MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.20 Algorithms: column addition

Teacher guide | Year 3

Teaching point 1:

Any numbers can be added together using an algorithm called '*column addition*'.

Teaching point 2:

The digits of the addends must be aligned correctly before the algorithm is applied.

Teaching point 3:

In column addition, the digits of the addends are added working from the least significant digit (on the right) to the most significant digit (on the left).

Teaching point 4:

If any column sums to ten or greater, we must '*regroup*'.

Teaching point 5:

The numbers within each column should be added in the most efficient order.

Overview of learning

In this segment children will:

- understand that we can add any numbers using an algorithm called '*column addition*'
- move from the use of quantity-value representations of place-value (e.g. Dienes) supporting additive calculation, to a column representation of place-value for additive calculation
- learn how to apply the column addition algorithm, including layout conventions of column addition (correct alignment of the digits of the addends) and working from the least significant digit to the most significant digit
- understand that when the digits in any column sum to ten or more, there is a need for regrouping.

This segment builds on previous work on the concepts of place value, addition using horizontal written, as well as mental, strategies, and partitioning. To gain efficiency when using column methods, children should already have mastered addition of two (or more) single digits using appropriate strategies. The segment introduces the idea that we can add any numbers using column addition, although, as the segment progresses, teachers should begin to discuss whether the column method is an efficient choice, and when other methods may be more suitable, based on the numbers involved.

Dienes are a familiar representation for children, and will support them in learning the correct layout for the column addition algorithm. However, Dienes emphasise a different understanding of place value compared to column methods – for example, a Dienes representation of forty-three shows forty ones and three ones (40 and 3), while a column representation (for example, a place-value chart or column algorithm) of forty-three shows four tens, indicated by a '4' in the tens column, and three ones. While children have done a lot of work on understanding place-value (see, for example, segment *1.17 Composition and calculation: 100 and bridging 100*), referring to whole Dienes tens rods with the language of unitising helps to move children towards the purely column understanding of place value which underpins column addition – for example, with the Dienes we might have four rods plus two rods is equal to six rods, representing 'four tens plus two tens is equal to six tens', this same calculation is represented in the column algorithm as four plus two is equal to six in the tens place ($4 + 2 = 6$, with each digit representing the number of tens, rather than $40 + 20 = 60$).

Once children have mastered applying the column addition algorithm without regrouping (no column sums greater than or equal to ten; *Teaching point 3*), they will build on their understanding of the equivalence between ten ones and one ten, and between ten tens and one hundred, to identify when regrouping is necessary, and how it is recorded (*Teaching point 4*).

In *Teaching point 3*, children are taught to add the digits of the addends working from the rightmost column to the leftmost column. At this stage there is no mathematical necessity for this; however, in *Teaching point 4*, when children apply the algorithm to calculations for which regrouping is required, it is important to work from the rightmost column, and it can be worthwhile to ask children whether they can now explain *why* we work from the right to the left.

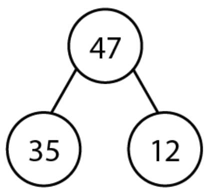
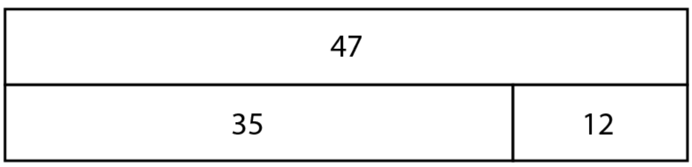
By the end of the segment children should have mastered addition of two or more two- and three-digit numbers using the column algorithm, as well as understanding when it is appropriate to use this method, and how to sense-check their answers. There is a focus on depth, and on the need for children to fully understand the underlying mathematics when they perform column addition.

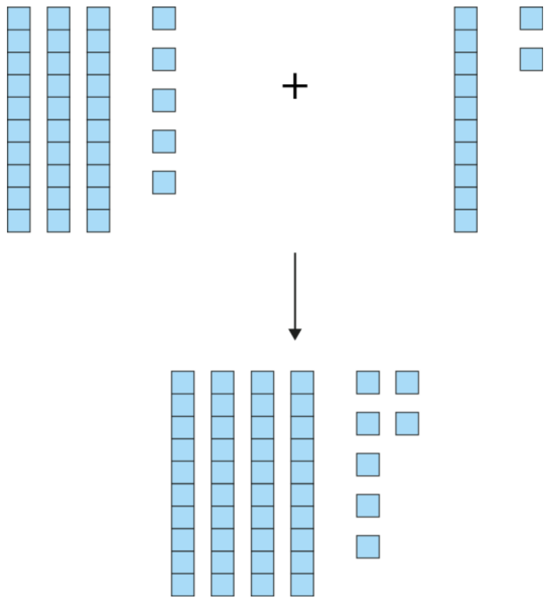
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Any numbers can be added together using an algorithm called '*column addition*'.

Steps in learning

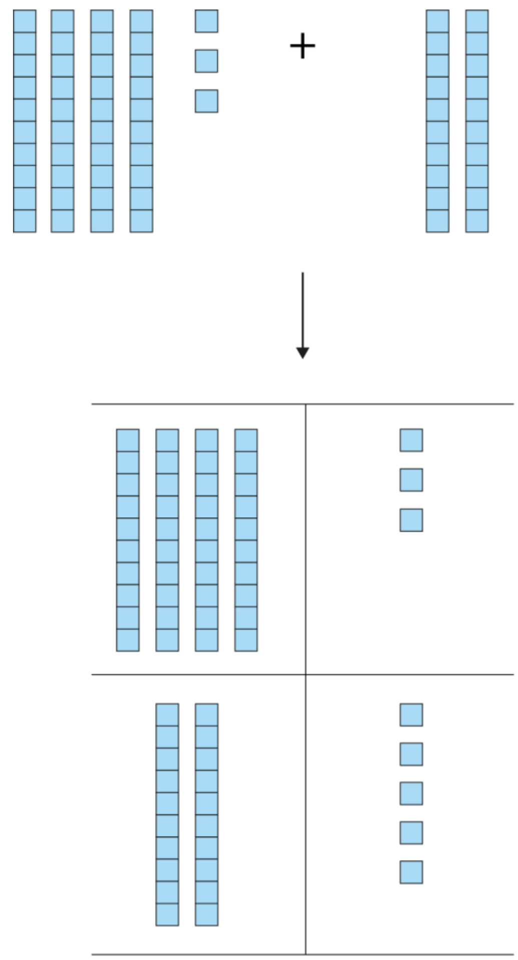
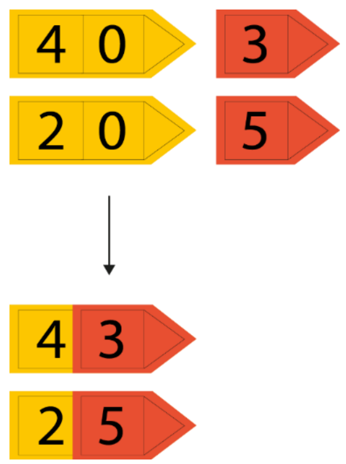
	Guidance	Representations
1:1	The purpose of this teaching point is for children to be able to recognise the addends and sum in a column addition calculation. The alignment of the digits will be covered in *Teaching point 2*. Before beginning work on column addition, it is important to ensure that children have already mastered writing equalities in the horizontal format, and that they are confident in using a range of mental strategies for addition (see segment *1.19 Securing mental strategies: calculation up to 999*). Review and encourage children to use the language of '***addend plus addend is equal to the sum***' or '***addend plus addend plus addend is equal to the sum***' (and so on for more addends); make sure that children can identify the addends and sum in a range of horizontally written calculations.	35 + 12 = 35 + 10 + 2 = 45 + 2 = 47 **35 + 12 = 47**
1:2	Begin to introduce the concept of column addition by working towards the identification of the addends and the sum in a column calculation. Use familiar representations of a given calculation alongside the column addition layout so that children see the relationship between the numbers. Familiar representations include: • part–part–whole (cherry or bar-model) • Dienes. To facilitate comparison of the representations, ask children: • *'What's the same?'* • *'What's different?'*	Part–part–wholes:  

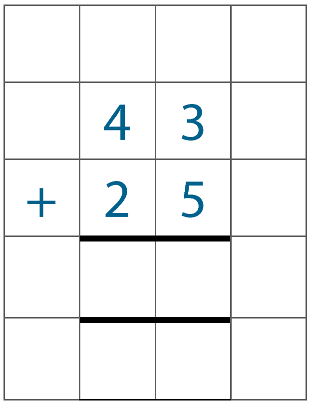
	• *'Which numbers are the addends?'* • *'Which number is the sum?'* To help children identify the sum in column addition, draw attention to the 'large equals symbol' that frames the sum.	Dienes:  Column addition: $\begin{array}{r} 3\ 5 \\ +\ 1\ 2 \\ \hline 4\ 7 \\ \hline \end{array}$
1:3	Repeat the comparison of representations for a range of two-digit, two-addend calculations. Then follow the same process for calculations involving the addition of three of more numbers (of varying digit size) to avoid children forming the preconception that column addition is solely used for adding two two-digit numbers.	

Teaching point 2:

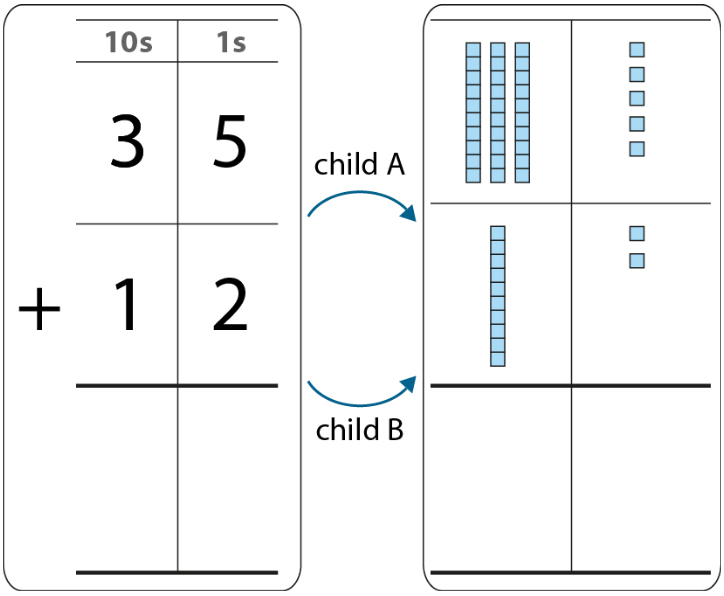
The digits of the addends must be aligned correctly before the algorithm is applied.

Steps in learning

	Guidance	Representations
2:1	In this teaching point, to keep the focus on the layout of the addends, show only the addends in the calculations, and not the sum. Begin by reminding children that, when we add using the strategy of partitioning both addends into tens and ones, we add like values together (tens with tens, ones with ones). Illustrate this with Dienes, using the language of unitising in preparation for the column layout, for example: • *'We add the ones; three ones plus five ones.'* • *'We add the tens; four tens plus two tens.'* Present the Dienes representation of the addends rearranged into tens and ones columns as shown opposite, but make sure that the columns are not labelled with place-value headings as this results in incorrect representation of value (for example, four tens Dienes rods placed in a column labelled '10s' would represent a value of forty tens, i.e. 400). Represent the same calculation using arrow cards, showing the 'tens' and 'ones' cards separately, then sliding the 'ones' cards over the top of the zeros on the 'tens' cards. Finally, show the same calculation laid out for column addition. Ask children to describe what each digit in the algorithm represents: • *'The three is in the ones column – it represents three ones; the five is in the ones column – it represents five ones.'*	Dienes:  Arrow cards: 

	• *'The four is in the tens column – it represents four tens; the two is in the tens column – it represents two tens.'* Present a range of calculations, with two two-digit addends, in this way. Although not essential until now, you should encourage children to form digits at a consistent size. You can use squared-paper exercise books, with a 'one digit in one square rule'. Sometimes the underdevelopment of children's fine motor skills (and therefore presentation of calculations) can lead to errors. If you are working with children who encounter these difficulties, they should still access the concept of column addition, and the significance of aligning the digits, by recording calculations using number cards.	Column addition on squared paper: 

2:2	Give children practice moving between representations until they are confident with the layout of the addends in column addition. You can provide children with pre-printed columns (without place-value headings for the Dienes, as discussed above) to arrange the Dienes or digits on; they can work in pairs, as illustrated opposite, describing the alignment of the digits using the following stem sentences (based on the language introduced in step *2:1*):
	• For Dienes:
	  • ***'We line up the ones; ___ one(s) plus ___ one(s).'***
	  • ***'We line up the tens; ___ ten(s) plus ___ ten(s).'***
	• For the column addition calculation:
	  • ***'The ___ is in the ones column – it represents ___ one(s); the ___ is in the ones column – it represents ___ one(s).'***	

	• ***'The ___ is in the tens column – it represents ___ ten(s); the ___ is in the tens column – it represents ___ ten(s).'*** Note: although the 'printed' cards opposite show space for the answers, for now ensure children focus only on laying out the addends correctly. The same cards can be used later when children progress to calculating the sum (see step *3:2*).	Child B writes out the calculation and says: • *'The five is in the ones column – it represents five ones; the two is in the ones column – it represents two ones.'* • *'The three is in the tens column – it represents three tens; the one is in the tens column – it represents one ten.'* Child A arranges the Dienes and says: • *'We line up the ones; five ones plus two ones.'* • *'We line up the tens; three tens plus one ten.'* 
2:3	Repeat steps *2:1* and *2:2*, now for calculations with: • addition of three two-digit numbers • addition of two or more three-digit numbers • cases where some of the digits are zero; ask children to notice what happens in these cases – they should recognise that there is 'nothing' in the column and the zero is a placeholder • addition of two numbers with different numbers of digits (e.g. a two-digit and a three-digit number, or a one-digit and a two-digit number)	

- examples where the smaller number is at the top of the column addition (such that children don't mistakenly believe that the larger number always comes first).

2:4

To increase depth of understanding, present children with a calculation laid out correctly and incorrectly. Ask children:

- *'Which of these calculations are laid out correctly?'*
- *'Which ones are not?'*
- *'What problems might this cause?'*

It may become more apparent to children, as the segment progresses, why the digits need to be aligned. This is particularly pertinent when totalling addends with different numbers of digits.

You can use a place-value chart to aid the alignment, as shown opposite.

'Which place-value chart correctly shows thirty-five plus twelve?'

100s	10s	1s
	3	5
1	2	

100s	10s	1s
	3	5
	1	2

'Which place-value chart correctly shows three hundred and five plus forty?'

100s	10s	1s
3	0	5
	40	

100s	10s	1s
3	0	5
	4	0

Which calculation correctly shows four hundred and ninety-two plus twenty-seven?

	4	9	2
+	2	7	

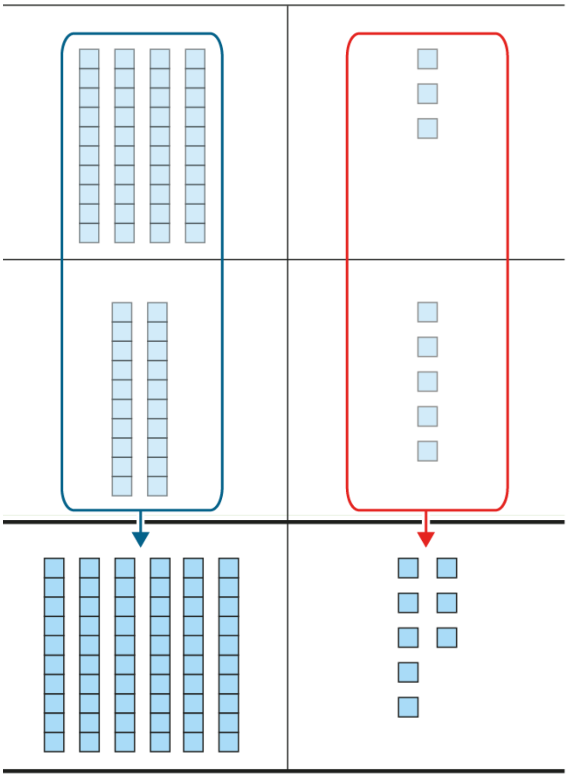
		2	7
+	4	9	2

Teaching point 3:

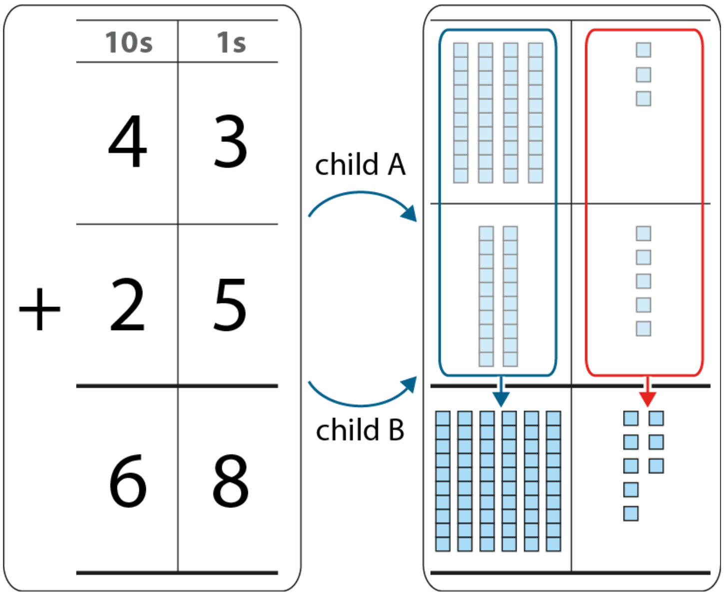
In column addition, the digits of the addends are added working from the least significant digit (on the right) to the most significant digit (on the left).

Steps in learning

	Guidance	Representations

3:1	Now move on to applying the column addition algorithm to find the sum. Throughout this teaching point only use calculations that require no regrouping (i.e. no column totals greater than nine).
	The language you use to describe application of the algorithm is important: it is common for children to be asked to memorise *'start with the ones/units column'*, but this leads to misconceptions. The statement is true when children first start to use column methods to add whole numbers, but when they later come to add decimal numbers they will not be starting with the ones column but sometimes the tenths, hundredths or thousandths column. A more useful and accurate generalised statement to use is:
	'In column addition, we start at the right-hand side.'
	Beginning with the addition of two two-digit addends, use Dienes laid out as for a column addition calculation, alongside the abstract representation. When using manipulatives, move all of the pieces in a particular column down into the answer space to form the sum for that column. Ensure that the manipulatives are used to highlight the structure, rather than as a tool for calculation; children should use known facts to find the sum for each column.
	As before, use the language of unitising to describe the addition of the Dienes, and encourage children to
		
		   4 3
		+ 2 5
		   6 8

	describe what each column in the abstract representation represents:
	• For Dienes:
	  ◦ *'Three ones plus five ones is equal to eight ones.'*
	  ◦ *'Four tens plus two tens is equal to six tens'.*
	• For the column addition calculation:
	  ◦ *'The ones column represents three ones plus five ones is equal to eight ones.'*
	  ◦ *'The tens column represents four tens plus two tens is equal to six tens.'*	

3:2	Give children practice completing some two-digit additions with the column algorithm, using Dienes for support. They can again work in pairs, as in step *2:2*, describing the addition of digits in each column using the following stem sentences (based on the language introduced in step *3:1*):
	• For Dienes:
	  ◦ ***'___ one(s) plus ___ one(s) is equal to ___ ones.'***
	  ◦ ***'___ ten(s) plus ___ ten(s) is equal to ___ tens'.***
	• For the column addition calculation:
	  ◦ ***'The ones column represents ___ one(s) plus ___ one(s) is equal to ___ ones.'***
	  ◦ ***'The tens column represents ___ ten(s) plus ___ ten(s) is equal to ___ tens.'***
		Child B writes out the calculation and says:
		• *'The ones column represents three ones plus five ones is equal to eight ones.'*
		• *'The tens column represents four tens plus two tens is equal to six tens.'*
		
		Child A arranges the Dienes and says:
		• *'Three ones plus five ones is equal to eight ones.'*
		• *'Four tens plus two tens is equal to six tens.'*
		
		

3:3	Once children understand how the algorithm works, remove the concrete apparatus. This should be done over a relatively short period of time, since the manipulatives only act as an aid to laying out the calculation correctly and a link to earlier additive work; moving children to only the abstract representation early helps to avoid them using the manipulatives as a tool for calculation.	
3:4	Repeat steps *3:1–3:3*, now for calculations with: • addition of three two-digit numbers • addition of three-digit numbers • cases where some of the digits are zero • addition of two numbers with different numbers of digits (e.g. a two-digit and a three-digit number, or a one-digit and a two-digit number) • examples where the smaller number is at the top of the column addition (such that children don't mistakenly believe that the larger number always comes first).	
3:5	To complete this teaching point, present varied practice for column addition without regrouping, including: • completing column addition calculations, including different combinations of numbers as listed in step *3.4* • laying out a given calculation for column addition • real-life problems, including measures contexts, for example: ◦ *'There are one hundred and seventy-two books in the book corner, and five hundred and sixteen books in the library. How many books are*	*'Complete the calculations.'* $\begin{array}{r} 32 \\ +\ 57 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 462 \\ +\ 205 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 335 \\ +\ 42 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 42 \\ 14 \\ +\ 33 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 123 \\ 432 \\ +\ 204 \\ \hline \\ \hline \end{array}$ $\begin{array}{r} 321 \\ 54 \\ +\ 120 \\ \hline \\ \hline \end{array}$

there all together?' (aggregation)

- *Sally has one pound and fifty-three pence, then her mum gives her another thirty-two pence. Then Sally finds twelve pence more. How much money does she have now?* (augmentation)

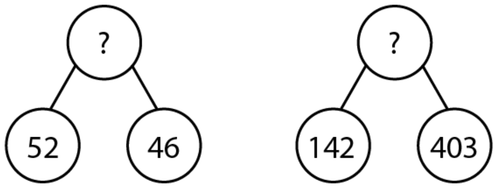
Initially, encourage children to draw part–part–whole diagrams to facilitate interpretation of the word problems, before expressing them as column addition calculations.

It is worth setting the expectation that children can incorporate simple conversions with which they should already be familiar, as illustrated in the second example word problem above. At this stage, children are not expected to write measures as decimals (for example, they have £1 which is equivalent to 100 p and another fifty-three pence, rather than £1.53). Application of the column algorithm to addition of numbers with tenths and hundredths, and money quantities written with a decimal point, will be covered in later segments.

To promote depth of understanding present dòng nǎo jīn problems such as those shown opposite. In each case ask children:

- *'What could the missing number be?'*
- *'What could it not be?'*
- *'How do you know?'*

'Write these as column addition calculations.'

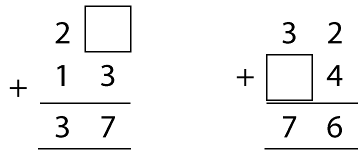


635 + 24

326 + 441 + 210

532 + 43 + 114

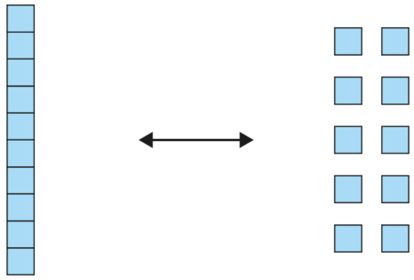
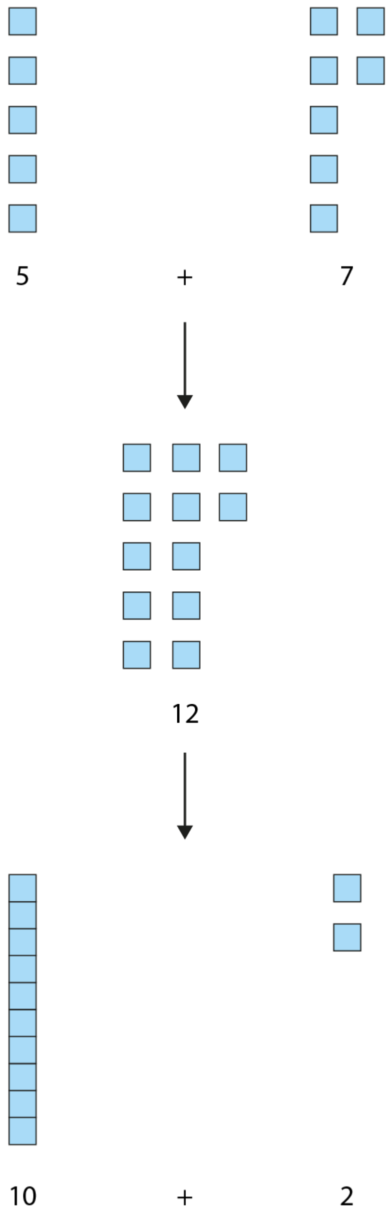
Dòng nǎo jīn:



Teaching point 4:

If any column sums to ten or greater, we must '*regroup*'.

Steps in learning

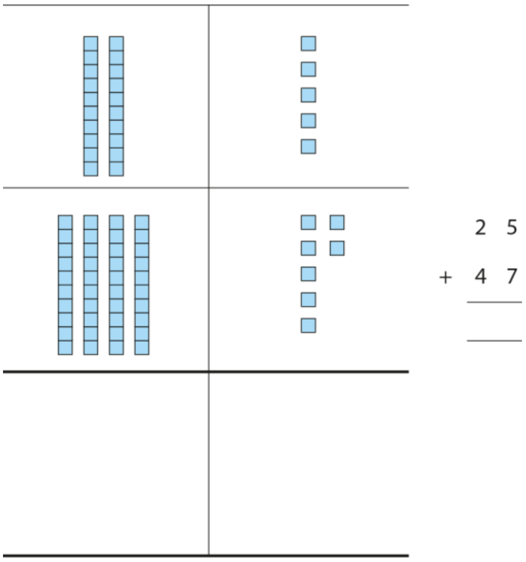
	Guidance	Representations
4:1	It is probable that children will take longer to master this teaching point than the previous ones in the segment. Begin by reviewing and extending previous work on grouping ten ones into one ten using manipulatives, such as Dienes.	
4:2	Spend some time performing addition calculations, and then regrouping, outside of the context of column addition, using Dienes. Encourage children to describe the process in full sentences using the language of unitising: • *'Five ones plus seven ones is equal to twelve ones.'* • *'Twelve ones is equal to one ten and two ones.'*	

4:3 Once it is clear that children have mastered the previous step, you can begin applying the column addition algorithm with regrouping in the ones column, demonstrating a consistent approach for recording the regrouping. You may wish to begin with an example in which the ones digits sum to exactly ten, before moving onto more general calculations with regrouping.

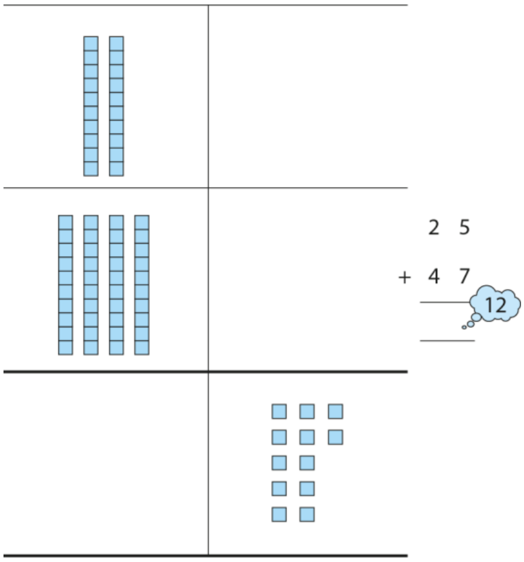
As in *Teaching point 3*, initially use Dienes alongside the abstract representation to help bridge the steps between concepts, and continue to use the language of unitising in ones and tens.

Encourage children to reason *why* a total of ten in the ones column should be exchanged for one ten and placed in the tens column, referring back to the previous steps. Work towards use of the generalised statement: ***'If the column sum is equal to ten or more, we must regroup.'***

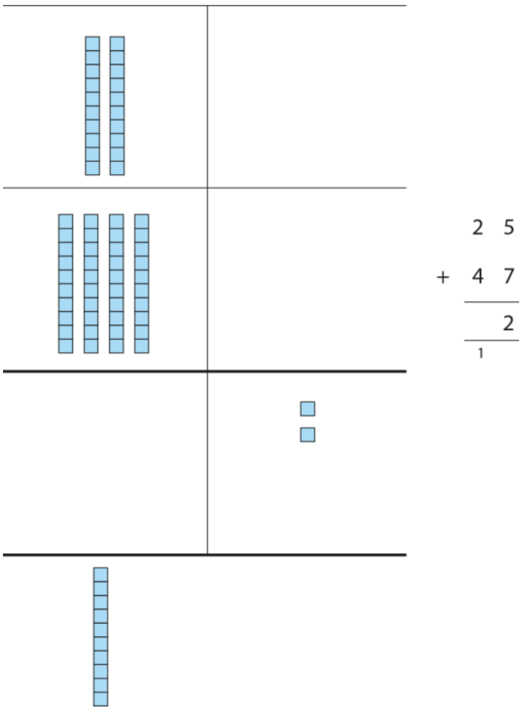
Step 1



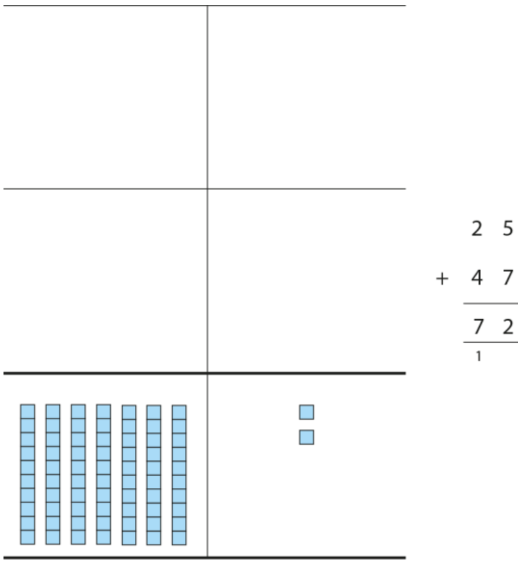
Step 2



Step 3



Step 4



4:4	Repeat steps *4:1–4:3* for regrouping ten tens into one hundred.	
4:5	Once children have experienced regrouping of both ones and tens, extend to include examples for which: • both the tens and ones columns require regrouping (both due to the original number values, e.g. *426 + 397*, or where regrouping of the tens is 'caused' by regrouping of the ones, e.g. *148 + 253*) • there are several addends which add up to a number greater than 20 in a given column (e.g. *18 + 36 + 29*) so children don't begin to believe that you can only 'carry a one' • the addends are two-digit numbers that sum to greater than 100; here we begin with only two columns in the algorithm, but end up with three in the sum, e.g. *87 + 42 = 129*.	
4:6	To promote depth, present children with a range of calculations and ask questions such as: • *'Which calculations require regrouping?'* • *'Which calculations require regrouping only once?'* • *'Which calculations require regrouping twice?'* Encourage children to look at the numbers involved and justify their answers in that way, rather than actually performing the calculations. Also present the following dòng nǎo jīn problems: *'Write down a column addition calculation that:* • *requires regrouping in the ones* • *requires regrouping in the tens* • *has a carrying digit of two in the ones.'*	124 + 233 = ____ 644 + 172 = ____ 366 + 277 = ____ 579 + 221 = ____ 791 + 163 = ____ 567 + 233 = ____

4:7	Provide varied practice similar to that in step *3:6*, but now with regrouping. Include calculations for which: • either the tens or ones columns require regrouping • both the tens and ones columns require regrouping • there are several addends that add up to a number greater than 20 in a given column • the addends are two-digit numbers that sum to greater than 100.	

Teaching point 5:

The numbers within each column should be added in the most efficient order.

Steps in learning

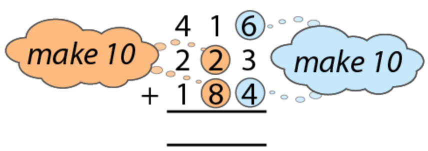
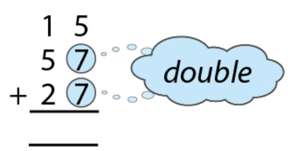
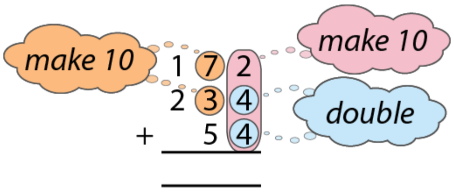
	Guidance	Representations
5:1	Before thinking about the order in which to add the digits in a column, it is worth spending some time thinking about when column methods are *not* necessary, or are *not* the most efficient method. In earlier segments children developed a range of mental strategies for addition, and these should not be abandoned in favour of always using column addition. For example, to calculate *180 + 20*, it would be more efficient for children to recognise bonds to 100 and calculate mentally, than to use the column algorithm. This is something to bear in mind generally as you continue to teach 'post column addition'. It is also important for children to maintain, and keep practising, mental addition since they will need these skills in everyday life. Here, spend some time examining different calculations, discussing for each whether it would better to use column addition, or to use a known mental strategy. Encourage children to explain their reasoning.	*'Which of these calculations are best suited to a column method and which could be better calculated mentally?'* 164 + 36 237 + 156 349 + 84 120 + 130
5:2	As already mentioned, when adding the digits in a given column, children should be using known facts and strategies, rather than relying on 'counting-on' methods. Now present some calculations with more than two addends, and ask children to choose the order in which to add the digits in each column, explaining their reasoning. Encourage use of efficient strategies such as:	 

Table for step 5:1 (Representations):

Use column addition	Use mental strategies

	• identifying when there are two addends that sum to ten • identifying doubles and near doubles. As shown opposite, begin with calculations in which the focus is on a single strategy, and then begin to combine strategies.	
5:3	Children should also be able to use known 'rules' to quickly check for errors. Present children with correct and incorrect calculations, asking them to check and spot the mistakes. Scaffold the discussion in order to help children to justify their answers; for example, ask *'Why do you know this calculation is incorrect?'*, and look for answers such as those shown opposite.	6 5 0 + 2 7 5 9 **3** 5 ✗ 1 *'The sum of two odd numbers is always an even number, so this can't be correct.'* 6 5 0 + 2 7 5 9 2 **0** ✗ 1 *'When zero is added to a number, the number remains the same, so this can't be correct.'*
5:4	Children should also be encouraged to estimate approximate answers as another way of sense-checking their answers.	*'Match each calculation to the correct estimated answer.'* 1 2 6 + 2 4 3 _____ just over 600 2 3 9 + 1 6 4 _____ between 300 and 400 1 8 7 1 4 3 + 1 4 9 _____ between 450 and 500 4 0 2 + 1 3 0 7 7 _____ between 450 and 500

5:5

To complete the teaching point, present varied practice, including calculations with and without regrouping, including:

- missing-number calculations, with variation in terms of the position of the missing number(s)
- calculations not already laid out as column addition (represented horizontally or as part–part–whole diagrams), including some problems for which column addition may not be the most efficient strategy; encourage children to always consider the addends rather than automatically writing out the calculation as column addition
- real-life problems, including measures contexts, such as the examples in steps *3.6* and *4.7*, again including some problems for which there may be a more efficient mental strategy.

Remember to include all of the following calculation types:

- addition of two-digit numbers
- addition of three-digit numbers
- addition of addends with different numbers of digits
- addition of more than two numbers
- cases where some of the digits are equal to zero.

Missing-number problems.
'Complete the calculations.'

$$\begin{array}{rrr} & 2 & 6 \\ + & 4 & 3 \\ \hline & & \\ \hline \end{array} \qquad \begin{array}{rrrr} & 3 & 2 & 9 \\ + & & 8 & 4 \\ \hline & & & \\ \hline \end{array}$$

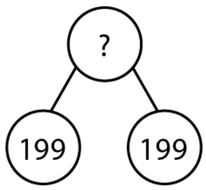
$$\begin{array}{rrrr} & 1 & 2 & 7 \\ & & 4 & 3 \\ + & & 4 & 9 \\ \hline & & & \\ \hline \end{array} \qquad \begin{array}{rrrr} & 4 & 0 & 2 \\ & 1 & 3 & 0 \\ + & & 7 & 8 \\ \hline & & & \\ \hline \end{array}$$

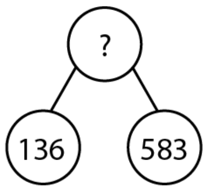
'Fill in the missing numbers.'

$$\begin{array}{rrrr} & 2 & 6 & 2 \\ + & 3 & \square & 1 \\ \hline & 5 & 8 & 3 \\ \hline \end{array} \qquad \begin{array}{rrrr} & 3 & 2 & 2 \\ + & 1 & 6 & \square \\ \hline & 4 & 9 & 1 \\ \hline \end{array} \qquad \begin{array}{rrrr} & 4 & 3 & 3 \\ + & \square & 4 & 9 \\ \hline & 6 & 8 & 2 \\ \hline \end{array}$$

$$\begin{array}{rrrr} & 1 & 2 & \square \\ + & 2 & \square & 4 \\ \hline & 3 & 4 & 6 \\ \hline \end{array} \qquad \begin{array}{rrrr} & 2 & 3 & 4 \\ + & 1 & \square & \square \\ \hline & 4 & 1 & 7 \\ \hline \end{array} \qquad \begin{array}{rrrr} & 1 & 3 & 6 \\ + & \square & \square & \square \\ \hline & 8 & 8 & 8 \\ \hline \end{array}$$

'Complete the following calculations. Choose carefully which method to use.'





175 + 25   63 + 89 + 42   50 + 250 + 300