

A Departmental Workshop

Fractions 1

This is a suggested plan for a professional development session. It has been written to support anyone wishing to lead such a session with a group of teachers and the green 'key points' sections are intended as a support specifically for such a facilitator in guiding discussions.

N.B. These workshops have been written to provide enough professional development activity and discussion for one session of approximately one hour with the option of further activity (as outlined in the 'Possible next steps' section at the end). This final section references the NCETM Secondary Mastery Professional Development Materials which can be found here www.ncetm.org.uk/secondarymasterypd

Overview

Fractions are notoriously difficult for some students. One of the difficulties stems from the fact that fractions have a number of different but connected meanings. They are numbers, but they are also operators on numbers. They express a multiplicative relationship and there is widespread research evidence that thinking multiplicatively, rather than additively, is a challenge for students.

This workshop explores the multiple images and meanings which fractions have and gives you the opportunity to work with other teachers and discuss:

- how to support students in having a deep and connected understanding of fractions in all their different forms
- what implications there might be for your future practice and curriculum development.

Activity 1: On a large sheet of flipchart paper, write $\frac{3}{4}$ in the middle and brainstorm, as a group, as many different ways of writing, drawing, writing a description of, or otherwise showing the meaning of, this fraction as you can think of.

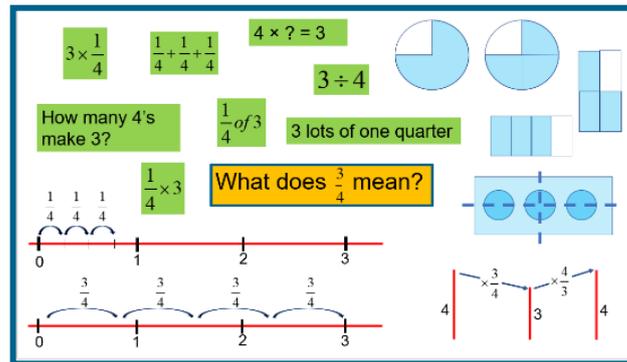
Discussion:

- Discuss what different meanings of $\frac{3}{4}$ each one of these descriptions conveys.

Key Point: $\frac{3}{4}$ could mean 3 lots of one quarter or one quarter of 3. It is a number between 0 and 1 and is also the multiplier that connects the integers 4 and 3.

What different diagrams might show these contrasting yet connected meanings?

Activity 2: Look at the 'Three quarters' handout and try to make sense of each expression, description and diagram. Which ones go together and show the same meaning? How is each one connected to all the others? Try to come up with a list of different meanings for a fraction.



Discussion

- Discuss which meanings of fractions you introduce in your own teaching.
- Are there some that are more prevalent than others?
- Are there some that you don't refer to at all?

Key Point: The part-whole image is very prevalent in many materials, textbooks and exercises related to fractions. This is the idea that, for example, $\frac{3}{4}$ can be represented by a shape divided into 4 equal pieces and 3 of those pieces shaded.

Such an image is helpful but if it is the only image that students have, it may limit their understanding. Some of the consequences of this are that:

- students see fractions as always less than 1 ('you can't have 5 quarters!')
 - they may understand the meaning of $3 \times \frac{1}{4}$ if they think of the '3' as the multiplier but can't understand how the $\frac{1}{4}$ can be a multiplier.
 - they fail to grasp the powerful idea that any integer can be transformed into any other integer by multiplying by a fraction
 - that multiplying by a fraction less than 1 makes the number smaller (and conversely, that dividing by a fraction less than 1 makes it bigger).
- Discuss other misconceptions your students might have about fractions and consider how a wider range of images and meanings for a fraction might help them overcome these.

Activity 3: Consider the addition (and subtraction) of fractions. What images and senses of fractions depicted in the 'Three quarters' handout might support students in understanding this?

Key Point: The image of $\frac{3}{4}$ as three lots of one quarter or $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ enables student to make sense of calculations like $\frac{3}{4} + \frac{7}{4}$ as just an exercise in 'counting quarters' (i.e. 3 quarters and 7 quarters equals 10 quarters) and so can help guard against the misconception of 'add the tops and add the bottoms'.

Additionally, being able to sub-divide a rectangle diagram like the one in the 'Three quarters' handout to show that $\frac{3}{4}$ can be written in an infinite number of different ways and yet still remain equivalent (i.e. $\frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}$) helps students to understand how to add fractions with different denominators.

Activity 4: Consider the multiplication (and division) of fractions. What images and senses of fractions depicted in the 'Three quarters' handout might support students in understanding this?

Key Point: For multiplication (and division) of fractions to make complete sense, students need to have an image of a fraction as a multiplier, and to realise that multiplying can make smaller when the multiplier is less than 1 (and, conversely, that division can make bigger).

Possible next steps

This session may have surfaced some more long-term developments that you and your department (or group of teachers you are working with) wish to take. This section offers a way of doing this at some point in a future session or series of sessions.

Have a look at 'Core Concept 2.1: Arithmetic procedures' from the [NCETM Secondary Mastery Professional Development Materials Theme 2](#).

In particular, look at the key ideas in:

- 2.1.3 *'Know, understand and use fluently a range of calculation strategies for addition and subtraction of fractions'*
- 2.1.4 *'Know, understand and use fluently a range of calculation strategies for multiplication and division of fractions (pages 11, 12 and 24 – 30).'*

Discuss:

- how these ideas might influence your own teaching of fractions in Key Stage 3
- how these ideas might support developments in your scheme of work.