# A Departmental Workshop

## Structural Arithmetic

*This is a suggested plan for a professional development session. It has been written to support anyone wishing to lead such a session with a group of teachers and the green 'key points' sections are intended as a support specifically for such a facilitator in guiding discussions.*

*N.B. These workshops have been written to provide enough professional development activity and discussion for one session of approximately one hour with the option of further activity (as outlined in the 'Possible next steps' section at the end). This final section references the NCETM Secondary Mastery Professional Development Materials which can be found here [www.ncetm.org.uk/secondarymasterypd](www.ncetm.org.uk/secondarymasterypd)*

**Overview**

Being fluent with arithmetic processes (including the efficient use of standard algorithms) is, of course, a vitally important part of being skilled in maths.

However, unthinking use of standard procedures and algorithms, using them when they are inappropriate or inefficient and having no idea why they work and what mathematical structures underpin them, does not represent a deep and connected understanding of maths.

This workshop introduces the idea of 'structural arithmetic' and gives you the opportunity to work with other teachers and discuss:

- how to support students in understanding some key structures underpinning calculation strategies and methods
- how this can be a powerful way of approaching algebra
- what implications there might be for your future practice and curriculum development.

**Activity 1**: Invite everyone in the group to solve the following problems by finding the missing number:

1. $\square + 17 = 15 + 24$
2. $99 - \square = 90 - 59$
3. $48 \times 2.5 = \square \times 10$
4. $3 \div 4 = 15 \div \square$

**Discussion**

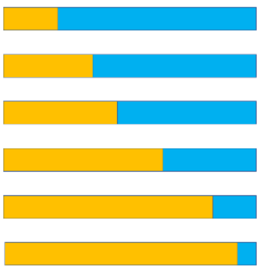
- How did you do the calculations and what might be the most efficient way of doing them?
- What particular structures of addition, subtraction, multiplication and division are revealed in these problems?
- If you gave these problems to a group of students, what sort of discussions might you have with them to help them see the structures involved?

**Key Point**: Tackling calculations by stepping back and thinking about the arithmetical structures which you might use to arrive at an efficient and elegant solution is sometimes called ‘structural arithmetic’.

By thinking of calculations in this way, students can become aware of the laws of arithmetic (e.g. the commutative, distributive, and associative laws) and the notions of additive and multiplicative identity and inverse – all important ideas in the development of number sense at Key Stage 3.

**Activity 2**: Work on each of the above calculations in turn and devise some diagrams which reveal the general arithmetic structure which is behind the calculation. For each diagram, show how this might be represented using algebraic symbols.

Note to facilitator: You may wish to show your colleagues the picture below, if they require a prompt * (available as a handout 1 or animated slide in the PowerPoint file) showing how the sum of two numbers remains the same when the same amount is added to one of the numbers and subtracted from the other. Alternatively, encourage them to come up with their own diagrams.



**Key Point**: By analysing calculations and the structures behind them, students can be introduced to algebra in a meaningful way.

They will see that algebra is generalised arithmetic, that the symbols stand for numbers, and that the various algebraic manipulations that we employ in algebra are the same as the manipulations that apply to numbers.

For example, $(a + c) + (b - c) = a + b + c - c = a + b$ indicates the general principle that might be used to solve the first question ($\square + 17 = 15 + 24$).

**Activity 3**: Use handout 2 - Calculations and work on the calculations together.

**A**
Calculate 25 × 16 × 125

**B**
Calculate 0.62 × 37.5 + 3.75 × 3.8

**C**
Calculate $\frac{2.25 \times 1.3}{39 \times 0.25}$

**D**
Do these calculations:
a) (7 × 6) + (7 × 4)
b) (4 × 9) + (9 × 6)
c) $8^2$ + double 8
d) $3^2$ + (7 × 3)
What do you notice?; why does this happen?

**E**
5542 ÷ 17 = 326. Explain how you can use this fact to find the answer to 18 × 326
(2016 Key Stage 2 Mathematics Paper 3: reasoning question 21)

**Discussion**

- Discuss the methods you have used, any connections that you see between the different examples, and the potential for using these tasks in lessons to explore certain arithmetic structures and to generalise the results with algebraic symbols.

**Key Point**:
Question A is solved more fluently (and elegantly) by noticing that 16 can be written as 4 × 4 and these 4s can be distributed between the 25 and the 125 to give 100 × 500. The associative law [$a \times bc = ab \times c$] is evident here.

In question B, use can be made of the associative law again (changing 0.62 × 37.5 into 6.2 × 3.75 and then applying the distributive law.

Question C relies on seeing multiples of 0.25 and 1.3 in the numerator and denominator respectively for a quick solution and the technique of ‘cancelling’ can be understood in terms of removing a common factor and realising that, for example, $\frac{1.3}{1.3} = 1$ (the multiplicative identity).

Question D shows some particular examples of ‘collecting like terms’ in algebra (i.e. 7a + 3a = 4a + 6a = 2a + 8a = 6a + 4a = 10a), which is also, in fact, the distributive law [ $a \times (b + c) = ab + ac$ ].

Finally, question E requires students to change the initial division to an equivalent multiplication and then apply the distributive law.

**Possible next steps**
This session may have surfaced some more long-term developments that you and your department (or group of teachers you are working with) wish to take. This section offers a way of doing this at some point in a future session or series of sessions.

Have a look at *‘Core Concept 2.1: Arithmetic procedures’* from the NCETM Secondary Mastery Professional Development Materials Theme 2.

In particular, look at the key ideas in 2.1.2 *‘Understand and use the structures that underpin multiplication and division strategies’* (pp10-11) and discuss:

- how these ideas might influence your own teaching of number and calculation in KS3
- how these ideas might influence your own teaching of algebra in KS3
- how these ideas might support developments in your scheme of work.