**Shaping the Year 7 Curriculum:**

**Building on Year 6**

**Key Idea 2: Understanding additive and multiplicative structures and using them to solve problems**

(6AS/MD–1 Quantify additive and multiplicative relationships; 6AS/MD–2 Derive related calculations; 6AS/MD–3 Solve problems involving ratio relationships; 6AS/MD–4 Solve problems with 2 unknowns).

Fundamental to having a really deep understanding of number structure is distinguishing between additive and multiplicative structures and to understand (and be able to use when solving problems) the relationships and connections within them.

**Additive structure**

Addend

Addend

Sum

Pupils should know that if, for example, 24 + 47 = 71 then the following are also true:

* 47 + 24 = 71 (the commutative property)
* 71 – 47 = 24 and 71 – 24 = 47 (inverse operation)

If any number is added to one addend and the same number is subtracted from the other addend, the sum remains the same e.g. 25 + 46, 34 + 37 and 21 + 50 are all equal to 71 (the compensation property).

**Multiplicative structure**

15

180

12

Similarly, for multiplicative relationships, pupils should know that if, for example, 12 × 15 = 180 then:

* 15 × 12 = 180 (commutative)
* 180 ÷ 12 = 15 and 180 ÷ 15 = 12 (inverse)
* If one factor is multiplied by a number and the other is divided by the same number, the product stays the same e.g. 6 × 30, 36 × 5 and 4 × 45 are all equal to 180 (compensation).

Area models for multiplication are useful to support pupils in understanding these relationships:

15

12

has the same area as

4

45

Pupils should be able to use this understanding of multiplicative relationships[[1]](#footnote-1) to solve a variety of one-to-many and many-to-many type ratio problems like the ones below:



 p38 DfE guidance ‘[Teaching mathematics in primary schools Y6’](https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/897805/Maths_guidance_year_6.pdf)

Pupils should also be able to apply these understandings to problems involving two unknowns where there are an infinite number of solutions (e.g. 5 × □ = 10 × □), more than one solution (e.g. Danny has some 50p coins and some 20p coins. He has £1.70 altogether. How many of each type of coin might he have?) and only one solution (e.g. The sum of 2 numbers is 48. One number is one-fifth times the size of the other number. What are the 2 numbers?).

**Progression to Key Stage 3**

Two key ideas develop in Key Stage 3 and build on this secure understanding of additive and multiplicative structures:

* **Re-arranging formulae**

The skills and awarenesses involved in being able to re-arrange additive and multiplicative number sentences are developed and generalised in Key Stage 3 when re-arranging formulae. For example, to make ‘a’ the subject of the formula:

 $v = u + at$

pupils must first ‘see’ the additive relationship and re-write it as:

 $at = v – u$

then to ‘see’ the multiplicative relationship and re-write as:

 $a=\frac{v-u}{t}$

* **Multiplicative reasoning**

While pupils in Year 6 need to be able to state a multiplicative relationship between two numbers where this is an integer multiplier, a significant shift in Key Stage 3 is to know that:

* + ***any two numbers*** can be connected by a multiplier
	+ the multiplier can be expressed as a fraction or a decimal
	+ when the multiplier is expressed as a fraction, the numerator and denominator of the fraction corresponds to the two numbers connected (e.g. the fractional multiplier connecting 2 and 3 is $\frac{2}{3}$ or $\frac{3}{2}$

This second idea supports many other areas in the Key Stage 3 curriculum including scaling, enlargements, percentage increase and decrease and ratio and proportion.

For further guidance on how to develop these important ideas in Key Stage 3, follow the links below to the relevant documents in the [NCETM Secondary PD Materials](https://www.ncetm.org.uk/teaching-for-mastery/mastery-materials/secondary-mastery-professional-development/).

**Theme Overviews:**

* [Theme 1: The structure of the number system](https://www.ncetm.org.uk/media/oconaxqx/ncetm_ks3_theme_1.pdf)

Theme 1 explores the key structures of number and the number system, such as place value, factors, multiples, and powers.

* [Theme 2: Operating on number](https://www.ncetm.org.uk/media/x2uj2qln/ncetm_ks3_theme_2.pdf)

Theme 2 is about students understanding the structures underpinning calculation using each of the four operations. It includes the solving of equations, which is essentially concerned with operations on as yet unknown numbers.

* [Theme 3: Multiplicative Reasoning](https://www.ncetm.org.uk/media/0mzbcnny/ncetm_ks3_theme_3.pdf)

Theme 3 addresses the idea that any two numbers can be connected by multiplication. This gives rise to ideas of ratio, proportionality, percentage increase and decrease, rates of change, enlargement, similarity, and trigonometric ratios.

**Core Concept documents:**

* [Core Concept 1.4: Simplifying and manipulating expressions, equations and formulae](https://www.ncetm.org.uk/media/zeeniedj/ncetm_ks3_cc_1_4.pdf)

This core concept concerns the generalisation of number structures, the use of algebraic symbols and techniques for their manipulation. The rearranging of formulae is also explored by linking it to the manipulation of number sentences.

* [Core Concept 2.1: Arithmetic procedures](https://www.ncetm.org.uk/media/xhqegzuq/ncetm_ks3_cc_2_1.pdf)

This core concept offers guidance on developing a strong understanding of the mathematical structures that underpin the standard procedures for calculation with decimals, fractions and directed numbers.

* [Core Concept 3.1: Understanding multiplicative relationships](https://www.ncetm.org.uk/media/mqfp3xb3/ncetm_ks3_cc_3_1.pdf)

This core concept explores fractions, percentages, ratio, and proportion (direct and inverse) as contexts in which multiplicative relationships are used.

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1. It is, of course, vital that pupils have rapid recall of their multiplication tables to make good progress with these ideas and this is an important aspect of being secondary ready. [↑](#footnote-ref-1)