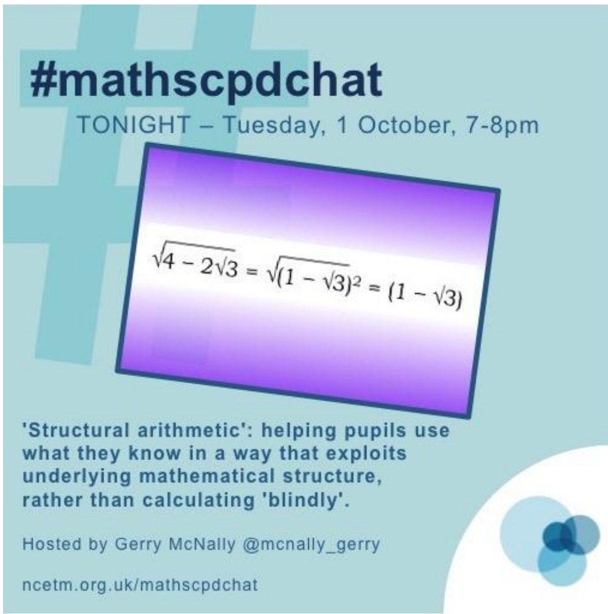
#mathscpdchat 1 October 2019

‘Structural arithmetic’: helping pupils use what they know in a way that exploits underlying mathematical structure, rather than calculating ‘blindly’.

Hosted by Gerry McNally

*This is a brief summary of the discussion – to see all the tweets, follow the hashtag **#mathscpdchat** in Twitter*



Some of the areas where discussion focussed were:

- that the term ***structural arithmetic*** as used in this discussion is explained, exemplified and discussed by **Tony Gardiner** in his book ***Teaching Mathematics at Secondary Level*** (links provided below);
- that ‘mental calculation work should not end with Key Stage 2 … it should continue in Y7, but should increasingly **use what pupils know in a way that exploits structure**, rather than calculating blindly’;
- that ‘from KS2 onwards, calculation should begin to move beyond bare hands evaluation, and should concentrate on developing an awareness of the algebraic

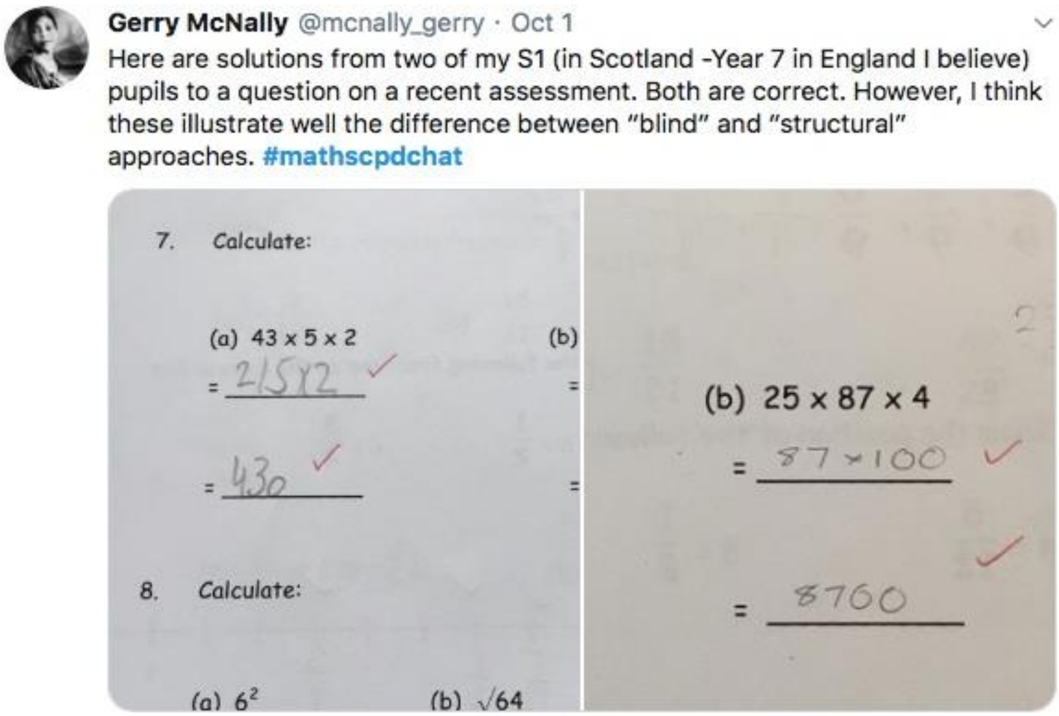
structure lurking beneath the surface of so many numerical or symbolical expressions';

- that **helping students** to 'calculate smart' rather than 'calculate blind' is not only about seeing lots of examples, but also **involves drawing attention to structural aspects of numbers and number combinations** … how to do that?
- **a real example (provided by the chat host)** of the actual steps used by two different Y7 pupils (during a diagnostic quiz) when they had to evaluate a product of three numbers … the example **illustrated the difference between 'blind' and 'structural' approaches**;
- that pupils' responses to items in a diagnostic quiz can give the teacher **'lots to work with going forward'**;
- pupils learning to **resist the temptation to evaluate every numerical expression that occurs part-way-through a problem-solving process** … learning to wait until they have a final unevaluated expression, and then to **use 'structure' to simplify that final unevaluated expression** … that foundation-level GCSE students who do this are at an advantage over students who evaluate expressions 'as-they-go';
- that, for students who are struggling in 'Foundation-level groups', **finding factors is hard… for example, they would struggle to see 25 × 16 as 25 × 4 × 4 = 100 × 4**;
- that pupils who have had **plenty of opportunities to notice the structure in arithmetical expressions** are more likely to use structure to simplify calculations than to calculate 'blindly';
- that challenges of the kind '**Find a quick way to calculate**, e.g. 4 × 51 + 3 × 102' are usually challenges to use 'structural arithmetic';
- challenging pupils to **work out mentally** lots of very simple examples (such as 14 × 5 as 7 × 10), and **share their mental methods**;
- that **when pupils share their methods for calculations** the object is not simply to 'say' the calculations but more to **reveal the structures they used**;
- students being in a position to be **surprised by less obvious ways of using structure** … for example, that **GCSE Higher-level students** should be able to see and do this kind of simplification-using-structure: $\sqrt{(4 - 2\sqrt{3})} = \sqrt{(1 - \sqrt{3})^2} = 1 - \sqrt{3}$;
- that pupils need to be **always on the lookout for ways to use structure** in arithmetic, so building for themselves a **sound base from which to learn (understand and use) algebra**;
- that '**algebra copies the structure of arithmetic exactly**, and **applies it to a new 'mixed universe'** of symbols (or letters) and numbers… it is **not the structure of arithmetic that is generalised, but the universe to which the old structure is applied**';

- **students benefitting from choosing their own numbers** (eg for dimensions) with the aim of making contextualised problems more manageable … that **students like being 'put in control'** rather than always doing the book's/teacher's examples;
- that KS1/2 pupils may learn to use structure in numerical expressions by engaging in **'Number Talks'**;
- that the **weekly #mathstratchat** (which originates in the USA on a Wednesday evening, USA time, and so can be conveniently reviewed in the UK on Thursday mornings) promotes and discusses 'creative approaches to processing numbers'.

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is part of a 'conversation' of tweets, about finding out to what extent pupils are using structural arithmetic naturally, making efforts to support pupils who struggle with it, and why facility with structural arithmetic lays foundations for understanding and using algebra. The conversation was generated by this tweet from Gerry McNally:


Gerry McNally @mcnally_gerry · Oct 1
Here are solutions from two of my S1 (in Scotland -Year 7 in England I believe) pupils to a question on a recent assessment. Both are correct. However, I think these illustrate well the difference between "blind" and "structural" approaches. #mathscpdchat

including these from Mary Pardoe, Gerry McNally and Heather Scott:


Mary Pardoe @PardoeMary · Oct 1
Replying to @mcnally_gerry
What have you done with your pupils, Gerry, to get the response on the right … the one using structure? #mathscpdchat

Gerry McNally @mcnally_gerry · Oct 1
Nothing is the honest answer, Mary. This was a diagnostic quiz intended to assess prior learning from primary school. It does give me lots to work with going forward, though. #mathscpdchat

Mary Pardoe @PardoeMary · Oct 1

I wonder how your pupils would approach this word-problem? #mathscpdchat

> *"I pack peaches in 51 boxes with 16 peaches in each box.*
> **How many boxes would I use if each box contained just 12 peaches?"**

- some pupils might calculate the total number of peaches and then divide by 12;
- one would prefer to see a more structural version of this representing the total number of peaches as "51×16" without evaluating, and the required number of boxes as $\frac{51 \times 16}{12}$ before cancelling

$$\frac{17 \times (3 \times 4) \times 4}{12} = 17 \times 4;$$

TEACHING MATHEMATICS AT SECONDARY LEVEL
ANTHONY D. GARDINER

Heather Scott @MathsladyScott · Oct 1

#mathscpdchat One of the most difficult thing for some students struggling in foundation group is finding factors. Working with a mid ability yr 7 group this week many had a problem with 35 x ___ = 35 I always find this fascinating and wonder where the difficulty comes from 🤔

Mary Pardoe @PardoeMary · Oct 1

Replying to @MathsladyScott

Yes. Maybe the difficulty comes from way back ... e.g. in KS2 not learning to work out 15 x 22 mentally as 3 x 5 x 2 x 11 = 3 x 10 x 11 = 33 x 10 = 330?

and these from Heather Scott and Mary Pardoe:

Heather Scott @MathsladyScott · Oct 1

#mathscpdchat - I think it is an interesting idea to explore in the classroom - to spend more time examining the structure of the arithmetic so then later one is more capable of finding solutions as there are more patterns to observe? 🤔

Mary Pardoe @PardoeMary · Oct 1

Replying to @MathsladyScott

Yes, and as a good foundation for algebra? This is another quote from Tony's book ... #mathscpdchat

> Elementary algebra does not really "*generalise* the structure of arithmetic" as suggested in the above official requirement: algebra **copies** the structure of arithmetic *exactly* (that is, the four rules, together with the commutative laws, the associative laws, and the distributive law) and *applies it to a new 'mixed universe'* of symbols (or letters) and numbers. Thus it is not the *structure* that is generalised, but the *universe* to which the old structure is applied.

TEACHING MATHEMATICS AT SECONDARY LEVEL
ANTHONY D. GARDINER

Heather Scott @MathsladyScott · Oct 1

#mathscpdchat I so agree with this - everything Tony Gardiner writes brings a deeper understanding to the mathematical situation - it should be compulsory reading for everyone teaching mathematics 😍

(to read the discussion-sequence generated by any tweet look at the ‘replies’ to that tweet)

Among the links shared were:

Teaching Mathematics at Secondary Level by Tony Gardiner which is a free downloadable copy of a very valuable, and influential book which offers a broad view of secondary mathematics. It should interest both seasoned practitioners and those at the start of their teaching careers. It was shared by Mary Pardoe

Teaching Mathematics at Secondary Level by Tony Gardiner which is a purchasable real-book copy of this very valuable, and influential book which clarifies certain crucial features of elementary mathematics and how it is learned – features which all teachers need to consider before deciding ‘How to teach’. It was shared by Mary Pardoe