National Centre for Excellence in the Teaching of Mathematics

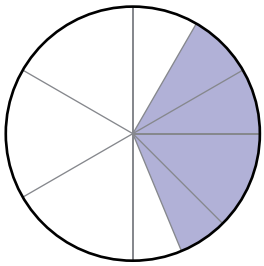
Professional development resource

Unit 1 - Deepening understanding of fractions
Lesson 1b: Pieces of a cake

Lesson summary

This lesson involves two activities in which a circular 'cake' is partitioned into a large number of equal parts or (see right) into several unequal parts which themselves are subdivided into equal parts. The purpose is to compare or combine fractions. An extension activity involves a similar task but uses the model of a double number line.

The focus is still on the part-whole view of fraction, though there are opportunities to overlay this with fraction as operator and as measure.



Focus of student learning

As with Lesson 1a, we explore students' understanding of the part-whole view of fractions.

- How familiar are students with the part-whole view of fractions?
- In estimating a part of whole, do students take account of the size of the whole (ie of the 'unit')?
- When partitioning a shape, do students appreciate the value of having equal parts?
- Where students use formal procedures, can they explain why they work?

Lesson preparation

- You might want to duplicate
  - the reference sheet for pupils for the Stage 2 task (page 11)
  - the worksheet for the Stage 3 Extension task (page 12).
- The file **UNIT-1b-slides.pdf** contains
  - slides of the Stage 1 sequence of diagrams
  - slides of the Stage 2 sequence of diagrams
  - slides of the Stage 3 Extension task diagram
  - slides of the four alternative Stage 1 diagrams.

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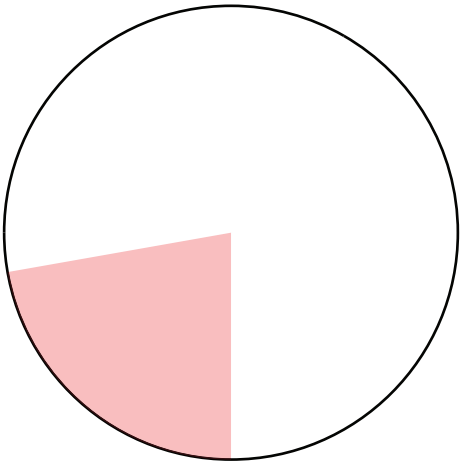
Lesson 1b, Stage 1

Stage 1: Work through this task as a class (with some pair-work if desired).

This cake is covered in white icing and pink icing:

a. Roughly, what fraction of the icing is pink?



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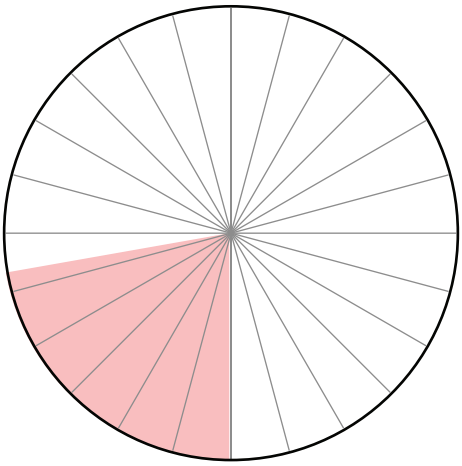
b. Karl wants to cut the cake into lots of equal pieces, in this kind of way:



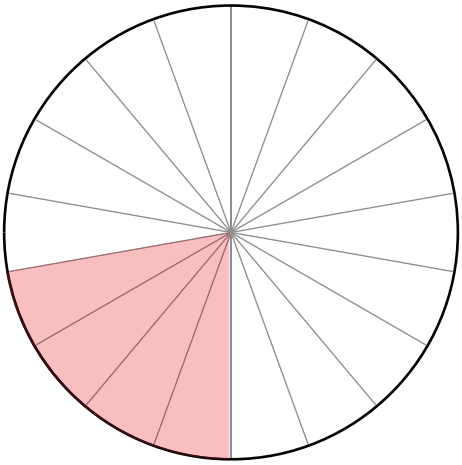
i. *Imagine* that he cuts the cake into 60 equal pieces. Roughly, what fraction of the icing is pink? Give your answer in 60ths.

.....

ii. Imagine that he cuts the cake into 100 equal pieces. Roughly, what fraction of the icing is pink? Give your answer in 100ths.

.....

c. In the end, Karl carefully cuts the cake into these equal pieces:
What fraction of the icing is pink?
Use the drawing to find the fraction as accurately as you can.



.....

d. Compare your estimates in parts a) and b) with your answer in c).

Find different ways of doing this.

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Commentary: Lesson 1b, Stage 1

It is not necessary to cover all the stages of this lesson (where practicable, omitted stages can be considered at another time). But stages should be considered in depth: be prepared to spend 10 – 30 minutes on any one stage.

a. It is likely that students will estimate the fraction as being 1/5, which is very close to its actual size of 2/9. (How close?!) It is unlikely that students will hit upon the actual size, though their later estimates might be very close, which should provide a strong incentive to consider *how* close their estimates are.

bi. The purpose of this drawing is purely to show 'lots of equal pieces' (there happen to be 24). Students are expected to *imagine* the 60 pieces rather than to draw them, in the hope that they will construct some kind of logical argument rather than over-interpret a drawing or sketch. Thus students might argue, "The pink icing will cover slightly fewer than 15 pieces (a quarter) so the fraction is about 14/60 (or 13/60, 12/60, etc)".

If students don't do so spontanously, you might want to relate the 60 pieces to minutes on a clockface. "Roughly how long would it take the minute hand of a clock to sweep over the pink region?"

bii. After a while, if students don't do so spontanously, you might ask, "What *percentage* of the icing is pink?"

You might want to use other equal partitonings, as well as or instead of 60 and 100, or ask students to come up with their own - easy or hard.

"What would we get if instead of 60 equal pieces we had 20? or 90?"

"What if instead of 100 we had 1000? or 25?"

"What if we had 4 equal pieces?"

c. Here, we can determine by counting that the cake has been cut into 18 equal pieces, 4 of which are covered by the pink icing. So the fraction is 4/18.

Some students might spontaeously reduce 4/18 to to 2/9:
"How can we tell from the diagram that these are equivalent?"

Others (or you) might suggest intermediate answers like 1/4 – 0.5/18:
"Can we use formal procedures to show that this is the same as 2/9?"

d. Students might try to compare the various estimates using 'common denominators', though this is not particularly nice for 60, 100 and 18.

Or they might use equivalent fractions whose denominators (or numerators) are *close* to one another. For example 2/9 is equivalent to 14/63 and 63 is quite close to 60, which suggests that 14/60 and 13/60 are both good estimates in part b)i. (In fact, 2/9 = 13.3recurring/60.)

Some students might argue that 14/60 = 21/90 (apply 'half as much again', or 'treble and divide by 2' to numerator and denominator), which is very close to 2/9 = 20/90.

Some might use decimals or percentages (perhaps with the aid of a calculator) to compare the fractions.

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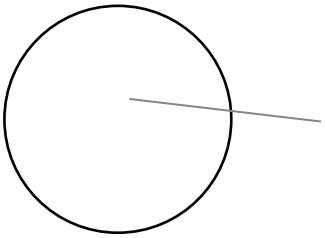
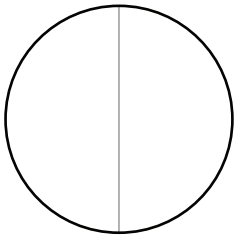
Lesson 1b, Stage 2

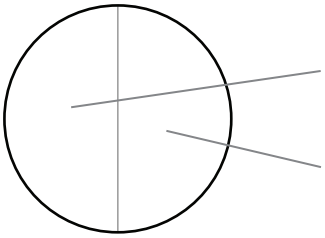
Stage 2: Work through this task as a class (with some pair-work if desired).



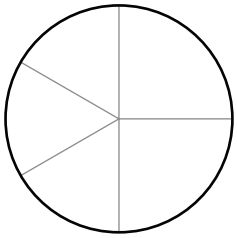
Rosa likes cutting up cakes.

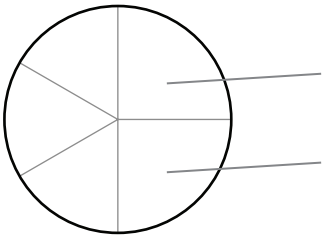
She cuts
this cake into 2 equal pieces. →





She now cuts
this piece into 3 equal pieces →
and
this piece into 2 equal pieces. →





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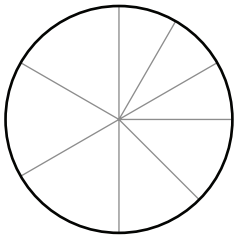

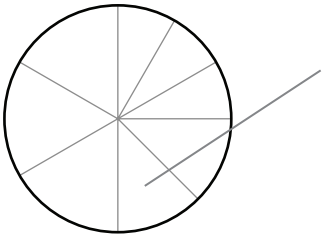

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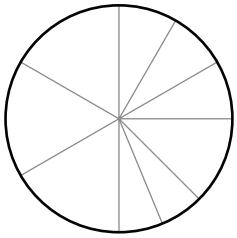
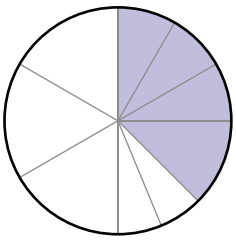


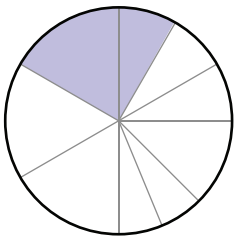
Rosa eats the part of the cake shown by the shaded region.

a. Estimate *roughly* how much of the cake she eats.

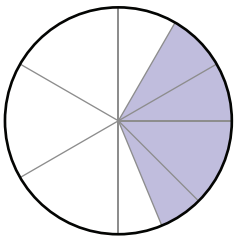
b. Find the *exact* amount.



A. Easy?



B. Hard?



C. Harder?

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Commentary: Lesson 1b, Stage 2

Stage 2: This task is about *using diagrams* to help students *mentally construct* equal parts of a whole.

Students try to name fractions and find appropriate equivalent fractions so as to express a composite region as a single fraction.

Where students resort to formal procedures, they should be asked to try to explain how these can be embodied in the diagrams.

You might want to use one or more of the composite regions A, B and C.
Or you could use other such regions or let students devise their own.

And you might want to partition the cake in other ways (simpler or more complex).

It can be illuminating to revisit the task using 'mediating' quantities such as 'price' or 'angle'.

In the case of price, a challenge is to find an 'easy' price. For example, for region A, the task is much easier if we assume the whole cake costs, say, £4 rather than £5.
A price of £8 might be easier still.

Of course, this relates to the notion of 'common denominator'.

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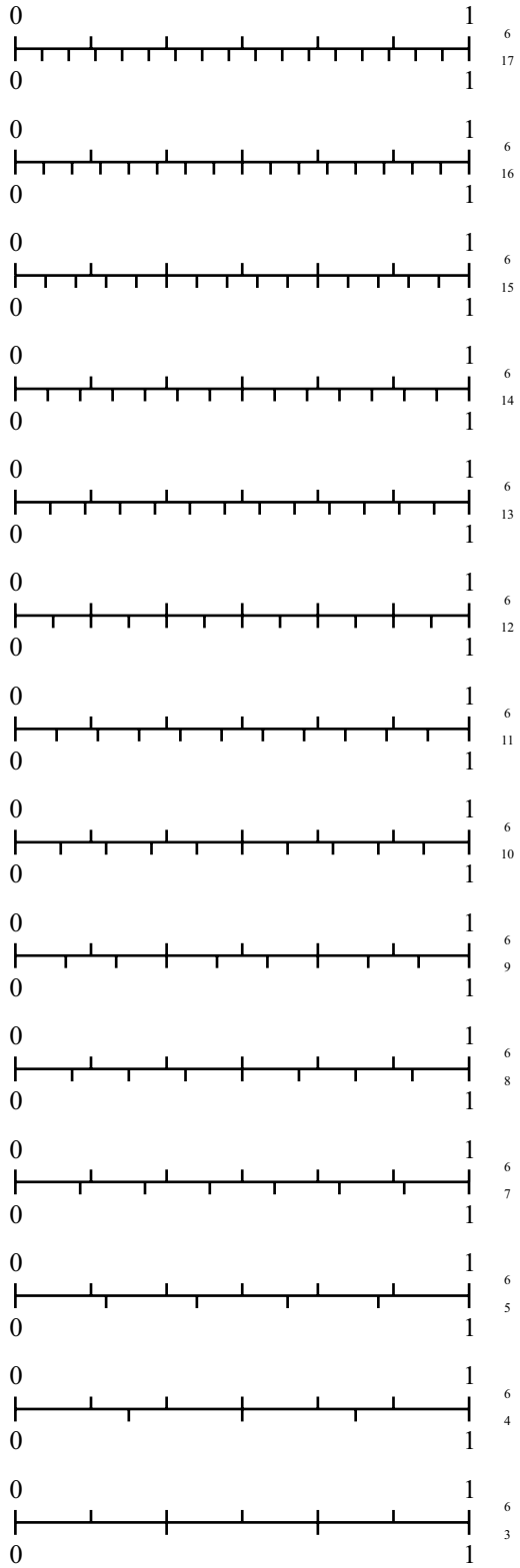
Lesson 1b, Stage 3 Extension

Stage 3, Extension:
You could work through this as a class, or set some of it as a homework task (after a brief introduction).

Each of these lines shows two sets of fractions.

For example, this line shows sixths and sixteenths:



a. Choose the line showing sixths and eighths.

Number each of the given marks (ie ⅙, ⅔, etc and ⅛, ⅜, etc).

b. Use the drawing to estimate the size of the 'smallest gap' between marks on the top and bottom scales (eg the gap between ⅚ and ⅞). .....

c. Explain how you can check your estimate.

.....

.....

d. Use the line to help you write some subtraction expressions involving sixths and eighths, for example ⅞ – ⅚ = ...
Give them to your partner to solve.

.....

.....

.....

.....

e. Repeat a. – d. for another line ...

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Commentary: Lesson 1b, Stage 3 Extension

Stage 3, Extension:

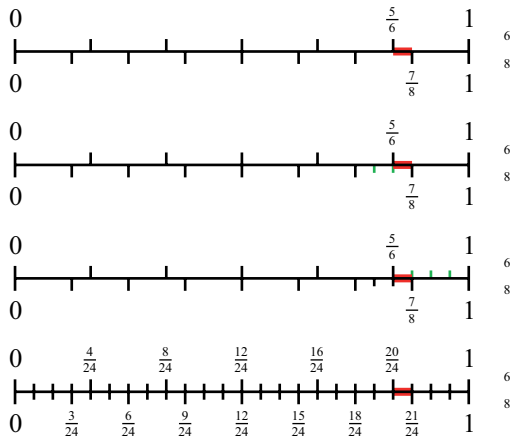
This task is intended to develop (or consolidate) the understanding of 'common denominator', although one might want to avoid the term (at least initially) since the idea is to use the diagram [a (double) number line] rather than a formal rule to the solve the task.

It can be seen from the diagram that for sixths and eighths the 'smallest gap' occurs between $\frac{1}{8}$ and $\frac{1}{6}$, between $\frac{2}{6}$ and $\frac{3}{8}$, between $\frac{5}{8}$ and $\frac{4}{6}$, and between $\frac{5}{6}$ and $\frac{7}{8}$. It turns out that this 'smallest gap' is a unit fraction of each of the given unit fractions (ie of $\frac{1}{6}$ and $\frac{1}{8}$), in this case $\frac{1}{4}$ of $\frac{1}{6}$ and $\frac{1}{3}$ of $\frac{1}{8}$. [Will this always be the case, whatever the two given unit fractions? (Is the answer surprising or obvious?!)]

It very much *looks* as though the 'smallest gap' is $\frac{1}{3}$ of $\frac{1}{8}$.

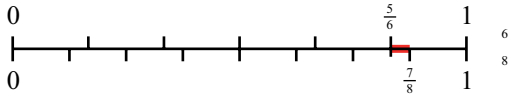
If this is true, then this means that the 'smallest gap' is also $\frac{1}{4}$ of $\frac{1}{6}$.

This gives a consistent result, since it means that this gap fits 3×8 = 24 times between 0 and 1 along the bottom scale and 4×6 = 24, which is the same number of times, between 0 and 1 along the top scale.

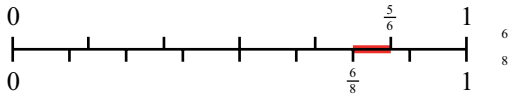


d. Here are some possible subtraction expressions (and answers). [The answers can be derived from the diagrams, which then provide scope for discussing how the diagrams embody rules/procedures involving equivalent fractions and common denominators.]

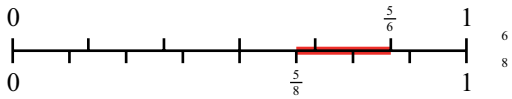
$\frac{7}{8} - \frac{5}{6} = \frac{1}{24}$



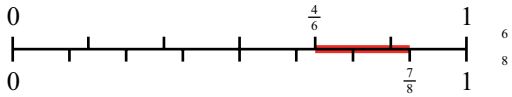
$\frac{5}{6} - \frac{6}{8} = \frac{2}{24} = \frac{1}{12}$



$\frac{5}{6} - \frac{5}{8} = \frac{1}{8} + \frac{2}{24} = \frac{5}{24}$



$\frac{7}{8} - \frac{4}{6} = \frac{2}{24} + \frac{1}{8} = \frac{5}{24}$



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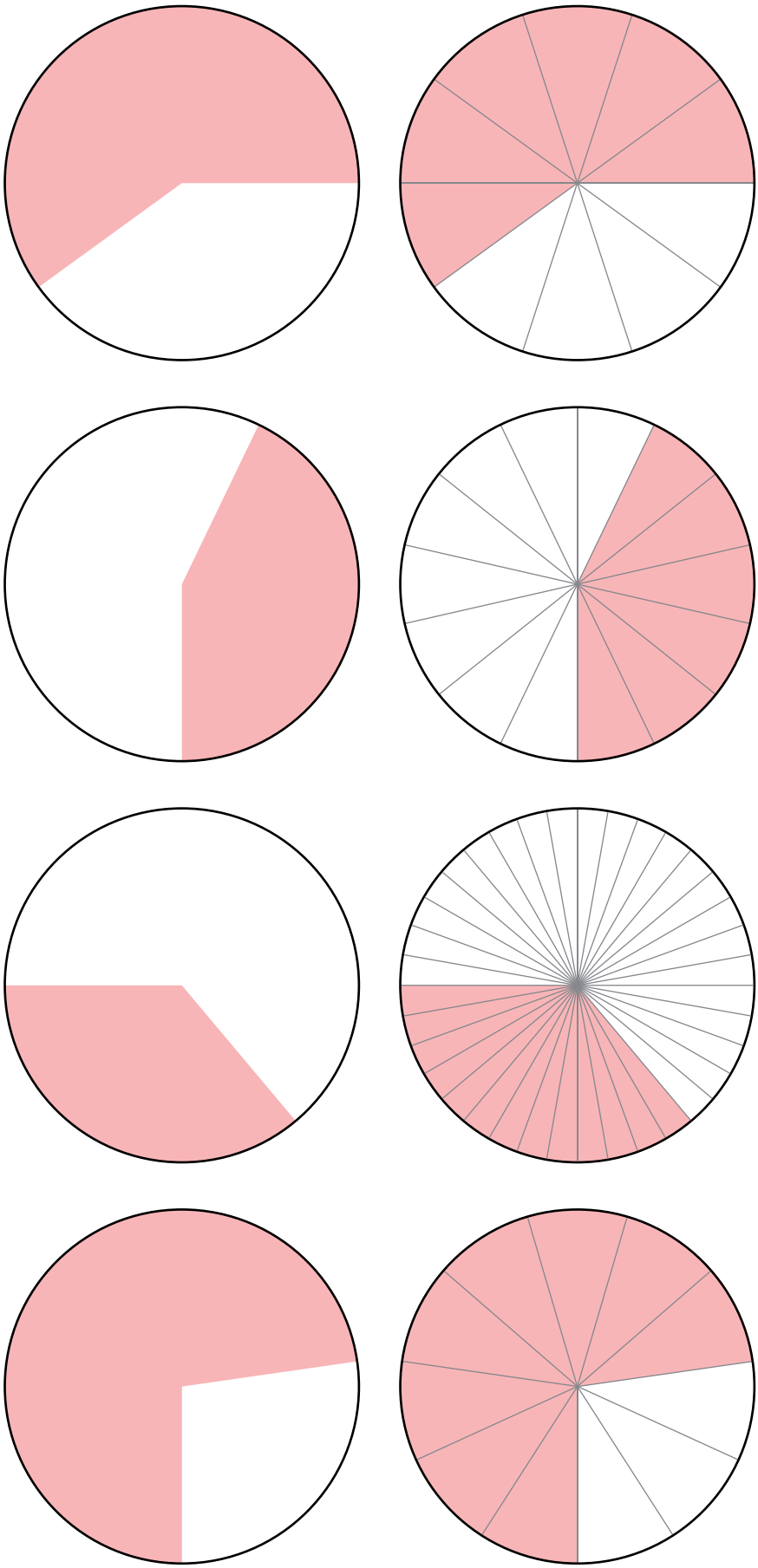
Lesson 1b, Stage 1 alternative diagrams

Stage 1, alternative diagrams:

You might want to choose one of the four fractions shown in these pairs of diagrams instead of (or as well as) the fraction (2/9) used in the Stage 1 activity.



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Commentary: Lesson 1b, Stage 1 alternative diagrams

Stage 1, alternative diagrams:

These pairs of drawings offer some alternative fractions that you might want to use.

Choose one pair of drawings.

For this pair,

Show the left-hand drawing of the cake;
estimate the fraction of icing that is pink;
imagine the cake is partitioned into a certain number of equal pieces;
express the fraction of pink icing in terms of the number of pieces;
repeat for other such imagined partitionings;
show the right-hand drawing to determine the exact fraction;
compare the various estimates to this fraction.

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Adapting the lesson

- The lesson outline and lesson commentary suggest several ways of modifying the tasks to make them more or less demanding.
- You can also vary the degree to which you focus on
  - informal estimates or on precise, analytic partitioning procedures
  - how much use is made of mediating quantities.
- Some alternative fractions for Stage 1 are shown by the pairs of diagrams on page 8.

Suggestions for lesson study

As with Lesson 1a, you might want to consider these aspects of students' thinking:

- How familiar are students with the part-whole view of fractions? Do they still make use of it?
- In estimating a part of whole, do students take account of the size of the whole (ie of the 'unit')?
- When partitioning a shape, do students appreciate the value of having equal parts?
- How flexible are students in finding alternative partitionings?
- Where students use formal procedures, can they explain why they work? In particular, can they express them using a part-whole representation?

You might also want to consider pedagogic aspects such as these:

- How did we (and how, if we teach the lesson again, could we find further ways to) elicit (and share) examples of students' thinking?
- How did we (and how, if we teach the lesson again, could we find further ways to) build on and interrogate students' thinking?

Background to the lesson

- This is a second lesson on the part-whole view of fraction. The approach is less exploratory than in the first lesson, though there is still plenty of room for informal methods. Students are presented with some already quite structured representations which they need to structure further in order to determine (or estimate) the size of a given fraction, or the size of a 'compound' fraction, or the difference between two fractions.
- The key ideas, which provide a basis for 'equivalent fraction', are that a partitioning of a whole can be transformed into an equivalent partitioning, and that we can compare parts of a whole (ie fractions) more easily when they are composed of parts of the same size.
- There is also an opportunity to make use of 'mediating quantities' such as 'cost' or 'number of minutes', which offers another (related) way of assigning a common measure to fractions so that they can be compared or combined easily. This, and the use of the (double) number line in the extension activity, brings in the ideas of fraction as operator and fraction as measure.
- Students might also represent the fractions as percentages or decimals.

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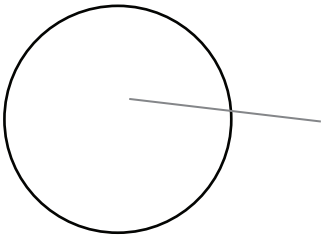
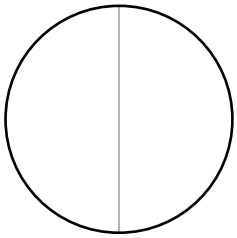
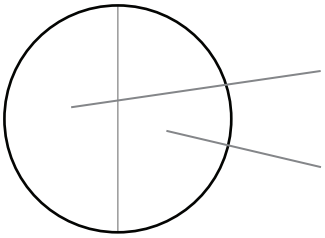
Resource: Stage 2 reference sheet



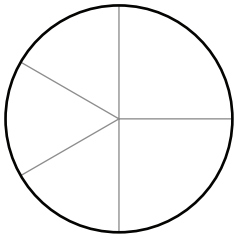
Rosa likes cutting up cakes.

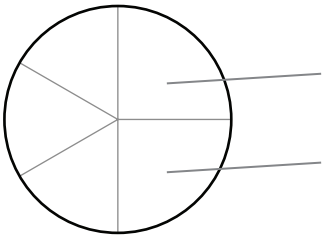
She cuts
this cake into 2 equal pieces. →





She now cuts
this piece into 3 equal pieces →
and
this piece into 2 equal pieces. →





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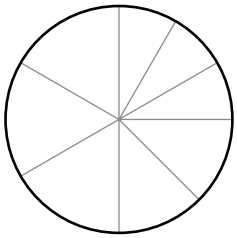

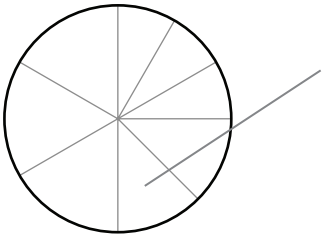

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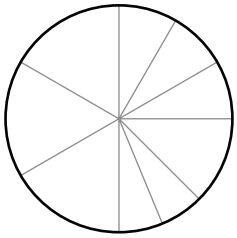
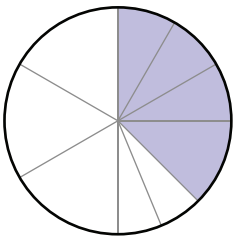


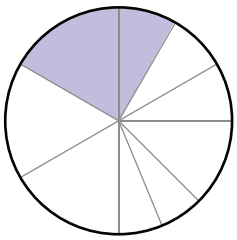
Rosa eats the part of the cake shown by the shaded region.

a. Estimate *roughly* how much of the cake she eats.

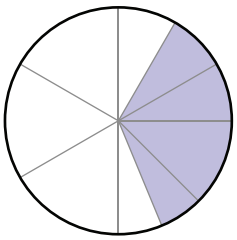
b. Find the *exact* amount.



A.



B.



C.

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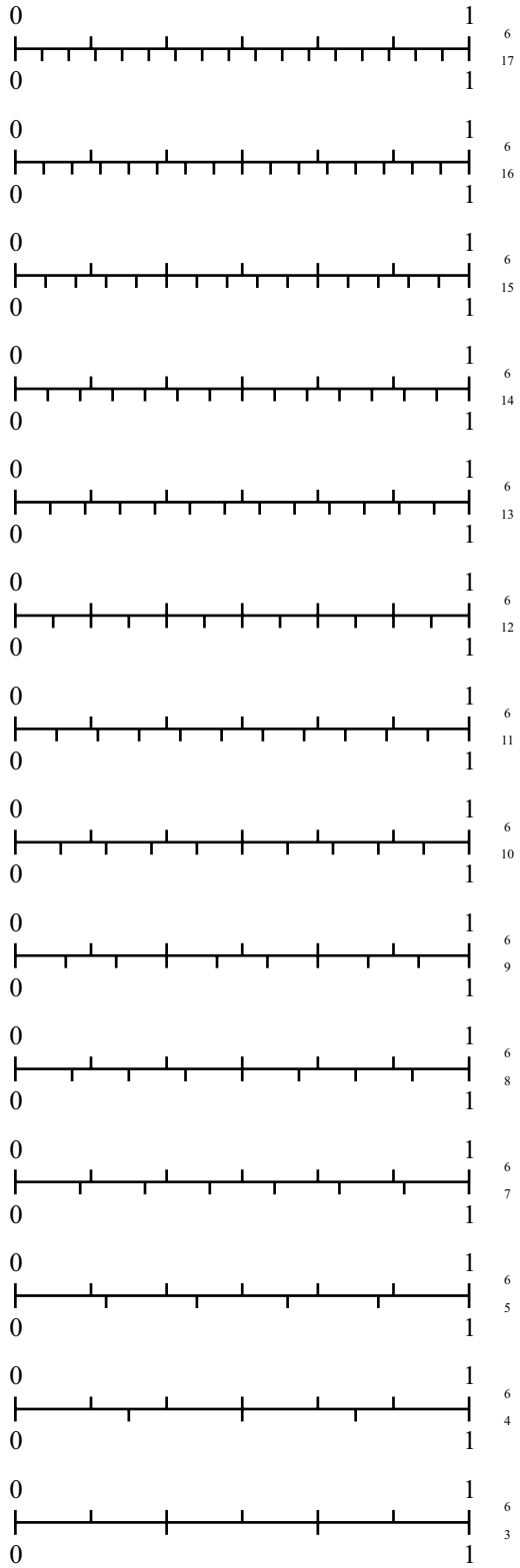
Resource: Stage 3 Extension, worksheet

Each of these lines shows two sets of fractions.

For example, this line shows sixths and sixteenths:



a. Choose the line showing sixths and eighths.

Number each of the given marks (ie ⅙, ⅔, etc and ⅛, ⅜, etc).

b. Use the drawing to estimate the size of the 'smallest gap' between marks on the top and bottom scales (eg the gap between ⅚ and ⅞). .....

c. Explain how you can check your estimate.

.....

.....

d. Use the line to help you write some subtraction expressions involving sixths and eighths, for example ⅞ – ⅚ = ...
Give them to your partner to solve.

.....

.....

.....

.....

e. Repeat a. – d. for another line.

dek