***'The whole has been divided into ___ equal parts.'*** • ***'One of these parts is one-___ of the whole.'*** The children may make one group with all 12 counters in. This could be described as follows: *'The whole is divided into one equal part. This part is $\frac{1}{1}$ of the whole.'* This is a correct written representation of the situation, but the concept that $\frac{1}{1}$ is equivalent to one (and that one can be written as $\frac{1}{1}$, two can be written as $\frac{2}{1}$, and so forth) is not covered in detail until secondary school.	Dividing 12 counters into equal groups: 

3:7	Next, provide children with further practice in identifying the fraction of a group that is circled or highlighted. At this stage, make sure that the objects are clearly arranged into groups of equal number, with one group highlighted, as shown in the examples opposite.	Example 1:  Example 2: 
3:8	Provide children with opportunities to identify whether images show a particular fraction or not. Encourage them to use their understanding of	

wholes and equal parts to explain their reasoning, using the stem sentences from the previous teaching points. Expect them to explain both why the correct answers are correct and why the incorrect answers are incorrect.

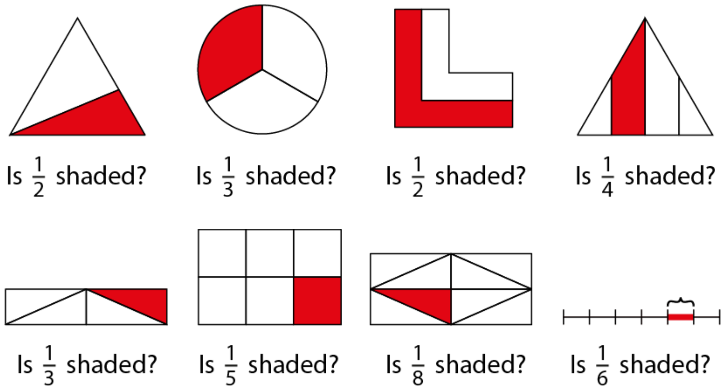
Where the fraction and picture do not match, prompt children to identify why this is. In some cases (first row, first, third and fourth images) it is because the parts are not equal; in others (second row, first and second images), the parts are equal, but the number of parts in the image does not correspond to the denominator.

Children's answers may reveal common misconceptions. In particular, if children think the first and second images in the second row are correct, they have probably misunderstood the denominator as the number of unshaded parts, rather than the number of equal parts that the whole has been divided into.

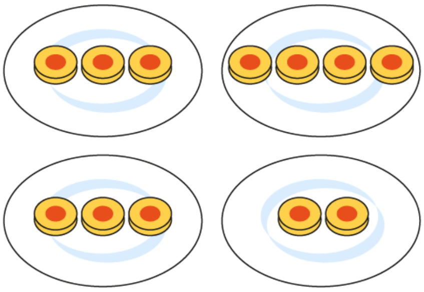
As before, it is important to include fractions of sets in the practice examples. Show children the image of biscuits opposite and encourage them to recognise that the groups need to be equal; although two of the plates have the correct number of biscuits, the other two do not, so each one is not one-quarter of the whole. As in segment *3.1 Preparing for fractions: the part–whole relationship*, step 2:2, you could ask *'Can we rearrange the biscuits so each plate has one-quarter of the biscuits?'*.

To promote depth of understanding, use a dòng nǎo jīn problem like the one shown opposite. Children may see that if they divide the top part into two equal parts, then the shape will be divided into four equal parts, so it is true that one-quarter is shaded.

- *'Does each shape show the given fraction?'*

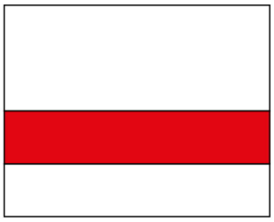


- *'Does each plate have one-quarter of the biscuits?'*



Dòng nǎo jīn:

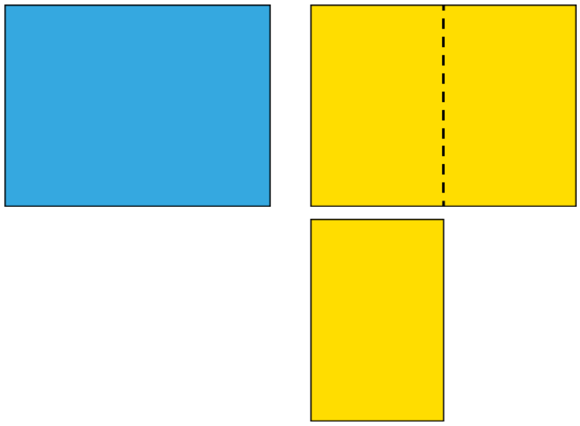
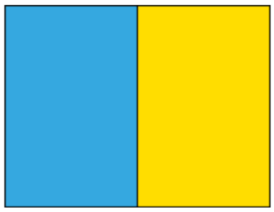
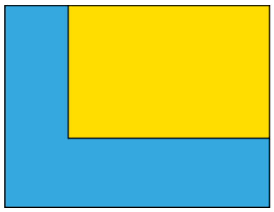
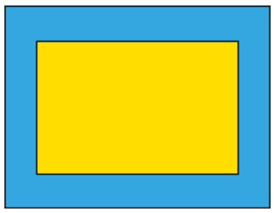
'Is one-quarter shaded?'

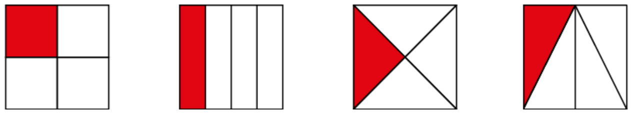
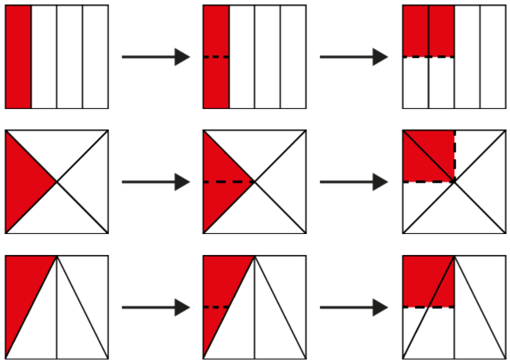
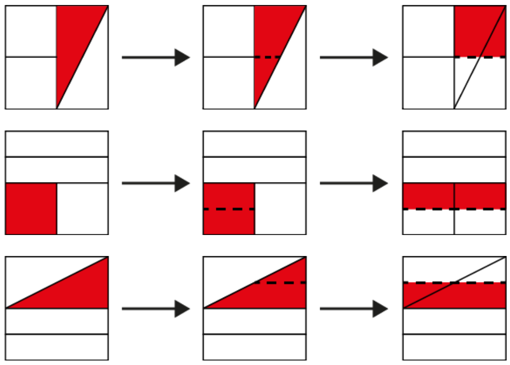


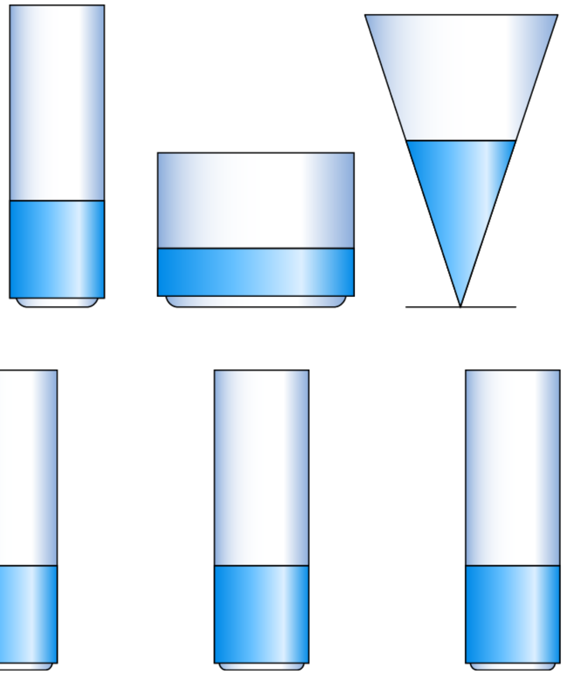
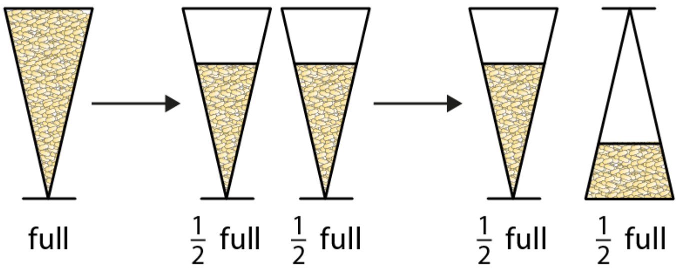
Teaching point 4:

Equal parts do not need to look the same.

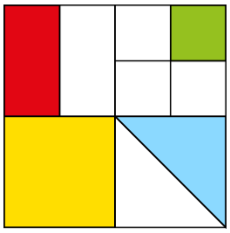
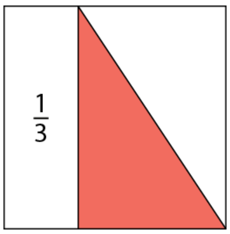
Steps in learning

	Guidance	Representations
4:1	Children have already been introduced to the idea that equal parts do not need to look the same. For example, biscuits within a part may be arranged differently. This step explores this concept further, in the context of area and volume. To illustrate this, take two different coloured pieces of paper that are the same size, and follow these steps: **1.** Fold one piece of paper in half. **2.** Place the half on top of the remaining whole so that it is obvious that the two colours are both one-half of the whole-sized sheet. **3.** Rotate the half and place it on top of the whole, lining up the top right corner. Discuss whether each of the two coloured areas are still one-half of the whole. **4.** Place the small rectangle inside the whole sheet so it is surrounded by the other colour. Again, discuss whether each of the two coloured areas are still one-half of the whole. **5.** Say *'The two colours have the same area. Is this true or false?'* Encourage children to explain why. Repeat with the yellow piece in different positions, finally returning to the side-by-side image with the open pieces of paper from step 1.	Equal parts may not look the same – area context: **1.**  **2.**  **3.**  **4.** 

4:2	Return to the four images of a square divided into four equal parts that children encountered in segment *3.1 Preparing for fractions: the part–whole relationship*, step *2:7*. Ask: • *'What is the same?'* • *'What is different?'* Children should recognise that each square has been divided into equal parts and that all the parts are the same size, even though they look different. To reinforce this, use the stem sentences: • ***'The whole has been divided into ___equal parts. '*** • ***'One equal part is shaded, so each equal part is ___of the whole.'*** Now ask the children to describe how we can show that all the parts are the same size, by dividing and moving parts of the quarters to form the same quarter as the first image. You could provide children with paper copies of the squares so they can physically cut out and move pieces into new positions.	• *'What is the same?'* • *'What is different?'*  • *'The whole has been divided into four equal parts.'* • *'One equal part is shaded, so each equal part is one-quarter of the whole.'* • *'Can we show that all the parts are the same size?'* 
4:3	Once children are comfortable with these different ways of representing one-quarter of a square, present them with squares divided into non-congruent quarters. This time, ask them to show that all the parts are the same size, by dividing and moving parts of the quarters to make four congruent parts within each square. Reiterate that the whole is the same each time, and ask '*If each shaded part is one-quarter of the whole square, are the shaded parts all the same size?*'. Children should realise that they must be, because they each represent the same fraction of the whole square: one-quarter.	• *'Can we show that all the parts are the same size?'* 

4:4	Step *4:3* showed visually that the non-congruent parts were the same size as each other. Another way of thinking about it is through mathematical reasoning such as: • *'If Shape A (triangle) is one-quarter of the whole square…'* • *'…and Shape B (small square) is one-quarter of the same whole square…'* • *'…then Shape A takes up the same space as Shape B.'*	
4:5	Another way to illustrate that equal parts do not need to look the same is by pouring coloured liquid into different-shaped glasses. For example, use three different-shaped glasses, each with a capacity of 300 ml (one tall and thin; one short and wide and one conical). Pour 100 ml of water or orange squash into each one, before showing them to the children. Ask them to estimate the fraction of each glass that is filled. Expect them to make different suggestions. Next, pour the liquid into three 300 ml glasses that are the same shape, pausing between each one to ask the children whether they think there is more or less liquid than in the previous glass. Ask the children what fraction of each glass is filled now (one-third). Now, pour the liquid back into the different-shaped glasses and ask what fraction of the glass is filled. Children should realise that the fraction is still one-third for each glass, even though they look different. Show the children that the whole is the same for each glass (300 ml), by pouring all the liquid into one glass at a time. If you cannot find suitable-sized glasses, an alternative is to use two identical conical glasses. Fill one of	Equal parts may not look the same – volume context: • Liquid  • Rice 

	them with rice, then pour some of the rice into a second glass until there is the same amount in each. At this stage you must have poured out half of the rice (as you have divided the whole into two equal parts). However, it looks like each glass is more than half full (in fact when a conical glass is half full by volume, the rice will come two-thirds of the way up the vertical height of the glass). Putting a piece of card on top of one glass and turning it upside down will show a 'half-full' glass, which looks very different. When children have explored this concept and are comfortable with the idea that equal parts can look different, agree on the generalisation: ***'Equal parts of the whole do not have to look the same.'***	

4:6	To deepen understanding of this concept further, present dòng nǎo jīn problems like the ones shown opposite.	Dòng nǎo jīn:
		• *'What fraction of the whole square is:'*
		  • *'green?'*
		  • *'red?'*
		  • *'light blue?'*
		  • *'yellow?'*
		
		• *'What fraction of the square is red?' 'Explain your reasoning.'*
		

Teaching point 5:

Unit fractions can be compared and ordered by looking at the denominator. The greater the denominator, the smaller the fraction.

Steps in learning

	Guidance	Representations
5:1	See guidance below	See representations below

5:1

Guidance

In *3.1, Preparing for fractions: the part–whole relationship, Teaching point 3*, children began to compare the relative size of parts. In this teaching point, they will compare unit fractions, first visually and then with reference to the denominator.

Begin practically by preparing pieces of ribbon, coloured card or paper, cut into equal-sized pieces from the same length whole (30 cm lengths of ribbon or strips of A4 will work well). Give each child a set of ribbons/cards/papers: one orange (whole), three red, four blue, five yellow, six green and ten purple.

Physically put them back together by sticking a set of pieces to a classroom wall or board, with the orange (whole) strip at the top. Ask children what they notice about each whole length, and agree that all the colours were the same length before they were cut up.

Ask:

- *'Which coloured strip has the most equal parts?'* (purple)
- *'What else do you notice about them?'* (They are the smallest.)

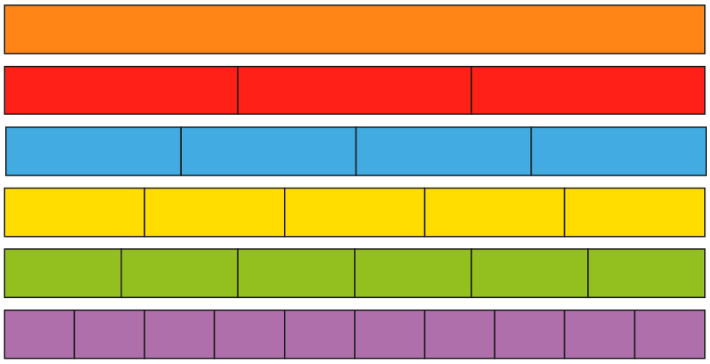
Introduce the generalisation: ***'When the whole is the same, the greater the number of equal parts, the smaller each equal part is.'***

Now ask:

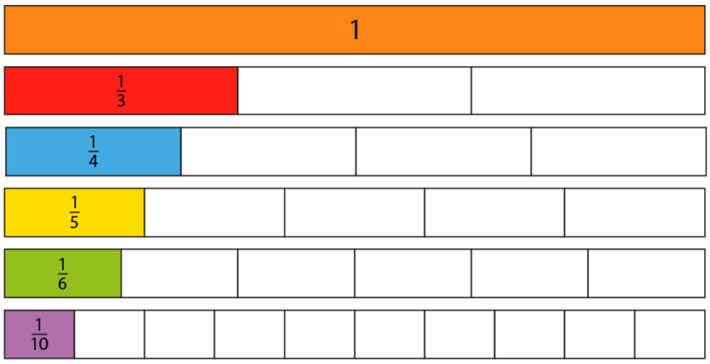
- *'Which colour has the fewest equal parts?'* (red)
- *'What else do you notice about them?'* (They are the largest.)

Representations

- *'Which coloured strip has the most equal parts?'*
- *'Which coloured strip has the fewest equal parts?'*



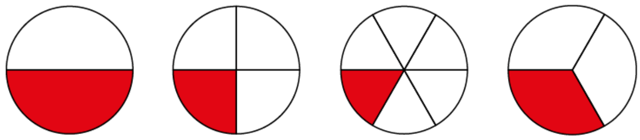
- *'What fraction is each piece of the whole length?'*



Ordering the fractions:

$\frac{1}{3} > \frac{1}{4} > \frac{1}{5} > \frac{1}{6} > \frac{1}{10}$

	You can now introduce the generalisation: ***'When the whole is the same, the smaller the number of equal parts, the bigger each equal part is.'*** This is implicit in the previous generalisation, but at this stage it is useful to make the point explicitly. Next, consider what fraction of the whole each coloured piece represents. Ask *'What fraction is each purple piece of the whole length?'* and encourage children to answer using the stem sentences: • ***'The whole is divided into ___ equal parts.'*** • ***'Each equal part is ___ of the whole.'*** Write '$\frac{1}{10}$' on the first purple piece. Repeat this for the red pieces, and then for the other colours. Label the first piece of each coloured ribbon with its unit fraction as you work through them. Again, notice the use of the '1' bar as a reference point for what the whole is. It is very important to constantly make reference to the whole, since a fraction only has a value in relation to the whole. The green area representing one-sixth, for example, would be a different value within a different-sized whole. Children should be reminded of this from time to time. Repeat that the red pieces are the largest and the purple pieces the smallest. Discuss what children notice about the unit fractions, leading to the following generalisation: ***'When comparing unit fractions, the greater the denominator, the smaller the fraction.'*** Write an inequality statement to show that the generalisation is true. Rearrange the unit fraction strips into a line above the inequality.	

5:2	Once children have grasped the concept, explore it further using a different area model. Show children the circles opposite, and ask them to identify the unit fractions that are shaded. Ask the children to order the fractions from smallest to largest, using the inequality sign, <, between the fractions. Challenge children's thinking by asking them to convince you that one-sixth is smaller than one-quarter. Encourage them to do this in two ways: • By reference to the picture: children should see that, given the same whole, one-sixth is smaller than one-quarter • By reasoning: for example, if we have a cake and share it equally between six people, each person will get less than if we share a cake equally between four people. In the first scenario, there are more people to share it between, so each person will get less. Reiterate the generalisation: ***'When comparing unit fractions, the greater the denominator, the smaller the fraction.'***	*'What fraction of each circle is shaded?'*  $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{3}$ Ordering the fractions: $\frac{1}{6} < \frac{1}{4} < \frac{1}{3} < \frac{1}{2}$
5:3	Pouring is a powerful method that you can use to encourage children to make approximate judgements, look at proportional relationships, and embed the knowledge that (for example) $\frac{1}{4} < \frac{1}{3}$. You will need some liquid such as water or squash, or you can use rice. You will need a straight-sided transparent container, such as a measuring cylinder or straight-sided glass vase. It is important that it is tall and thin so that children have a very clear view.	

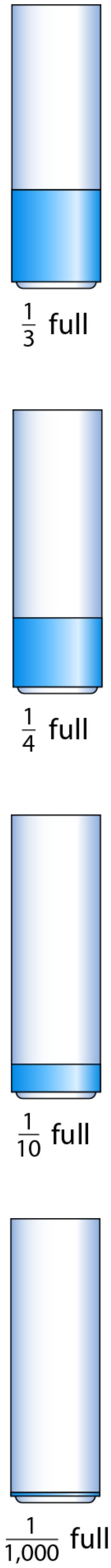
Say to the children *'I want to pour liquid/rice in so that it is about one-third full. Shout 'stop' when it's about right. How will we know when it is one-third full?'* Agree that if you had that amount three times it would fill the container – you need to imagine the container divided into three equal parts. Pour in the liquid/rice until the container is about one-third full and the children shout *'stop'*. Ask a child to shade the approximate amount on a picture on the board, so that you have a record of what one-third looks like in this context.

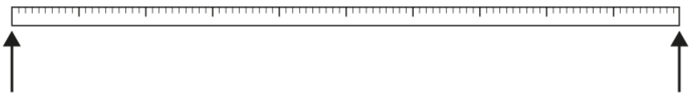
Empty out the container and tell children that this time you want them to shout *'stop'* when it is one-quarter full. Before you begin pouring in the liquid/rice, ask them to discuss with a partner whether it will be more or less liquid/rice than last time and to justify their opinion. Once they have identified that there are more equal parts this time, they will realise that each part must now be smaller, as discussed in step *5:1*.

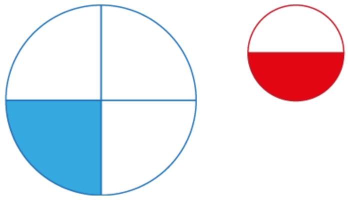
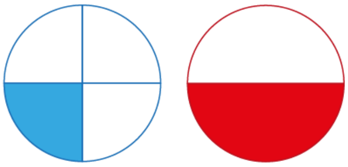
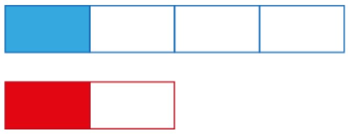
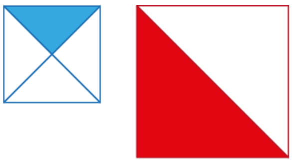
Pour in the liquid/rice, and when the container is about one-quarter full, say *'Convince me that it is about one-quarter full.'* Agree that if you had that amount four times it would fill the container – you need to imagine the container cylinder divided into four equal parts, or replicating the part that is filled four times. Again, ask a child the shade the approximate amount on a picture on the board, alongside the existing picture of one-third.

Empty out the container and repeat for one-tenth full, again asking children whether it will be more or less liquid/rice than last time. You can refer to the previous discussion, where you identified that the more equal parts a whole is divided into, the smaller each equal part is.

Comparing fractions by pouring:



	Finally, ask the children what one-thousandth full might look like. Some children may initially say that it will overflow, latching onto the ‘thousandness’ and feeling instinctively that this is a lot. Some *will* disagree, realising that this is in fact a tiny amount. The children will probably take great delight in shouting *‘Stop!’* almost as soon as you start pouring. Of course, this discussion can be extended to one-millionth full, etc. to really emphasise the point; taking a context to an extreme is a useful strategy to support generalisation of a concept. You can also use a metre rule to show what $\frac{1}{2}$ m, $\frac{1}{4}$ m, $\frac{1}{10}$ m etc look like, and then what $\frac{1}{100}$ of a metre looks like (1 cm), or even $\frac{1}{1000}$ of a metre looks like (1 mm). With the metre rule in front of them, ask the children to show a distance of $\frac{1}{2}$ m with their hands (you can go round and measure with the metre rule), then $\frac{1}{4}$ m, $\frac{1}{10}$ m and $\frac{1}{100}$ m, drawing children's attention to the fact that they show a smaller amount each time. Now ask children to look at the pictures of the container and metre rule and ask *‘What do you notice?’* before again reiterating the generalisation: ***‘When comparing unit fractions, the greater the denominator, the smaller the fraction.’*** This could be turned around: ***‘When comparing unit fractions, the smaller the denominator, the greater (or bigger) the fraction.’***	Comparing fractions on a metre rule:  Ordering the fractions: $\frac{1}{3} > \frac{1}{4} > \frac{1}{10} > \frac{1}{100} > \frac{1}{1000}$ $\frac{1}{1000} < \frac{1}{100} < \frac{1}{10} < \frac{1}{4} < \frac{1}{3}$

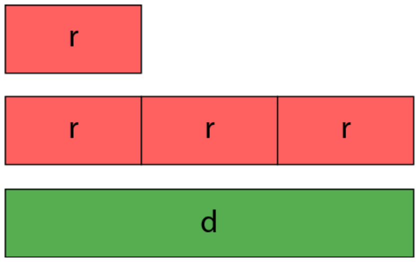
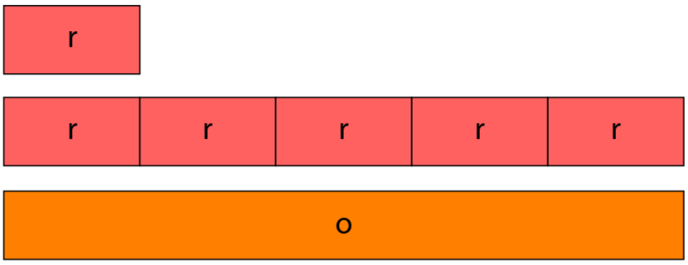
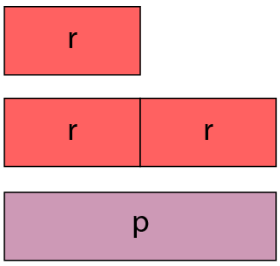
	Children should be able to order fractions into inequality statements, both from smallest to largest and from largest to smallest. Provide examples, such as those shown opposite, and ask children to order them.	
5:4	Throughout this teaching point, children have compared fractions of various different wholes. It is important at this point that they understand that fractions can only be compared if they refer to the same whole. Look at the first pair of images and Emma's statement about them, and discuss as a class. It is true that one-quarter of the blue circle is bigger than one-half of the red circle. However, a fraction expresses a comparison between the part and its whole. In these examples the blue part is a smaller part of its whole (the blue circle) then the red part is of its whole (the red circle). If we want to compare them to each other as fractions and demonstrate that a quarter is smaller than a half, then the wholes need to be the same size. Now look at the blue bars and Zainab's statement. What is true about what Zainab has said? It is true that one-quarter of the blue bar is equal to one-half of the red bar, but it does not prove that one-quarter is equal to one-half, because they don't refer to the same whole. Finally look at Femi's diagram and statements. Children may recognise that Femi's statement, $\frac{1}{4} < \frac{1}{2}$ is true. However, note that the other part of Femi's statement (that these diagrams prove this fact) is not true. Again, because these are not the same whole, they can't be used to prove that $\frac{1}{4} < \frac{1}{2}$.	• *'Emma looks at these two diagrams. She says that they prove that* $\frac{1}{4} > \frac{1}{2}$. *Do you agree or disagree?'*  • *'Disagree: to compare fractions, the wholes must the same.'*  • *'Zainab says that these two diagrams prove that* $\frac{1}{4} = \frac{1}{2}$. *Do you agree or disagree with Zainab?'*  • *'Femi says that these two diagrams prove that* $\frac{1}{4} < \frac{1}{2}$. *Do you agree or disagree with Femi?'* 

	Note that giving children correct statements but invalid proof of a statement is a good way to promote depth of thinking. Discuss the sequence of images with the children and agree that the problem is that they do not refer to the same whole, leading to the generalisation: ***'When we compare fractions, the whole has to be the same.'***	

Teaching point 6:

If the size of a unit fraction is known, the size of the whole can be worked out by repeated addition of that unit fraction.

Steps in learning

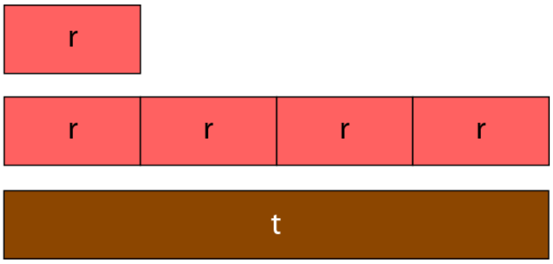
	Guidance	Representations
6:1	In *3.1 Preparing for fractions: the part–whole relationship, Teaching point 4*, children were given one part and the number of equal parts in the whole, and asked to construct a whole. They can now be asked to construct a whole given one part and the fraction of the whole that each part represents. Cuisenaire® rods are an excellent tool for exploring this relationship. Give each pair of children a red rod. Say that you are thinking of a whole and the red rod is *one-third* of your whole. Ask the children to work out what your whole is, by making it with Cuisenaire® rods. Encourage the children to justify their responses – the by now very familiar stem sentence can be used to support reasoning: '***The whole is divided into ___ equal parts. Each equal part is ___ of the whole.'*** They should reason along the lines of *'I know that the whole is divided into three equal parts, and this is one of them, so there must be three of these parts in the whole.'* Encourage them to construct the whole first, by taking more red rods; they may then identify that the green rod is the same length as three red rods. To draw out the multiplicative relationship, also ask children to express the relationship between the red and green rod using the sentence: *'The <u>green</u> rod is <u>three</u> times as long as the red rod.'*	• *'The red rod is one-third of the whole. What is the whole?'*  • *'The red rod is one-fifth of the whole. What is the whole?'*  • *'The red rod is one-half of the whole. What is the whole?'* 

Now tell the children you are thinking of a new whole. This time the red rod is one-fifth of the whole. Ask *'What does the whole look like now?'* As before, they should work together to make the whole first, by taking more red rods. They may then identify that the orange rod is the same length as five red rods. Use the sentence: *'The <u>orange</u> rod is <u>five</u> times as long as the red rod.'*

Repeat this twice more, with the red rod as one-half of the whole, and with the red rod as one-quarter of the whole.

Through this activity, it becomes clear that the red rod does not have an intrinsic value in and of itself. It only has a value with relation to the whole. When the whole changes, the fraction that the red rod is of that whole also changes.

- *'The red rod is one-quarter of the whole. What is the whole?'*



6:2

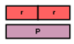
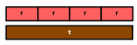
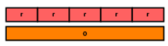
Use a table to summarise what you have discovered, filling it in with the children and discussing each step. Then look at the relationship between the part as a fraction of the whole and the whole as a multiple of the part.

Introduce the following statements:

- *'If one-<u>half</u> is a part, then the whole is <u>twice</u> as much. Take <u>two</u> parts and put them together to make one whole.'*
- *'If one-<u>third</u> is a part, then the whole is <u>three times</u> as much. Take <u>three</u> parts and put them together to make one whole.'*

Ask children to complete the stem sentence: ***'If one-ninth is a part, then the whole is___ times as much. Take ___ parts and put them together to make one whole.'***

Ask children to construct their own examples using these generalised sentences.

Part	Part as a fraction of the whole	Number of equal parts in the whole	Whole
	$\frac{1}{2}$	2	
	$\frac{1}{3}$	3	
	$\frac{1}{4}$	4	
	$\frac{1}{5}$	5	

6:3

Now, give children practice using the part and fraction to define the whole, using a simple shape (for example, a square) as the part.

The following stem sentence will help children to identify the number of equal parts, and hence the whole, and support them in completing the table:

'If one-____is a part, then the whole is ____ times as much. Take ___ parts and put them together to make one whole.'

Part	Part as a fraction of the whole	Number of equal parts in the whole	Whole
	$\frac{1}{3}$	3	
	$\frac{1}{4}$		
	$\frac{1}{5}$		
	$\frac{1}{6}$		
	$\frac{1}{7}$		
	$\frac{1}{8}$		

6:4

Next, show children a linear model with two line segments of the same length, but labelled with different unit fractions. The rest of each whole should be covered over.

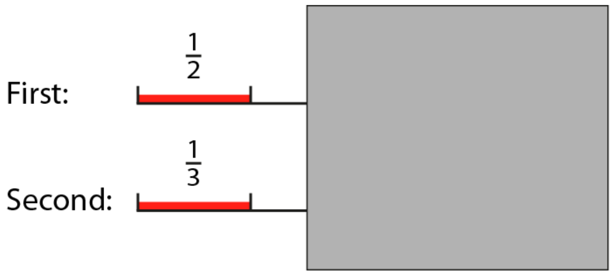
Ask the children to reason which line must be the longer. They should realise that the two parts shown are the same length and use this knowledge to explain which whole is the longer. Encourage them to use the stem sentence from step *6:3* to help them:

'If one-___is a part, then the whole is ___ times as much. Take ___ parts and put them together to make one whole.'

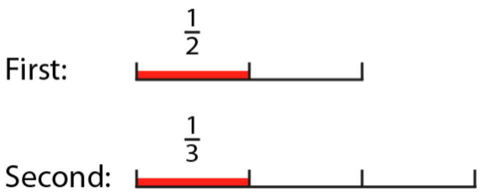
They should conclude that the second line is the longer.

Provide children with similar examples using different unit fractions, so they can practise this reasoning.

'Which line is longer?'



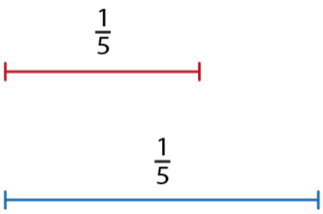
- First: *'If one-half is a part, then the whole is two times as much. Take two parts and put them together to make one whole.'*
- Second: *'If one-third is a part, then the whole is three times as much. Take three parts and put them together to make one whole.'*



6:5

The linear model shown opposite can also be used for comparison. This time, the fractions are the same, but the line segments are different lengths.

Ask the children to explain which whole would be longer, again using the stem sentences. They should conclude that in the blue line, each part is longer than in the red line, and so the whole blue line will be longer than the whole red line.



- Red line: *'If one-fifth is a part, then the whole is five times as much. Take five parts and put them together to make one whole.'*
- Blue line: *'If one-fifth is a part, then the whole is five times as much. Take five parts and put them together to make one whole.'*

6:6	Similar problems involving sets or groups of objects or people can be used to draw out the same reasoning about generating the whole given one equal part.
	
	Show children two groups of four people, representing different unit fractions of a class, and ask them to calculate how many people will be in each whole class. Begin by asking *'What is the same? What is different?'*. Children should see that both groups have four people, but the fractions that each group represents are different. Then ask *'Which class has more students?'*. As before, encourage the children to use the stem sentences from step *6:3* to help them: ***'If one ____ is a part, then the whole is _____ times as much. Take ___ parts and put them together to make one whole.'***
		Finding the wholes – same size part, different fraction:
		• *'What is the same?'*
		• *'What is different?'*
		 
		This is $\frac{1}{5}$ of Class A. This is $\frac{1}{6}$ of Class B.
		• *'Which class has more students?'*
		  ◦ Class A: *'If one-fifth is a part, then the whole is five times as much. Take five parts and put them together to make one whole. Each part has four students. So, there are 5 × 4 = 20 students.'*
		  ◦ Class B: *'If one-sixth is a part, then the whole is six times as much. Take six parts and put them together to make one whole. Each part has four students. So, there are 6 × 4 = 24 students.'*
		  ◦ *'Class B has more students.'*

6:7	Next, show children two groups that both represent one-fifth of a class (the same fraction), but have different numbers in each part. Again, ask: • *'What is the same?'* • *'What is different?'* and then ask children to work out which class has more students, using the stem sentences to help them.	Finding the wholes – different size part, same fraction: • *'What is the same?'* • *'What is different?'*   This is $\frac{1}{5}$ of Class C. This is $\frac{1}{5}$ of Class D • *'Which class has more students?'* ◦ Class C: *'If one-fifth is a part, then the whole is five times as much. Take five parts and put them together to make one whole. The part has six students. So, there are 5 × 6 = 30 students.'*

		• Class D: *'If one-fifth is a part, then the whole is five times as much. Take five parts and put them together to make one whole. The part has five students. So, there are 5 × 5 = 25 students.'* • *'Class C has more students.'*

6:8	Finally, show children two groups that represent different fractions of a class and have different numbers in the part.
	Again, ask:
	• *'What is the same?'*
	• *'What is different?'*
	and then ask children to work out which class has more students, using the stem sentences to help them.
	Work through further similar examples. By now, children should be able to solve similar problems involving any unit fraction.
		Finding the wholes – different size part, different fraction:
		• *'What is the same?'*
		• *'What is different?'*
		 
		This is $\frac{1}{7}$ of Class E. This is $\frac{1}{6}$ of Class F.
		• *'Which class has more students?'*
		  • Class E: *'If one-seventh is a part, then the whole is seven times as much. Take seven parts and put them together to make one whole. The part has four students. So, there are 7 × 4 = 28 students.'*
		  • Class F: *'If one-sixth is a part, then the whole is six times as much. Take six parts and put them together to make one whole. The part has five students. So, there are 6 × 5 = 30 students.'*
		  • *'Class F has more students.'*

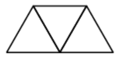
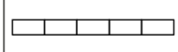
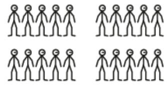
6:9 Finally, summarise these contexts, and any others you have used, in a table. In segment *3.1 Preparing for fractions: the part–whole relationship,* we saw that, given one part and the number of equal parts, we can construct the whole. In this teaching point we have seen that, given one part and the fraction that each equal part is of a whole, we can also construct the whole.

Look back at the table you used in segment *3.1*, step *4:5* (which you may have supplemented with your own examples). Now we can add another column, showing the fraction of the whole that each equal part represents.

Add additional rows to the table, using some of the examples worked on with the class. Provide information in some columns, and ask the children to use the information to help complete the other columns. Notice that providing the information in the first two columns is sufficient to complete the other two columns.

Discuss the table with the class, showing that we can move both from part to whole, and from whole to part. The following sentences will support the children to describe this two-way relationship. For example, for the triangles you might say:

- *'This is the whole. The whole is divided into three equal parts. Each part is one-third of the whole. This is one of the parts.'* (Move from the right-hand column, via the middle columns, to the left-hand column).
- *'This is a part. Each part is one-third of the whole. There are three equal parts in the whole. This is the whole.'* (Move from the left-hand column, via the middle columns, to the right-hand column).

Part	Part as a fraction of the whole	Number of equal parts in the whole	Whole
		3	
		5	
		4	
	$\frac{1}{5}$		
	$\frac{1}{7}$		