Additive Reasoning

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

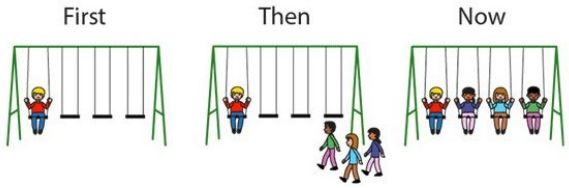
Question 4

How confident are you that you understand and can support children to explain commutative and associative laws and the importance they play in additive reasoning?

1 ☐ 2 ☐ 3 ☐ 4 ☑

How would you respond …?

a. Sarah says the augmentation structure for addition is not commutative as the story has to be in a 'first, then, now' format. Will disagrees. Who do you agree with and why?



b. How and why can the associative law of addition be applied to this question?

Responses

Note your responses to the questions here before you engage with the rest of this section:

Did you notice that…?

a. Due to the first, then, now structure of augmentation, it can be more difficult to see that commutativity applies here. However, it doesn't matter in which order the story is told, the same sum will be reached. For example:

- *'At first, there is one child on the swings and then three more arrive. Now, there are four children on the swings.'*
- *'What would happen if the children arrived in a different order?'*
- *'At first there are three children on the swings and then one more arrives. Now, there are four children on the swings.'*

The sum of the children on the swings was not changed by the order in which they arrived; this demonstrates the commutative law. Therefore, Will is right; augmentation is commutative.

b. In associative law, it does not matter in which order the numbers are added, the same sum will be reached. In this example, children may find it easier to look for the complement, reordering the calculation so that the 26 and 14 can be paired, before adding this to 533: **14 + 533 + 26 = (14 + 26) + 533 = 40 + 533 = 473**

Commutative and associative law

This section focuses on the commutative and associative laws of addition, considering how they can support children in applying the laws and developing efficient calculation strategies.

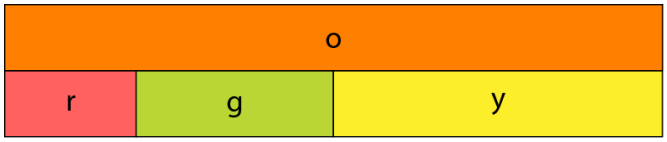
Exploration of ordering addends should lead the children to a generalisation for both the commutative and associative laws that, *'if the order of the addends is changed, the sum still stays the same'*. This understanding will allow children to manipulate the order of addends to make them easier to calculate.

For example, when there are more than two parts, the associative law can be expressed in the following ways:

o = r + (g + y)

o = (r + g) + y

o = (r + y) + g



Whichever pair of addends are added first, the whole will remain the same.

When calculating informally, people often use commutativity without being explicitly aware they have used it; it is an important strategy to develop. For example, if children are adding to calculate how long something will take, they may automatically look for minute complements to 60, using the hour as a bridge.

When presented with an expression, children can reorder the numbers, adding them together in an order that they prefer. For example, during mental calculations, it may be preferable to reorder by putting the largest number first, or to add together two numbers that complement each other, as shown below.

Example A – largest number first

36 + 252 + 14 = 252 + 36 + 14

Example B – complements to 100

172 + 48 + 28 = (172 + 28) + 48

This understanding will also help when developing mental calculation strategies, as children will be able to reorder numbers to support strategies, such as bridging through 10, near doubles or compensation.

Commutative and associative law applies to addition; however, it does not apply to subtraction. It is important that children have the opportunity to develop an understanding that subtraction is not commutative and cannot be reordered. 14 – 6 does not equal 6 – 14.

Common errors in this area may include:

- children applying the commutative and associative laws to subtraction
- children not transcribing the original numbers accurately when applying commutativity
- children reordering without a reason, making the calculation less efficient.

What to look for

Can a child:

- manipulate addition calculations, making them more efficient?
- explain how commutativity applies to both aggregation and augmentation structures for addition?
- discuss reasons why commutativity has made a calculation more efficient?

Links to supporting materials:

NCETM Primary Professional Development materials, Spine 1: Number, Addition and Subtraction:

- Topic 1.7: Addition and subtraction strategies within 10
- Topic 1.11: Addition and subtraction: bridging 10

Notes:

Key learning from support material and self-study:

What I will focus on developing in my classroom practice: