MathsHUBS
Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development
6 Geometry

6.4 Constructions

Guidance document | Key Stage 3

Making connections

The NCETM has identified a set of six 'mathematical themes' within Key Stage 3 mathematics that bring together a group of 'core concepts'.

The sixth of these themes is *Geometry*, which covers the following interconnected core concepts:

6.1   Geometrical properties

6.2   Perimeter, area and volume

6.3   Transforming shapes

6.4   **Constructions**

This guidance document breaks down core concept *6.4 Constructions* into two statements of knowledge, skills and understanding:

6.4.1   Use the properties of a circle in constructions

6.4.2   Use the properties of a rhombus in constructions

Then, for each of these statements of knowledge, skills and understanding we offer a set of key ideas to help guide teacher planning.

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Overview

In Key Stage 2, students will have learnt about the properties of certain geometric shapes and used these properties to compare and classify shapes. They will also have had experience of drawing certain shapes using a ruler and angle measurer, but the use of compasses to construct shapes will be a new idea at Key Stage 3. In this core concept, students will learn the ruler and compass constructions of:

- triangles of given lengths
- a perpendicular bisector of a line segment
- a perpendicular to a given line through a given point
- an angle bisector.

An important awareness is that these constructions are based on the geometrical properties of a few key shapes (a circle, an isosceles triangle and a rhombus). A deep understanding and awareness of these geometrical properties will support students in gaining a conceptual overview of these constructions. It will help guard against constructions being learnt mechanically as a set of procedural steps.
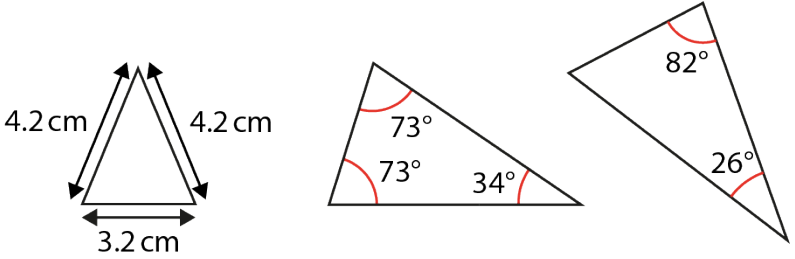
Prior learning

Before beginning to teach *Constructions* at Key Stage 3, students should already have a secure understanding of the following from previous study:

Key stage	Learning outcome
Upper Key Stage 2	- Draw 2D shapes using given dimensions and angles - Compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons

You may find it useful to speak to your partner schools to see how the above has been covered and the language used.

Checking prior learning

The following activities from the [NCETM primary assessment materials](#)[1] offer useful ideas for assessment, which you can use in your classes to check whether prior learning is secure:

Reference	Activity
Year 6 page 35	*Which of these triangles are isosceles?* *Explain your decisions.*

| Year 6 page 35 | *Accurately draw two right-angled triangles with sides of different lengths.* |
| | *Compare them and describe what's the same and what's different about them.* |

Key vocabulary

Term	Definition
altitude of a triangle	A line segment through a vertex and perpendicular to the side opposite the vertex.
arc	A portion of a curve. Often used for a portion of a circle.
bisector	A point, line or plane that divides a line, an angle or a solid shape into two equal parts.
	A perpendicular bisector is a line at right angles to a line-segment that divides it into two equal parts.
construction	A construction in geometry is the act of drawing geometric shapes using only a pair of compasses and a straightedge. No measuring of lengths or angles is permitted.
line	A set of adjacent points that has length but no width. A straight line is completely determined by two of its points, say *A* and *B*. (see 'line segment')
line segment	The part of a line between any two of its points is a line segment. (see 'line')
locus	A locus of points is the set of points, and only those points, that satisfies given conditions.
	Example: The locus of points at a given distance from a given point is a circle.
perpendicular	A line or plane that is at right angles to another line or plane.

Collaborative planning

Below we break down each of the two statements within *Constructions* into a set of key ideas to support more detailed discussion and planning within your department. You may choose to break them down differently depending on the needs of your students and timetabling; however, we hope that our suggestions help you and your colleagues to focus your teaching on the key points and avoid conflating too many ideas.

Please note: We make no suggestion that each key idea represents a lesson. Rather, the 'fine-grained' distinctions we offer are intended to help you think about the learning journey irrespective of the number of lessons taught. Not all key ideas are equal in length and the amount of classroom time required for them to be mastered will vary, but each is a noteworthy contribution to the statement of knowledge, skills and understanding with which it is associated.

The following letters draw attention to particular features:

D   Suggested opportunities for **deepening** students' understanding through encouraging mathematical thinking.

L   Examples of shared use of **language** that can help students to understand the structure of the mathematics. For example, sentences that all students might say together and be encouraged to use individually in their talk and their thinking to support their understanding (for example, *'The smaller the denominator, the bigger the fraction.'*).

R   Suggestions for use of **representations** that support students in developing conceptual understanding as well as procedural fluency.

V   Examples of the use of **variation** to draw students' attention to the important points and help them to see the mathematical structures and relationships.

PD   Suggestions of questions and prompts that you can use to support a **professional development** session.

For a selected key idea, marked with an asterisk (*), we exemplify the common difficulties and misconceptions that students might have and include elements of what teaching for mastery may look like. We provide examples of possible student tasks and teaching approaches, together with suggestions and prompts to support professional development and collaborative planning. You can find these at the end of the set of key ideas.
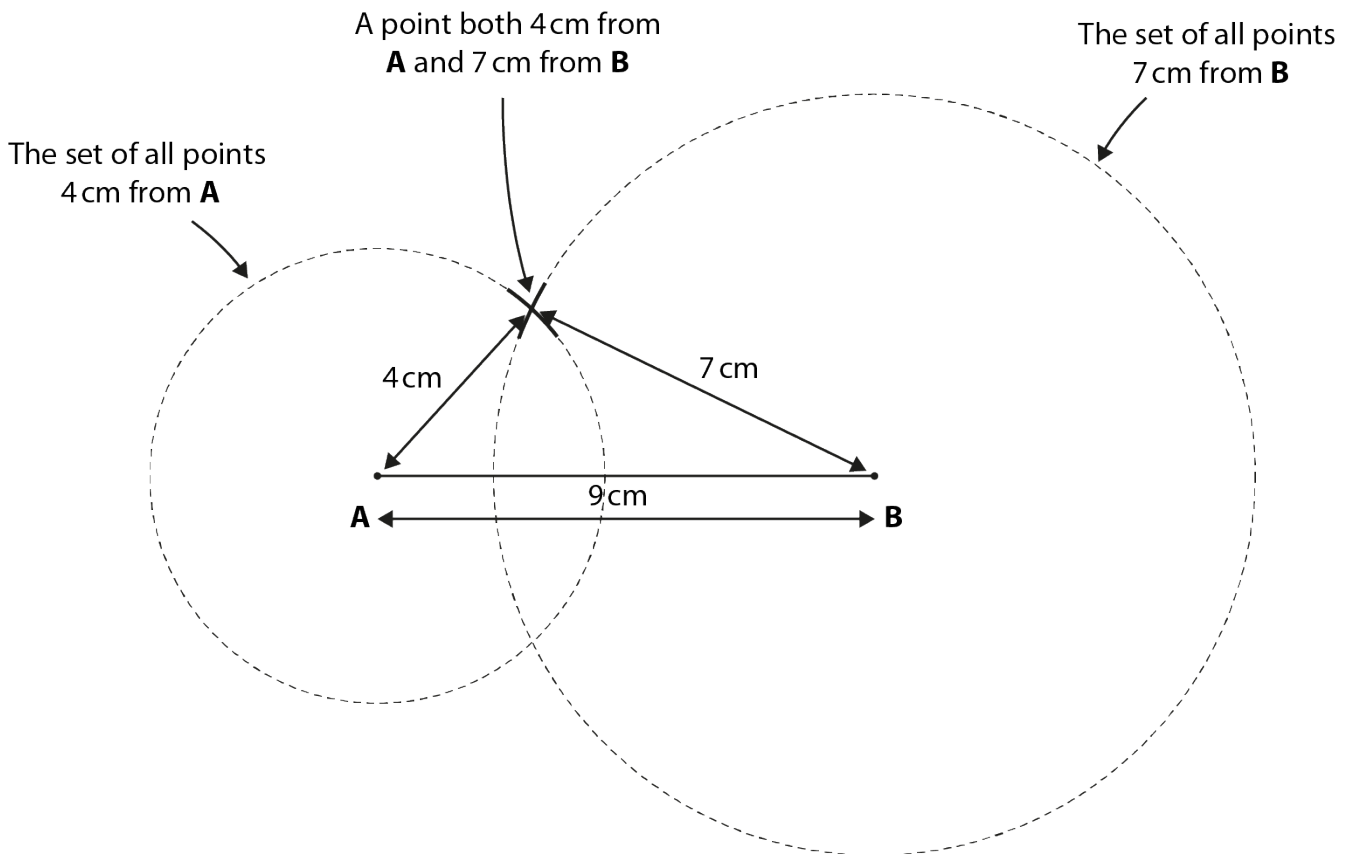
Key ideas

6.4.1   Use the properties of a circle in constructions

When faced with the problem of constructing a triangle with lengths of, for example, 4 cm, 7 cm and 9 cm, students' intuition may be to do so using a ruler. Trying to solve this problem using a ruler will be a useful exercise, as it draws students' attention to the challenge of finding a point that is a specified distance from one point and, simultaneously, a specified distance from another, as shown below:

4 cm from **A**
and
7 cm from **B**

9 cm

A   **B**
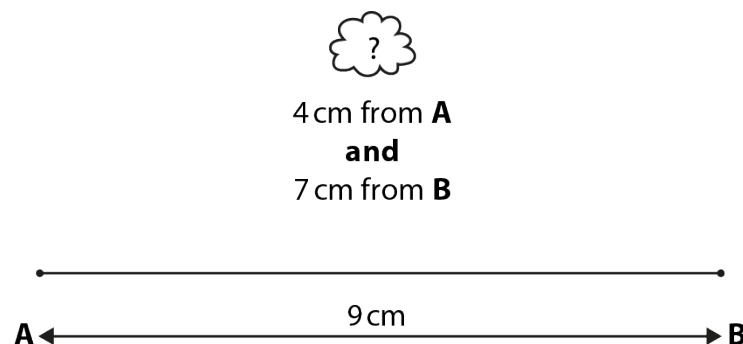
A key awareness is that drawing a circle creates an infinite set of points, all of which are equidistant from its centre. Students will need plenty of experience of using a ruler and a pair of compasses to appreciate the nature of the construction, and to explore the different ways that drawn circles can be used to identify points that are a specified distance away from one or more points. Students should also become aware that drawing full circles is not necessary – the drawing of carefully placed arcs is more efficient.

The set of all points
4 cm from **A**

A point both 4 cm from
A and 7 cm from **B**

The set of all points
7 cm from **B**

4 cm

7 cm

9 cm

A   **B**

Once students can construct a scalene triangle with ease and efficiency, they can be challenged to construct other shapes (for example, equilateral and isosceles triangles and rhombuses). This will not only provide opportunities for students to become fluent with the construction processes, but also, importantly, to engage in some early discussions about the basic properties of these shapes. These discussions can then be extended when focusing on the ruler and compass constructions considered in 6.4.2, below.

6.4.1.1   Understand a circle as the locus of a point equidistant from a fixed point

6.4.1.2   Use intersecting circles to construct triangles and rhombuses from given lengths

6.4.2   Use the properties of a rhombus in constructions

Using their previous understanding of how to use arcs of circles to construct isosceles triangles and rhombuses, students can explore more closely the properties of these shapes. For any isosceles triangle, the altitude of the triangle bisects the base at right angles and bisects the angle at the vertex. Consequently, any rhombus (essentially two isosceles triangles put together) has diagonals which perpendicularly bisect each other and bisect their associated internal angles.
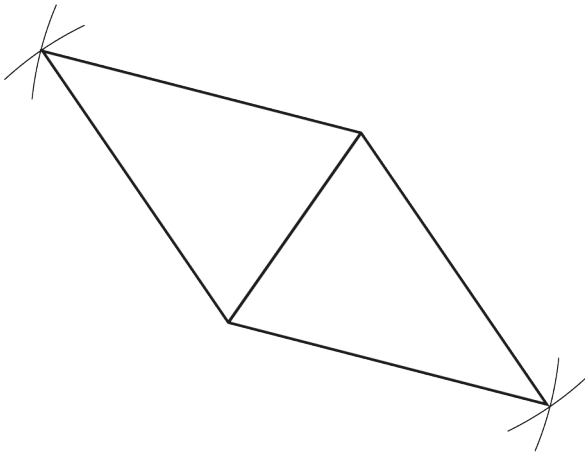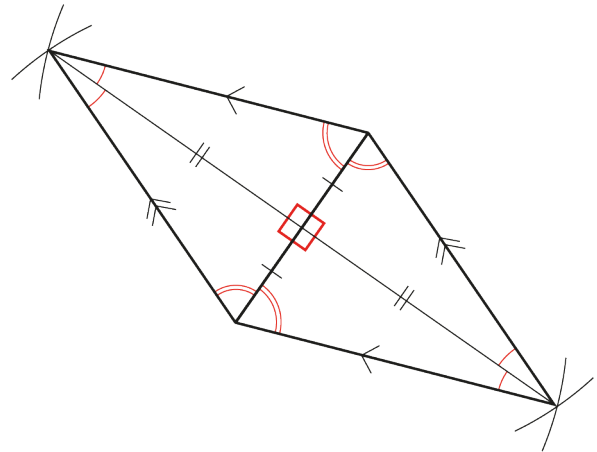
Fig 1: Construction of a rhombus

Fig 2: Diagram showing how the diagonals of a rhombus bisect each other at right angles and bisect the internal angles

The key awareness here is that, when a rhombus is constructed, other constructions have also necessarily been produced. Students can then be challenged to use this awareness to create, for themselves, a method of producing the standard ruler and compass constructions efficiently and fluently.

6.4.2.1   Be aware that the diagonals of a rhombus bisect one another at right angles

6.4.2.2   Be aware that the diagonals of a rhombus bisect the angles

6.4.2.3* Use the properties of a rhombus to construct a perpendicular bisector of a line segment

6.4.2.4   Use the properties of a rhombus to construct a perpendicular to a given line through a given point

6.4.2.5   Use the properties of a rhombus to construct an angle bisector

Exemplified key idea

6.4.2.3   Use the properties of a rhombus to construct a perpendicular bisector of a line segment

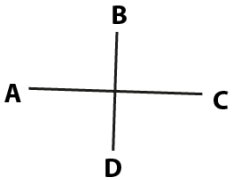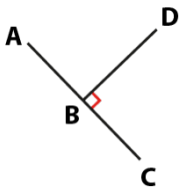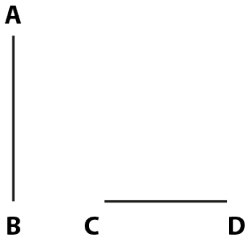| Common difficulties and misconceptions |
| --- |
| Some students may experience difficulties surrounding the mathematical language used in this key idea. They must understand the difference between drawing a figure accurately by eye, and producing a construction based on the geometrical properties of the figure. It is possible to draw a perpendicular bisector by using a ruler to determine the midpoint of a line and a protractor (or using squared paper) to judge a right angle, but this is not what is required in this key idea. In a construction, it is geometrical properties, not measurement, which are used to produce the required result. |
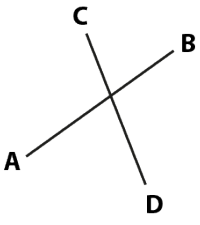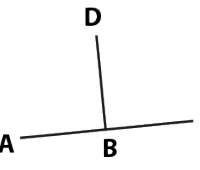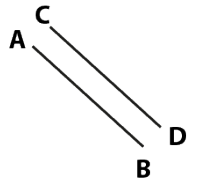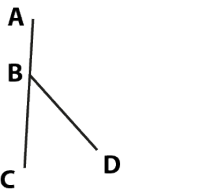
Students should be given time to practise using construction equipment accurately. It is likely that they will not have used a pair of compasses frequently (if at all) during Key Stage 2, and so students often, initially, lack coordination. They may need support in setting up their equipment and should be encouraged to check their working, with the aim to be within ±2 mm and ±2° of the required measurements.

This key idea is heavily linked to the geometric properties of both circles and rhombuses.

PD You may wish to consider your schemes of learning as to when this key idea is introduced.

If there is significant time between work on the properties of 2D shapes and these constructions, then it is worthwhile revising the relevant geometric properties before you begin to introduce constructions. Students must be able to identify the geometric properties of circles and rhombuses, and understand how these properties enable key constructions.

Difficulties occur for students when they are trying to memorise the various steps in the construction of circles and rhombuses, with no link to other knowledge they might have about geometrical properties. Students will be helped considerably in this if they are aware that constructing a perpendicular bisector of a line segment is not an isolated concept but linked to the properties of circles and rhombuses.

What students need to understand	Guidance, discussion points and prompts
Understand key mathematical language associated with constructions. *Example 1:* *Which lines are perpendicular?* *Explain how you know.* a) b) c) d) e) f) g)	**V** In *Example 1*, only one pair of lines is marked with a small square to indicate that the lines meet at a right angle and are, therefore, perpendicular. It is likely that students will identify this pair of perpendicular lines first. Asking students whether they can identify another pair may prompt them to eliminate those that they know are definitely not perpendicular and lead them to three further pairs which look like they could be, but this can only be confirmed by measuring. One pair of lines has been deliberately drawn as parallel to assess a common error, which is the confusion between parallel and perpendicular. **L** This example provides an opportunity to encourage accurate language when describing geometrical properties, and students should be confident in using letters to describe lines. For example: *'AC is perpendicular to BD'* and *'AB and CD are parallel.'*

Example 2:

Find the midpoint of each line segment.

Bisect each line segment and mark it with an ×.

a)

A

6 cm

B

b)

C

44 mm

D

c)

F

3 cm

E

d)

G

5.2 cm

H

e)
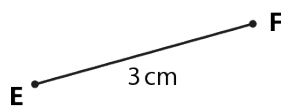
I

73 mm
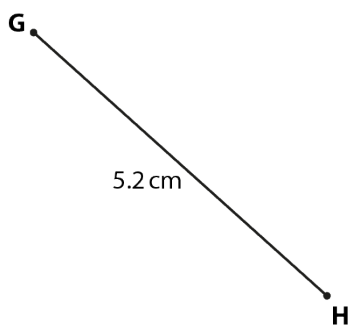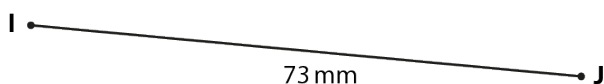
J
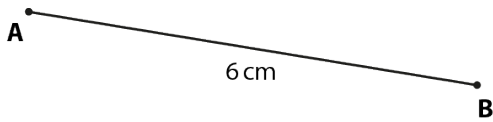
In *Example 2*, students are asked to find the midpoint of some line segments, which will provide an introduction to bisecting line segments.

V   It is likely that students will notice the line segments in parts a) and b) have even measurements, which are easily halved to find the midpoint.

The line segments in parts c), d) and e) need to be measured accurately in order to calculate and mark the midpoint.

The line segment IJ in part e) is 73 mm long, so this may prompt discussions with students about accuracy during constructions.

Example 3:

a) *Accurately construct a rhombus with sides of 6 cm.*

b) *Explain how you have used the properties of a circle to help produce your construction.*

Example 3 gives students an opportunity to understand the strong connections between the properties of a circle, the construction of a rhombus and the properties of a rhombus, which result in the construction of a perpendicular bisector.

Students may need some support in considering how to construct a rhombus. They should reflect on the properties of a circle that allow them to draw arc lengths, indicating all the points that are equidistant from each end of their starting line.

D It is likely that students will construct a range of rhombuses all with sides of 6 cm, but with diagonals of differing lengths. It is worth discussing that the length of the sides is not enough to produce congruent rhombuses, and for students to consider what information would be required for these to be constructed.

Students may realise that the length of their initial line segment (which will end up being a diagonal of their rhombus) is not critical, and they could investigate the maximum and minimum lengths of the initial line segment for which a rhombus of side 6 cm could be constructed.
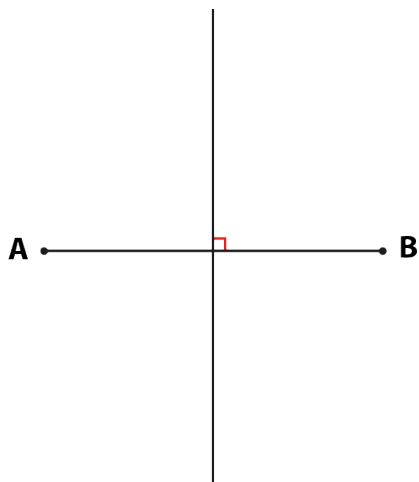
Example 4:

Which of these diagrams show an accurate construction of the perpendicular bisector of the line segment AB?

Explain how you know.

a)

A ●————————————● B

b)
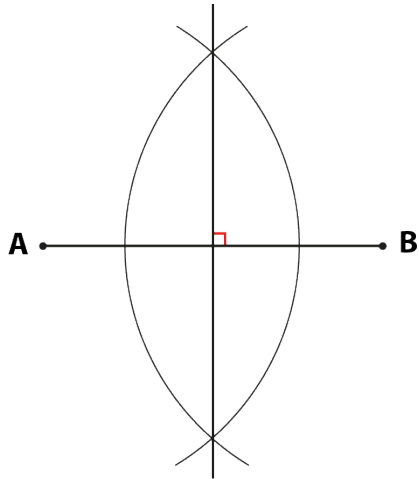
A ●————————————● B

In *Example 4*, students are provided with four diagrams and asked to comment on which of them show an accurate construction of a perpendicular bisector.

Students must apply previous learning of the key terms 'perpendicular' and 'bisector' to identify which diagrams show the perpendicular bisector of line segment AB.

V   Part b) shows a drawing rather than a mathematical construction. Students must be aware of the difference and understand the need to leave construction markings on their diagrams to show their method.

Parts a), c) and d) all show perpendicular bisectors of the line segment AB. These diagrams could support class discussion as to which diagram is the most efficient in terms of space.

Students should explore how the diagrams have been constructed (with arcs of different sizes from points A and B). They may need support to notice that the rhombuses constructed do not need to have sides of equal length to the line segment (the diagonal of the rhombus) as in parts c) and d), nor do full circles need to be drawn, only intersecting arcs.

c)

d)

e)
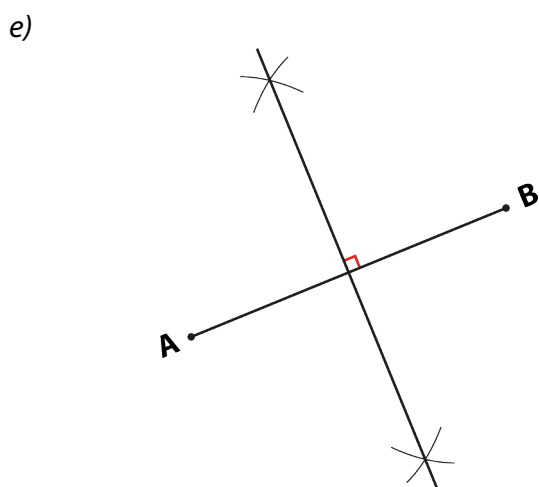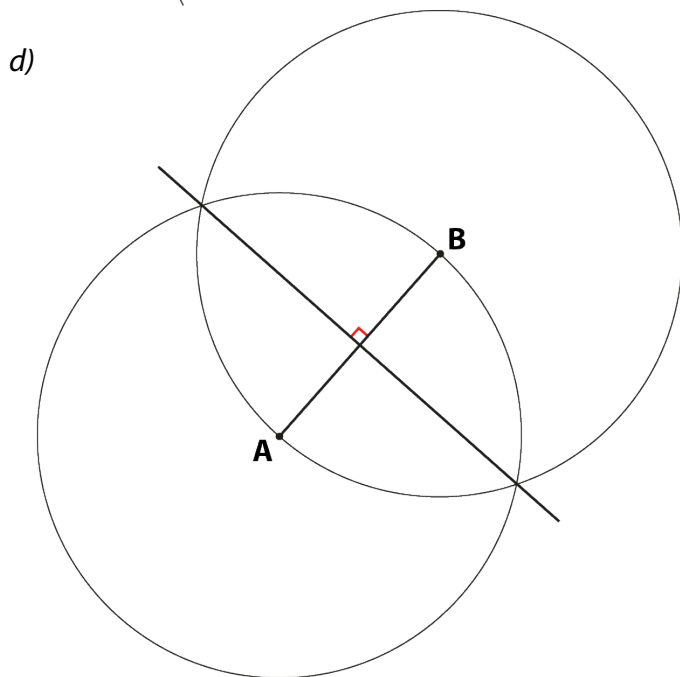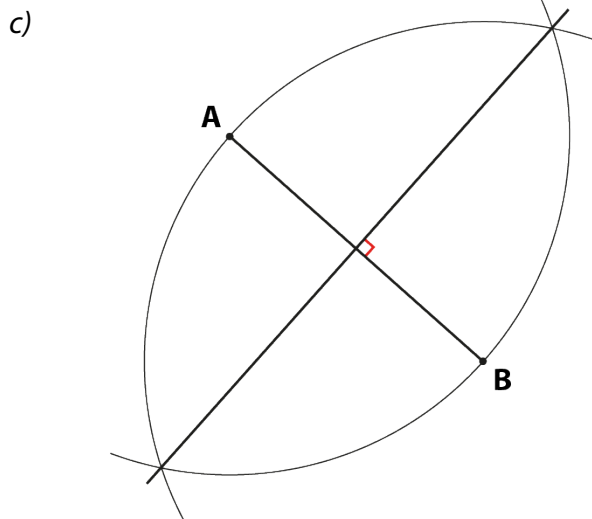
Understand how the geometric properties of key shapes are used in standard constructions.
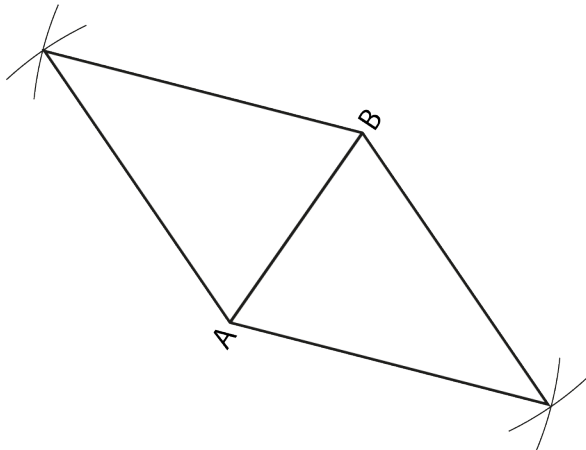
Example 5:

This diagram is a construction of an isosceles triangle, drawn from either end of line segment AB.
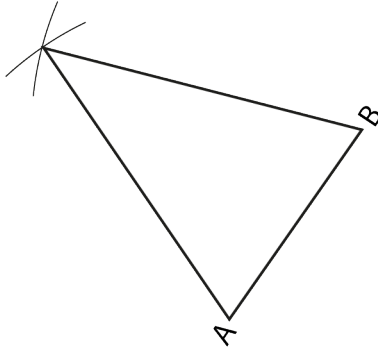
a) Write down (or indicate on the diagram) as many properties of this triangle as you can.

b) Draw in an altitude of this triangle. Are there any other properties that you can now state?

Encourage students to use such symbols as:

$<$ or $\lll$ to indicate pairs of parallel sides

$\diagdown$ or $\diagdown\diagdown$ to indicate pairs of sides of equal length

$\llcorner$ to indicate perpendicularity

$\lhd$ or $\lhd\rhd$ to indicate angles of equal size

as well as written descriptions when completing *Example 5*.

Example 6:

Add another identical isosceles triangle to the diagram in Example 5, like this:

a) What shape have you made?

b) Write down (or indicate on the diagram) as many properties of this shape as you can.

c) Draw in the longest diagonal of this shape. Are there any other properties that you can now state?

R The representation below (a rhombus with all its properties indicated) is a key image.
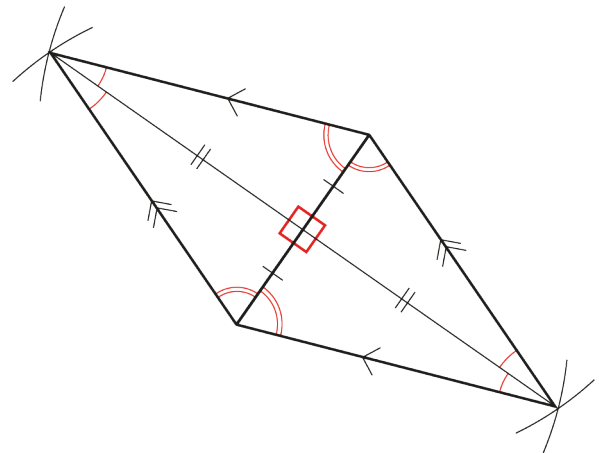
Once students have constructed this, it will be important to discuss the relationships embedded in this diagram, so that all students appreciate them. You could offer prompts, such as:

- 'Can you see two lines that bisect each other at right angles?'
- 'Can you see an angle that has been bisected? And another, and another, …?'

	D	For a deep and connected understanding of the ruler and compass constructions, students could imagine a rhombus and construct the part of it that will produce the required construction. For example, if required to construct a perpendicular bisector to the line AB, encourage students to imagine the line AB as being a diagonal of a rhombus and then proceed to draw the rhombus (or part thereof).
Construct the perpendicular bisector of a line segment. *Example 7:* *Do this activity in pairs.* *Use a ruler and a pair of compasses to construct a rhombus.* *Mark a diagonal and label it AB.*	**R**	A key aspect of *Example 7* is to support students in visualising the construction of a rhombus around any line AB, and to know that this will result in obtaining the perpendicular bisector of AB by identifying the other diagonal. Having such an image in their mind when producing any of the constructions will support students in seeing the geometrical properties on which the constructions are based, and guard against using a set of routines with no understanding. It will be important to discuss with students what the minimum number of construction lines is for this to be possible. This will support them in being as efficient as possible.
Example 8: *The line segment AB is 8 cm long.* a) *Which of these methods could be used to construct the perpendicular bisector of AB?* b) *Which is the best method?* *Explain your reasoning.* (i) *Constructing arcs which are 3 cm from both A and B* (ii) *Constructing arcs which are 4 cm from both A and B* (iii) *Constructing arcs which are 5 cm from both A and B* (iv) *Constructing arcs which are 8 cm from both A and B*	**V** **D**	In *Example 8*, students must decide which of the four options could result in the construction of the perpendicular bisector of the line segment AB. The size of the arcs has been carefully chosen to prompt students to consider which would intersect both above and below the line segment AB and allow an accurate construction of the perpendicular bisector. In part (i), arcs of 3 cm would not intersect. In part (ii), the arcs of 4 cm would only bisect the line segment at the midpoint. Parts (iii) and (iv) could both be used, but arcs of 8 cm – part (iv) – are unnecessarily large. In this task, students should be initially encouraged to consider the construction conceptually, but some may need to actually construct using the options in order to fully understand which will work. Students could

be asked to explore other viable options (including arcs with non-integer radii), before generalising the method in terms of the radius of the arc lengths required to construct a perpendicular bisector.
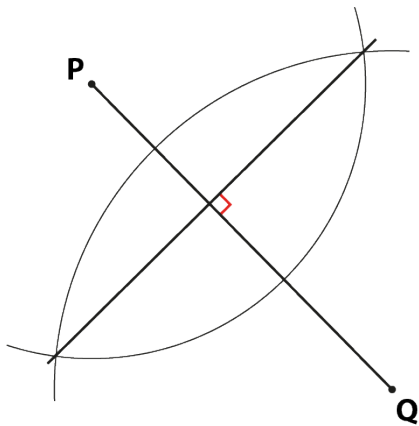
Example 9:

Complete this construction of the perpendicular bisector of line segment XY.
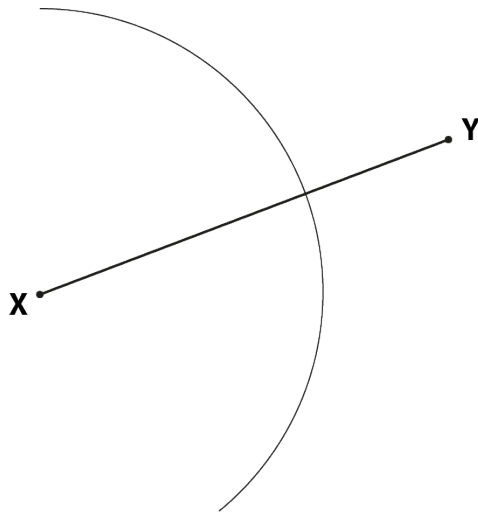
In *Example 9*, students must apply their awareness of the properties of a rhombus to complete an already started construction. They should understand that to construct a rhombus, and hence the perpendicular bisector, the arcs drawn from X and Y need to be of equal radius.

Example 10:

Tinashe has attempted to construct the perpendicular bisector of the line segment PQ.

Comment on his construction.

a)   *Which quadrilateral has he constructed?*

b)   *Why might this have happened?*

c)   *What do the properties of this shape indicate about the diagonals?*

V   *Example 10* shows an incorrect construction. At first glance it may look like the perpendicular bisector has been constructed, but students should identify that the arcs drawn do not have equal radius and so a kite has been constructed instead of a rhombus. The diagonals of a kite are perpendicular but do not bisect each other, so a perpendicular bisector has not been constructed. This reinforces the need to keep the pair of compasses open to the same length in order to construct a rhombus.

It will be important to use non-standard examples of such constructions and not to present students with only prototypical ones, where the line to be bisected is horizontal or vertical.

Weblinks

1   NCETM primary assessment materials
     https://www.ncetm.org.uk/resources/46689