A Departmental Workshop

Proportional Reasoning

This is a suggested plan for a professional development session. It has been written to support anyone wishing to lead such a session with a group of teachers and the green 'key points' sections are intended as a support specifically for such a facilitator in guiding discussions.

N.B. These workshops have been written to provide enough professional development activity and discussion for one session of approximately one hour with the option of further activity (as outlined in the 'Possible next steps' section at the end). This final section references the NCETM Secondary Mastery Professional Development Materials which can be found here www.ncetm.org.uk/secondarymasterypd

Overview
Multiplicative structures and relationships underpin much of the key content in the secondary mathematics curriculum, and can be seen across number, algebra, and geometry. A deep understanding of multiplicative reasoning will open up and unify the curriculum for students.

In this workshop, you will have the opportunity to:

- think deeply about proportional/multiplicative reasoning
- challenge and extend some of your own understandings
- discuss with colleagues the implications for what you teach and how.

Activity 1

Here is part of a 'times table' square:

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20

Thinking proportionally is having an understanding about the relationships between the rows and columns. For example, each number in the second column is two times the corresponding number in the first column, numbers in the third column are three times numbers in the first column, etc.

Describe some other relationships on the table.

How would you describe the relationship between numbers in column 2 and the corresponding numbers in column 3? What about the relationship between numbers in columns 3 and 2?

Discussion

- Why might students find some of these relationships difficult to describe?
- Do you ask your students to use terminology such as 'multiply by $\frac{2}{3}$ ' ?

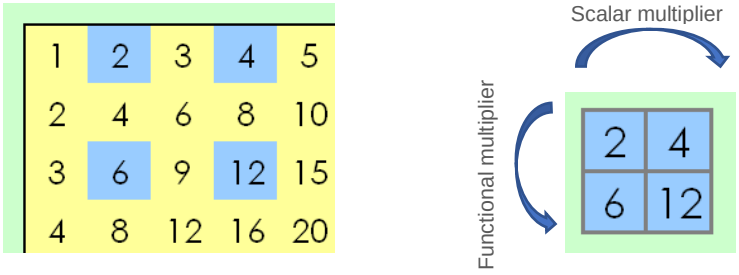
Key Point: *Any* two numbers can be connected via a multiplicative relationship.

It is important that students have experience of a range of carefully chosen examples to help them to appreciate this.

Be aware that when thinking about relationships between numbers, students often tend to want to add rather than multiply – especially when the quantities are not whole number multiples of each other.

Activity 2

Look at the handout and at the squares coloured in the grid.

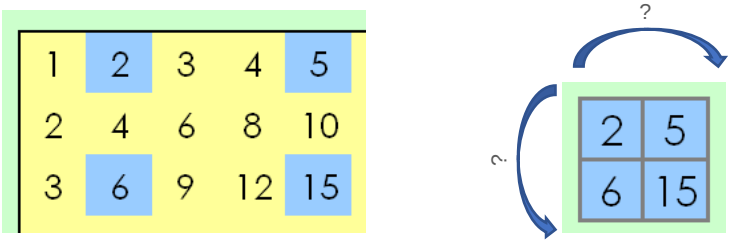


What can you say about the relationships between these numbers:

– looking horizontally (scalar multiplier)?

– looking vertically (functional multiplier)?

What about these?



Key Point:

In the first example, the scalar multiplier is 2. Moving from left to right, numbers are multiplied by 2. Moving from right to left, numbers are divided by 2 (or multiplied by $\frac{1}{2}$).

The functional multiplier is 3. Moving from top to bottom, numbers are multiplied by 3. Moving from bottom to top, numbers are divided by 3 (or multiplied by $\frac{1}{3}$).

In the second example, the scalar multiplier is $\frac{5}{2}$ or 2.5. Moving from left to right, numbers are multiplied by $\frac{5}{2}$ or 2.5. Moving from right to left, numbers are divided by $\frac{5}{2}$ (or multiplied by $\frac{2}{5}$).

Representations like these are known as 'ratio tables' and they can help students to reason and understand questions involving proportional/multiplicative structures. They can be used very flexibly.

Activity 3

Ratio tables can be used if thinking in terms of unitary method.

4 oranges cost 92p. How much do 7 oranges cost?

4	1
92	**?**

Step 1: Cost of 1 orange is 23p (divide by 4)

1	7
23	**?**

Step 2: Cost of 7 oranges is 161p = £1.61 (multiply by 23)

Discussion

- Do you use ratio tables in your own teaching?
- How might this representation help students to organise their thinking? What are the limitations?

Key Point:
When solving ratio problems, it may sometimes be easier to use the scalar multiplier to work out a missing value; in other cases the functional multiplier will be easier. It is important for students to know and understand that, for any ratio relationship a : b = c : d, there will always be these two multipliers to choose from.

Possible next steps

This session may have surfaced some more long-term developments that you and your department (or group of teachers you are working with) wish to take. This section offers a way of doing this at some point in a future session or series of sessions.

Have a look at *'Core Concept 3.1 Understanding Multiplicative Relationships'* from the NCETM Secondary Mastery Professional Development Materials Theme 3.

1. Read and discuss the exemplification of multiplicative relationships on pp8-13.

2. The double number line representation supports understanding of multiplicative relationships by including aspects of scale and continuity. Read and discuss the exemplification of use of double number line on pp16-18.

3. Discuss
   - how these ideas might influence your own teaching of these concepts
   - how these ideas might support developments in your scheme of work.