

#mathscpdchat 13 June 2023

How do you introduce students to trigonometry?

Hosted by Lizi Pepper

This is a summary of the discussion - to see all the tweets, follow the hashtag #mathscpdchat in Twitter



The links shared during this discussion were:

<u>MEI Deeper Maths</u> which are free resources from MEI. The <u>MEI Deeper Maths Trigonometry Topic</u>

<u>Overview</u> can be downloaded from the bottom of the page. It was shared by <u>Lizi Pepper</u> and <u>Mary</u>

<u>Pardoe</u>

<u>Geogebra Trigonometric Ratios (1)</u> which is an interactive Geogebra applet in which users explore relationships between side lengths of a right-angled triangle as they move a slider to enlarge the triangle. It was shared by <u>London South East Plus Maths Hub</u>

<u>Geogebra Trigonometric Ratios (2)</u> which is another interactive Geogebra applet in which users explore relationships between side lengths of a right-angled triangle. The triangle is subtended from a one-unit line segment which the user rotates (about one of its ends) using a slider. It was shared by <u>London South East Plus Maths Hub</u>



Key Ideas in Teaching Mathematics which is a part of the Nuffield Foundation website. It provides research-based guidance and classroom activities for teachers of mathematics, organised around seven key mathematical 'ideas'. These resources accompany the book *Key Ideas in Teaching Mathematics* by Anne Watson, Keith Jones, and Dave Pratt, to which there is a link on the page. There are also links to relevant online activities and resources for teachers to use with their students. It was shared by Mary Pardoe

<u>Trigonometry</u> which is a blog by <u>Paddy MacMahon</u>. It is a plan for an introductory lesson on trigonometry ('measuring triangles'), with a focus on the techniques needed, rather than formal language. It was shared by <u>Paddy MacMahon</u>

<u>Trigonometry Pile Up!</u> which is a PDF file showing a single problem. The challenge is to find the side length of a triangle in a colourful image of right-angled triangles arranged 'in a pile' with sides touching. It was shared by <u>Lizi Pepper</u>

<u>Sine, Cosine and Tangent Ratios</u> which is another single-page PDF file presenting a task. The challenge is to provide missing information (side lengths of right-angled triangles and numbers in trigonometric ratios shown as fractions) in a two-way table. It was shared by <u>Lizi Pepper</u>

An illustrated summary of the discussions in this #mathsCPDchat follows.



The host posted this message shortly before the discussion began ...



Lizi Pepper @mathspeptalk · 22h

What a sunny day! #mathsCPDchat starts at 7pm - I'm really looking forward to hosting for the first time, discussing how we introduce trigonometry! Remember to include #mathsCPDchat in your responses and I'll do my best to reply! See you soon! @NCETM @mathscpdchat



... then at 7pm she welcomed contributors with a tweet which included her first main question ...



Lizi Pepper @mathspeptalk · 22h

Welcome to #mathscpdchat where we are kicking straight off with the big question! (Remember to include the hashtag!)

Q1: It is the first time a class is meeting trigonometry. What does the first lesson look like? How do you then carve out the rest of the journey from there?

... and then immediately prompted thoughts by showing, and providing links to, parts of three past Twitter threads relevant to that question. First, she showed the next sequence indicating how a teacher might engage students in activity that reveals relationships and develops understanding:



Lizi Pepper @mathspeptalk · 22h

twitter.com/Yorkshire_Stex. a great thread to encourage small steps to build a solid understanding by @Yorkshire Steve

Norkshire Steve @Yorkshire_Steve · Aug 13, 2018 Replying to @Yorkshire Steve @bennewmark and 2 others only latterly have I joined the dots that the successful approaches were often not rushing,



Yorkshire Steve @Yorkshire Steve · Aug 13, 2018

building up the learning incrementally - very small steps very thoroughly embedded





Yorkshire Steve @Yorkshire Steve · Aug 13, 2018

so for Trig - get class to draw lots of 30/60/90 right angled triangles and work out Opp/Hyp



Yorkshire Steve @Yorkshire_Steve · Aug 13, 2018

embed that for triangles of this type opp/Hyp is always the same.



Yorkshire Steve @Yorkshire_Steve · Aug 13, 2018

give loads of triangle sketches and ask 'if drawn accurately will the angle be 30 degrees?'



Yorkshire Steve @Yorkshire_Steve · Aug 13, 2018

move on to get a pair doing 5/85/90 triangles to find the magic number for opp/Hyp for 5/85/90 etc.



Yorkshire Steve @Yorkshire Steve - Aug 13, 2018

collect magic numbers in and end up with (effectively) a sine table in 5 degree increments.



Yorkshire Steve @Yorkshire_Steve · Aug 13, 2018

then loads of sketches - students to find missing angle (work out opp/Hyp and look up in table).



Yorkshire Steve @Yorkshire Steve · Aug 13, 2018

progress to some not in table ("it's between 35 and 40 sir") and share an old trig table.



Yorkshire Steve @Yorkshire_Steve · Aug 13, 2018

only once the link between opp/Hyp and the angle is firmly embedded bring in calculator, sin etc.

In the next sequence of tweets that the host quote-retweeted the teacher describes how she has used resources that she created, and which she shared in the thread:



Lizi Pepper @mathspeptalk · 22h

And @karenshancock shared her thoughts earlier this year, and again yesterday on how she has approached trigonometry #mathscpdchat



Saren @karenshancock · May 30, 2022

Belated have uploaded the trigonometry resources I used to introduce it this year.

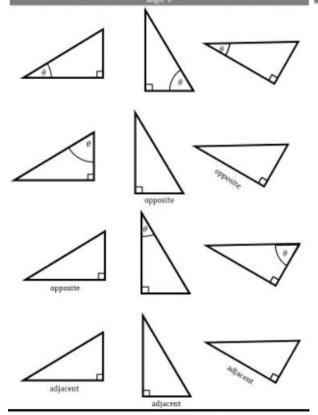
After a brief overview of labelling we did this task involving pythagorean triples:

Show this thread



TASK: In each of these triangles label the opposite, adjacent, hypotenuse and the

TASK: Use Pythagoras' Theorem to complete this table of side ratios

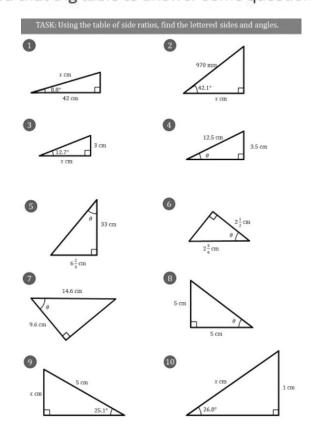


Angle (#)	одр Бур	ad) 1979	oup oup
8.8"	13 85		
10.4*	11 61		
11.4*		90 101	
12.70		40 41	
14.31			16 63
16.3*			7 24
18.9*	12 37		
22.0"	5 18		
25.1*		77 85	
26.0*		80	
28.1*			B 15
30*			√ <u>1</u>
30.5°	33 65		
31.0°	28 53		
33.4"		91 109	
36.9*		4 5	
41.1*			48 55
42.1°			65 72
43.61	20 20		
45*	$\frac{\sqrt{2}}{2}$		



Karen @karenshancock · May 30, 2022

Then we used that trig table to answer some questions:



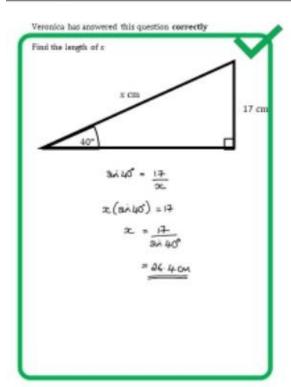


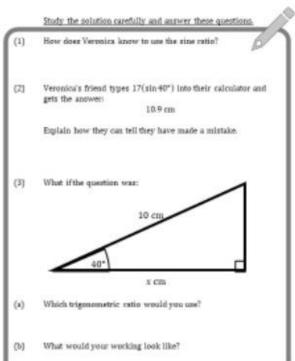


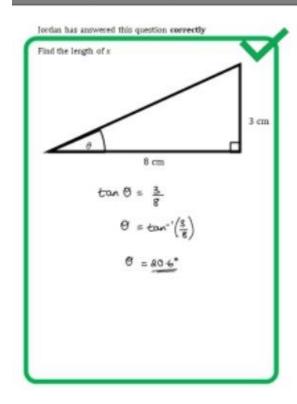
Karen @karenshancock · May 30, 2022

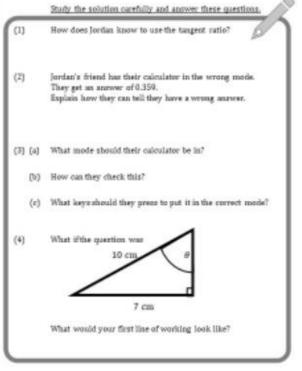
And then some worked examples and self-explanation prompts:

WORKED EXAMPLE - TRIGONOMETRY - FINDING LENGTHS









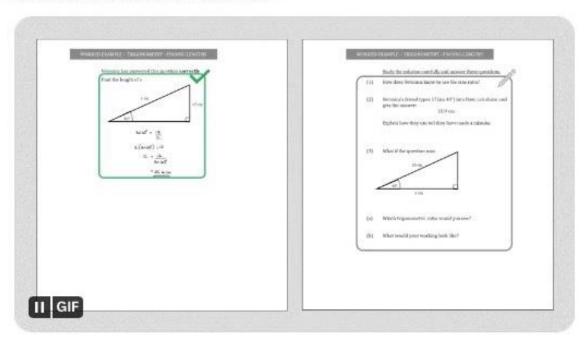




Karen @karenshancock · May 30, 2022

The whole PowerPoint is here for download:

kshancock.co.uk/lessonresource...



The thread above that the host showed and linked to at the start of this #mathsCPDchat discussion prompted the following reply, which included the sharing of two Geogebra interactive resources:



London South East+ Maths Hub @LSE_MathsHub · 21h Looks almost identical to something I cobbled together to pair with a geogebra activity I created geogebra.com/classic/cvmzjb...

And

Geogebra.com/classic/n74sfa..

(Links to these resources are provided at the top of this summary.)



London South East+ Maths Hub @LSE_MathsHub · 21h Et voila

**



Coherence in Trigonometry Worksheet

Angle	SF	C'D'	A'D'	C'D'
		A'D'	A'C'	A'C'
1"	1			
5°	1			
10°	1			
15°	1			
15°	1.5			
15°	2			
15°	0.5			
30°	1			
30°	1.5			
30°	2			
30°	0.5			
45°	1			
45°	1.5			
45°	2			
45°	0.5			
60°	1	1.73	0.5	0.86
60°	1.5			
60°	2			
60°	0.5			
75°	1			
75°	1.5			
75°	2			
75°	0.5			
80°	1			
85°	1			
899	1			

Reflections:

- 1. What do you notice about the ratios as the scale factor changes?
- 2. What do you notice about the ratios as the angle changes?
- 3. What happens to $\frac{C \cdot B^{+}}{A^{+}C^{-}}$ as the angle gets closer to 90°? What about 0°?

 4. What happens to $\frac{A^{+}D^{+}}{A^{+}C^{-}}$ as the angle gets closer to 90°? What about 0°?

 5. What happens to $\frac{C \cdot B^{+}}{A^{+}D^{-}}$ as the angle gets closer to 90°? What about 0°?
- 6. Do you spot any patterns?

sin 25° ≈



Coherence in Trigonometry Worksheet



This is the third past Twitter thread quoted by the host at the start of the chat includes the sharing of more resources:



Lizi Pepper @mathspeptalk · 22h

Which gave us this wonderful thread from @nathanday314 a month or so latertwitter.com/nathanday314/s...#mathsCPDchat



🚳 Nathan Day @nathanday314 · Jun 4

New Tasks: Introducing Trigonometry

interwovenmaths.com/introducing-tr...

Here are some tasks I used when introducing my Year 10s to Rightangled Trigonometry last half term.

Editable PPT + PDF with Answers available at the link.

Details below!

(1/6)

The host having quote-retweeted the three Twitter threads shown above, her first main question (repeated here) ...

> Q1: It is the first time a class is meeting trigonometry. What does the first lesson look like? How do you then carve out the rest of the journey from

... generated three conversations. This was a short discussion in which a teacher explains why he prefers to start with the Tan ratio:



Paddy MacMahon @paddymac_maths · 21h

Tan only, without necessarily even calling it that. Pupils start by drawing triangles.

#mathscpdchat



paddymacmahon.com paddy macmahon - pre-a level Problems and resources that I've used with years 7-11.





Paddy MacMahon @paddymac_maths · 21h

I particularly like that you can ask lots of kinds of questions very quickly:

Questions can now be set on finding sides (including the hypotenuse if pupils have done some Pythagoras) and finding angles where the legs are in one of the ratios from the table above. Pupils get an idea of the processes required in formal trigonometry and in the idea of inverting a function. Win!



Lizi Pepper @mathspeptalk · 21h

Interesting- this was going to be a question later. How do we support students who find trigonometry tough? I've seen some teach just sine and other information can be found with pythagoras and angles in triangles. Why tan out of interest? #mathscpdchat



Paddy MacMahon @paddymac_maths · 21h

I suppose because pupils will naturally draw triangles in an orientation where the adjacent and opposite are horizontal and vertical, and the measurements are easier. But it's a minor point, and probably boils down to "that's what I've always done"! #mathscpdchat

In the second conversation in response to Q1 using the unit circle was discussed briefly \dots



MrHawesMaths @HawesMaths · Jun 13

Similar triangles with the same angle 30 degrees. Then look at the ratio of their side lengths.



MrHawesMaths @HawesMaths · Jun 13

Like this #mathscpdchat





Mr Mattock FCCT NPQSL @MrMattock · Jun 13

I do something similar but dont specify the angle, that leads nicely into being able to use whatever ratios I like, and then the full unit circle.



Lizi Pepper @mathspeptalk · Jun 13

I like this as it really highlights the fact it is about the ratio of side lengths! #mathsCPDchat



Lizi Pepper @mathspeptalk · Jun 13

Do you link this to similar shapes and scale factors? The unit circle starts with hypotenuse one and then multiplies up, I wonder if these approaches are similar? Could we use ratio tables? #mathscpdchat



Mr Mattock FCCT NPQSL @MrMattock · Jun 13

Yes you can use ratio table for trig, set the exchange rate between the opposite and hypotenuse to be equal to sine theta (for example)



MrHawesMaths @HawesMaths · Jun 13

I only tend to introduce unit circle in year 10 when we look at trig graphs. #mathscpdchat. I can see how an early introduction would be beneficial.



Lizi Pepper @mathspeptalk · Jun 13

It has been surprising how well my y9 dealt with the unit circle, and then they generated the formula themselves so know what they mean! It was a bit daunting the first time, but I can really see the benefit! #mathsCPDchat



Mary Pardoe @PardoeMary · Jun 13

This paragraph (from here too:global.oup.com/academic/produ..) re using the circle is interesting ...

#mathscpdchat

Their conclusion was that a procedural method was more efficient in enabling students to solve these kinds of question, but the questions were indeed limited to those that required procedural knowledge. It is not clear, from their study, which students could adapt their knowledge to the more mathematically-sophisticated roles of trigonometry. If the unit circle method merely provides a diagram from which the right-angled triangle has to be extracted and dealt with separately, then all it has achieved is extra transformations to carry out. Yet if the unit circle is used to present 'sine' as the multiplier which, when applied to a radius, evaluates height, no transformation is necessary. Lakoff and Núñez (2000) argue that the unit circle provides a 'natural' understanding of trigonometric ideas which can be developed beyond 360 degrees, a further advantage of using this image.



This tweet (shown above) ...

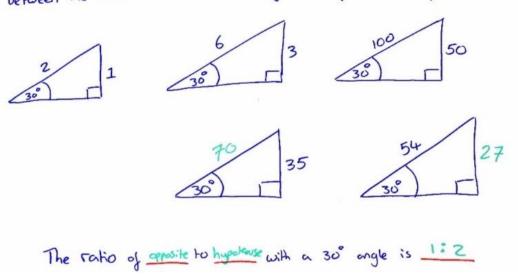


MrHawesMaths @HawesMaths · Jun 13

Like this #mathscpdchat



When you have a triangle (right angled) with a specific angle. The ratio between the sides remain the same regardless of the size of the triangle.



... also prompted the following comment:



Mary Pardoe @PardoeMary - Jun 13

Great! 'Further teaching experiments ... would be useful! From this again: global.oup.com/academic/produ.. #mathsCPDchat

of using this image.

Other writers have reported success in engendering robust understandings of trigonometric ideas, beyond what is needed for solving right-angled triangles, by combining unit circle, similarity, and exploratory approaches. For example, Steer, de Vila and Eaton (2009a, 2009b, 2009c) report a series of lessons designed by Jeremy Burke which move from a mathematical description of trigonometry, through a set of classroom activities (using dynamic geometry tools to permit exploration) which combine ratio and unit circle methods. Students developed their own accounts of trigonometry, using these tools, over four lessons. Further teaching experiments of this type would be useful.

Classing trigonometric functions

The host's Q1 ...



Q1: It is the first time a class is meeting trigonometry. What does the first lesson look like? How do you then carve out the rest of the journey from there?

... also prompted the following observations:



Mary Pardoe @PardoeMary · 22h

There are lots of possible approaches. This is from 'Key Ideas in Teaching Mathematics' ... there is a link to the book from the website here: nuffieldfoundation.org/students-teach...
#mathscpdchat

While the illusion of clear boundaries can lead to narrow procedural teaching, a variety of approaches to introducing trigonometric ideas are taken in practice (Watson, 2009b).

- Similarity: students find that the ratio of pairs of sides in sets of similar triangles is the same.
- Functions: students plot values of the height of a point moving on a unit circle and relate this to the right-angled triangle it forms with the radius and x-axis.
- Multiplier: students relate heights of various right-angled triangles to a fixed-length hypotenuse using 'sine' on a calculator.
- Ratio: students use ratios to solve scaling problems and are then introduced to the names for these ratios.
- Procedural resolution of triangles: students are presented with trigonometric ratios as tools to use, alongside Pythagoras, to find missing values.
- Exploration of calculator functions: students explore what the function does by controlling input variables and plotting data or recording output in a spreadsheet.



Lizi Pepper @mathspeptalk · 22h

I've been guilty of teaching trigonometry procedurally!. Our amazing KS5 lead introduced us to the @MEIMaths deeper Maths resources (free) which introduce trig via the unit circle and Geogebra, first sine and cosine together then tangent after. Has anyone else? #mathsCPDchat



 $\textbf{Mary Pardoe} @ Pardoe Mary \cdot 22h$

MEI Deeper Maths Trigonometry Overview is here: mei.org.uk/resource/b5f92...



mei.org.uk Resource – MEI



The host's second main question ...



Lizi Pepper @mathspeptalk · Jun 13

On a slightly side note:

Q2: When do you think it is best to introduce trigonometry to students? What year and term roughly? Why? #mathsCPDchat

... generated five threads of various lengths. This one included comments about students' positive attitudes ...



MrHawesMaths @HawesMaths - Jun 13

I have introduced it to my year 8s top set as a 'triangle project' where we look at right angles triangles. Area, angles totalling 180, Pythagoras and then trig. Summer term. #mathscpdchat



Lizi Pepper @mathspeptalk · Jun 13

It feels like a summery topic! We teach in year 9 in summer term #mathsCPDchat how do your year 8s cope with the challenge?



MrHawesMaths @HawesMaths · Jun 13

They love it. They feel super smart doing it. It enables us to have the conversation of 'if we see a right angled triangle what are our options?' They like the challenge of trying to find the right ratio. Once found they can substitute and solve. #mathscpdchat



Lizi Pepper @mathspeptalk · Jun 13

I love when students find topics empowering – mine didn't know they were doing trigonometry for ages and when I accidentally said trigonometry they were like "THIS is trigonometry? It's so easy!" #mathscpdchat

... this included acknowledgement of the knowledge and understandings on which a true grasp of trigonometry depends, and a reference to trigonometric functions ...



Mr Mattock FCCT NPQSL @MrMattock · Jun 13

Never see any need to introduce before year 10. Gives time for knowledge of similarity to be taught and mature which is a crucial start point.





Sandfroid @ben_jolley2 · Jun 13

Worth mentioning that it's in the KS3 national curriculum and so should be seen before Year 10. Got called out on this in an Ofsted inspection not long ago. I know that's not the point of the OP though.



Mr Mattock FCCT NPQSL @MrMattock · Jun 13

Interesting, we had Ofsted and no one mentioned anything about it. There is a lot of nonsense in the KS2 and KS3 national curriculum that has no business being there. One of the few benefits to being in an academy means we don't have to bother with it.





Lizi Pepper @mathspeptalk · Jun 13

It is certainly a topic with many pre-requisites! Even simple rearranging car be a barrier! What other skills do you hope they would have mastered first? #mathsCPDchat



Mr Mattock FCCT NPQSL @MrMattock · Jun 13

Need a knowledge of inverse functions, and reciprocals for the way I approach it. #mathscpdchat



Paddy MacMahon @paddymac_maths · Jun 13

It's an ideal opportunity to start thinking about functions and inverse functions. E.g. in this triangle the opposite leg is 0.36 times the adjacent leg - what must the angle be? #mathscpdchat

... this prompted a reference to a resource book ...



Mr Mann @MrMannMaths · 21h

If students are capable of rearranging formulae to solve equations with fractions, then they are capable. Generally, this would be Y8 or Y9.



alex smith @noise random · 19h

Trigonometric Delights by Eli Maor - brilliant book



Lizi Pepper @mathspeptalk · 18h

Sounds like an interesting read-thanks for sharing!

... this was another reminder of connections/dependency-relationships between trigonometry and other mathematical ideas ...



Paddy MacMahon @paddymac_maths · Jun 13

Year 8, about now. Fits in nicely with work they've done on similar triangles and scale factors (not just in geometry, but also in percentages: increase such and such by 23% by using a scale factor of 1.23; solve reverse percentage problems with a scale factor...) #mathscpdchat



Paddy MacMahon @paddymac_maths · Jun 13

I really like to stress the tan ratio as a scale factor: it's what you multiply the adjacent by to give the opposite.

So finding the adjacent? Divide the opposite by your scale factor. No formula triangles... #mathscpdchat

... and this involved backwards time travel:





RobotMaths @robotmaths · Jun 14

Basic trigonometry is Level 7, so most Year 9s will need to see it before their SATs. Top set Year 8 can tackle it in the Summer term as an investigation, and then all but bottom set Year 9 in the Spring term alongside Pythagoras.

This tweet was brought to you by the year 2004.



Lizi Pepper @mathspeptalk · Jun 14

Thanks for your reply- not everyone in agreement as when is the best time! It's interesting to hear thoughts on why people teach it when they do. I think there are quite a few key concepts that need to be embedded before we introduce trig-that's probably more important when.

The host's, Lizi Pepper's, third main question ...



Lizi Pepper @mathspeptalk · Jun 13 Touched on this earlier, but...

Q3. How do you scaffold trig so all students understand what can be a tricky concept?

For example I've seen some schools who have just used sine, then use Pythagoras' theorem and angles in triangles to find other sides/angles #mathsCPDchat

... prompted three conversations and three single replies. This one included the expression of thoughts about avoiding practices that are likely to result in students following procedures without developing real understanding ...



Mr Mattock FCCT NPQSL @MrMattock · Jun 13

People doing that are not making sure kids understand trig...#mathscpdchat



Lizi Pepper @mathspeptalk · Jun 13

Interesting - I agree and it isn't something I would do! But from an exam point of view, I see why potentially reducing the cognitive load and simplifying the process may help some students? (playing devil's advocate!) #mathsCPDchat



 $\textbf{MrHawesMaths} @ \textbf{HawesMaths} \cdot \textbf{Jun 13}$

Also steer clear of the formula triangle!!!





Mary Pardoe @PardoeMary · Jun 13 Yes!

#mathscpdchat

Groundwork prior to trigonometric functions

In preparation for learning about trigonometric functions, students need to be familiar with a range of ideas across algebra and geometry. Analyses of the mathematical ideas involved in trigonometric situations, even fairly simple ones, show that students who cannot appreciate a ratio given as a number, or manipulate a multiplicative relation, are at a clear disadvantage to those who can (Weber, 2005, 2008). This could underlie why it can be tempting to provide

shortcuts to remembering how to use various transformations of the formulae for sine, cosine, and tangent, and to remembering which ratio involves which sides. One example of this is the popular mnemonic 'SOH-CAH-TOA' for the trigonometric ratios.1 A further assumption about students in terms of pretrigonometry is that they can readily identify right-angled triangles in atypical situations; this is not always the case (Byers, 2008).

On top of all this, trigonometry is often the first context in which students meet functions that are not polynomials in x and that are represented using a name. For students who have used the f(x) notation this involves replacing f by 'sin' but if their previous experience is solely of notations like equations 'y =' or like mappings ' $x \rightarrow$ ', then using a name for a function can be mystifying.



Lizi Pepper @mathspeptalk · Jun 13

Mary these book snippets are brilliant! You'll have to let us know what the book is/books are 😝 #mathscpdchat



Mary Pardoe @PardoeMary · Jun 13

Key Ideas in Teaching Mathematics

The link is in my earlier tweets ... here: global.oup.com/academic/produ...



global.oup.com

Key Ideas in Teaching Mathematics

Big ideas in the mathematics curriculum for older school students, especially those that are hard to ...



... this again included reference to connections within mathematics ...



Mr Mann @MrMannMaths · 21h

- Labelling sides.
- Identifying sin/cos/tan kahoot/quizizz is good for this.
- Finding unknown side
- 4. Finding unknown angle
- Extension to multiple step trig questions or 3D trig.



Mr Mann @MrMannMaths · 21h

Also very useful to give students practice of solving equations with fractions before moving to Step 3



alex smith @noise random · 20h

And link immediately to the rotation of a radius around a circle. Must establish the relationship with circles straight away.

... and this included the sharing of a resource in which the controversial 'SOH CAH TOA' mnemonic (mentioned in a guidance-resource shared in a previously shown conversation) appears:



MrHawesMaths @HawesMaths · Jun 13

I start with lots of angle calculations so students can get used to labelling and applying trig then I head to finding missing lengths. Half the battle I find is the naming element of trig. And selecting the correct ratio. #mathscpdchat



Lizi Pepper @mathspeptalk · Jun 13

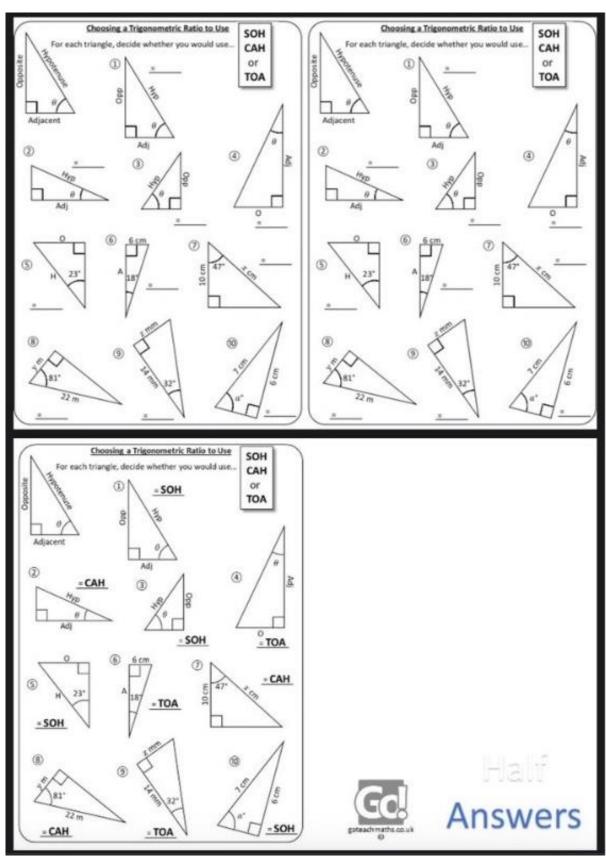
I love this resource from @draustinmaths to practice labelling and identifying correct ratios draustinmaths.com/_files/ugd/7ac..



MrHawesMaths @HawesMaths · Jun 13

I like this one from @goteachmaths





These were the three 'single' replies to Q3:





I think something like this is actually a very good way to start - my first school did tan ratios using tables of values to look them up and combined with Pythag problems. Of course we came back the next term and added in sine and cosine as well and used calculators too. These were students who would almost all go on to top grades but in principle I'd think doing it this way could give any student the chance to focus on understanding. Tan is a nice place to start as you can relate it to steepness of the hypotenuse easily too and a table of increasing values for increasing angles can be more intuitive.



Tracy # 2 (1) @tracyblig · Jun 14

In all my years of teaching maths, it never entered my head to limit to only one trig function... It's an interesting idea for those who struggle most with choosing which one to use



Janie Mac 🥻 @beancanleveller · Jun 14

I'm not a maths teacher, but have been teaching level 3 engineering principles this year. A common problem that I found was that my students couldn't access trig, because they couldn't identify the opposite and adjacent sides of a triangle. So I started there.

Because there was little time left, and contributors were responding to questions posted earlier, the host's last main question ...



Lizi Pepper @mathspeptalk · Jun 13

Final question for the night, everyone loves getting resource inspiration! Q4. Finally, what are your favourite trigonometry resources? #mathsCPDchat #mathsCPDchat

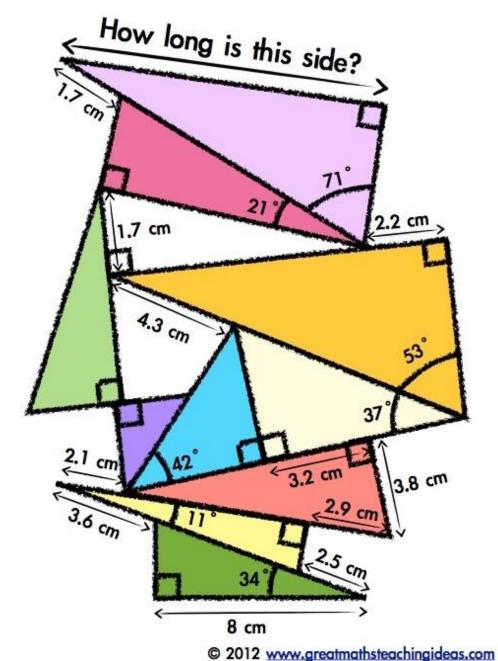
I can't find it but I love the trigonometry pile up! Whoever made that is a genius!

... received only one reply:





Trigonometry Pile Up!



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Lizi Pepper @mathspeptalk · Jun 13

Well that's all we have time for!

Thank you to everyone who contributed, I've definitely got some great ideas from the discussion- I hope others have too! #mathsCPDchat