KS3 Progression Map: Ratio, proportion and rates of change

This progression map expands upon the statements of subject content in the DfE document *Mathematics programmes of study: Key Stage 3* published September 2013. Suggested allocation of material to Years 7, 8 and 9 is given as starting points for writing schemes of work, but the implicit chronology is not intended to be prescriptive or restrictive; indeed, the programme of study is explicit that "Decisions about progression should be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content in preparation for key stage 4. Those who are not sufficiently fluent should consolidate their understanding, including through additional practice, before moving on". The NCETM fully endorses these principles, and will be developing further this progression map to help teachers achieve them.

Furthermore, although the map is organised by content, this is only for ease of reference and use. In the classroom, links between topics on the map, and between different maps, should be looked for and explored at every opportunity, so that "by the end of Key Stage 3, pupils … know, apply and understand the matters, skills and processes specified". Throughout Y7-9 pupils should have regular and opportunity and developmental feedback that helps them to **develop fluency**, to

- consolidate their numerical and mathematical capability from Key Stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]

- use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability and statistics;

to **reason mathematically**, to

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally;

and to **solve problems**, to

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

The NCETM will be developing further resources to support the development and embedding of these skills.

Year 7	Year 8	Year 9
Multiplicative relationships		
change freely between related standard units, for example: time (4 hours = 4 × 360 seconds), length (7 mm = 7 × 0.1 cm), area (9 m^2 = 9 × 10000 cm^2), volume/capacity (3 mm^3 = 3 x 0.001 cm^3) , mass (5 kg = 5 × 1000 g)	change freely between related standard units, for example speed (m per sec to km per hour and vice-versa)	change freely between related standard units, for example acceleration
express one quantity as a whole-number multiple of another, and by reversing the expression of the same relationship express one quantity as a unit fraction of another	express one quantity as a fraction of another, where the fraction is less than 1 and where it is greater than 1	given the expression of quantity A as a non-unit fraction of quantity B know immediately how to express quantity B as a fraction of quantity A
Ratio notation and number multipliers		
understand that a multiplicative relationship between two quantities that can be expressed as a ratio of the form 1 : n where n is an integer can also be expressed as the unit fraction $^1/_n$	understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction	understand that a multiplicative relationship between two quantities can be expressed as a ratio, fraction or decimal
use ratio notation, including reduction to simplest form	use ratio notation, including deriving the fraction A / (A + B) from the ratio A : B in appropriate contexts	use ratio notation to express relationships between side-lengths of right-angled triangles
use scale factors of scale diagrams and maps in everyday contexts	use scale factors when constructing similar shapes by enlargement	use scale factors when solving problems involving similar shapes
relate the language of ratios and the associated calculations to the arithmetic of fractions	relate the language of ratios and the associated calculations to gradients	relate the language of ratios and the associated calculations to linear functions
relate dividing a given quantity into two parts in a	divide a given quantity into two parts in a given	distinguish between contexts involving

given part:whole ratio to finding a fraction of a quantity; relate part:part ratios of quantities to the corresponding part:whole ratios	part:part or part:whole ratio; express the division of a quantity into two parts as a ratio	comparisons expressed using a : b notation in which the idea of ‘part’ is a helpful model and contexts in which the idea of ‘part’ is not a helpful model
Percentage change		
	solve problems involving percentage change, including: percentage increase, decrease and original value problems	solve problems involving simple interest in financial mathematics
Direct and inverse proportion		
	solve problems involving direct proportion, including graphical and algebraic representations	solve problems involving direct and inverse proportion, including graphical and algebraic representations
Compound units		
use the idea of compound units (A ‘per’ B), as in unit pricing, to solve problems	use familiar compound units, such as speed, to solve problems	use compound units, such as density, to solve problems