

Mastery Professional Development

5 Statistics and probability



5.3 Probability

Guidance document | Key Stage 3

Making connections

The NCETM has identified a set of six 'mathematical themes' within Key Stage 3 mathematics that bring together a group of 'core concepts'.

The fifth of these themes is *Statistics and probability*, which covers the following interconnected core concepts:

- 5.1 Statistical representations and measures
- 5.2 Statistical analysis
- 5.3 **Probability**

This guidance document breaks down core concept 5.3 *Probability* into three statements of knowledge, skills and understanding:

- 5.3.1 Explore, describe and analyse the frequency of outcomes in a range of situations
- 5.3.2 Systematically record outcomes to find theoretical probabilities
- 5.3.3 Calculate and use probabilities of single and combined events

Then, for each of these statements of knowledge, skills and understanding we offer a set of key ideas to help guide teacher planning.

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Overview

Students will encounter probability in many aspects of their daily lives, from sporting events to weather reports. However, probability is an area of mathematics that students can find perplexing. Students may feel that their lived experiences do not reflect calculated mathematical likelihoods. For example, rolling a six on a dice in order to win a board game often ‘feels’ far less likely than any of the other outcomes. The introduction of probability at Key Stage 3 will offer students a way to quantify, explore and explain likelihood and coincidence, and to reason about uncertainty.

Students should have the opportunity to engage in experiments and develop a feel for likely, unlikely, even, certain and impossible chances, before starting to quantify probabilities and the likelihood of different outcomes. An understanding of equally likely outcomes is key to this. Often, students mistakenly believe that an event with only two possible outcomes has an ‘even’ chance of happening, or that the probability of one event occurring when there are n possible outcomes is ‘one in n ’. Students should be exposed to examples of when this is true and when this is not true (for example, whether it will rain or not rain tomorrow, whether the school football team would beat Brazil if they were to play them, whether the teacher choosing a student from a class of 30 has a probability of 1 out of 30) and discuss what’s the same and what’s different about the situations.

Furthermore, students need to appreciate that predictions of likelihood do not predict individual events. Rather, experimental data will tend towards this theoretical value (for example, knowing that flipping a head or a tail on a coin has an even chance of occurring does not mean these outcomes will occur an equal number of times).

As they start to quantify outcomes, students should be exposed to different ways to systematically organise and represent possible results, including lists, tables, grids and Venn diagrams.

Specific and precise language is key to working with probability. For example, students should understand the distinctions between an event (for example, flipping a coin) and an outcome (for example, a coin landing on heads) or between probability and possibility (for example, it is possible that it will snow in summer, but not probable).

Prior learning

Before beginning to teach *Probability* at Key Stage 3, students should already have a secure understanding of the following from previous study:

| Key stage | Learning outcome |
|-------------|--|
| Key Stage 3 | <ul style="list-style-type: none"> 3.1.3 Understand that fractions are an example of a multiplicative relationship and apply this understanding to a range of contexts 3.1.4 Understand that ratios are an example of a multiplicative relationship and apply this understanding to a range of contexts <p>Please note: Numerical codes refer to statements of knowledge, skills and understanding in the NCETM breakdown of Key Stage 3 mathematics.</p> |

You may find it useful to speak to your partner schools to see how the above has been covered and the language used.

You can find further details regarding prior learning in the following segments of the [NCETM primary mastery professional development materials](#)¹:

- Year 3: 3.2 Unit fractions: identifying, representing and comparing
- Year 3: 3.3 Non-unit fractions: identifying, representing and comparing
- Year 6: 2.27 Scale factors, ratio and proportional reasoning

Checking prior learning

The following activities from the [NCETM primary assessment materials](#)² offer useful ideas for assessment, which you can use in your classes to check whether prior learning is secure:

| Reference | Activity |
|----------------|--|
| Year 6 page 20 | Put the following numbers on a number line: $\frac{3}{4}$, $\frac{3}{2}$, 0.5, 1.25, $3 \div 8$, 0.125 |
| Year 6 page 23 | You can buy 3 pots of banana yoghurt for £2.40. How much will it cost to buy 12 pots of banana yoghurt? A child's bus ticket costs £3.70 and an adult bus ticket costs twice as much. How much does an adult bus ticket cost? To make a sponge cake, I need six times as much flour as I do when I'm making a fairy cake. If a sponge cake needs 270 g of flour, how much does a fairy cake need? |

Key vocabulary

| Term | Definition |
|----------------------------------|--|
| combined event | A combined (or compound) event is an event that includes several outcomes. Example: In selecting people at random for a survey, a combined event could be 'girl with brown eyes'. |
| conditional probability | The conditional probability of an event B is the probability that the event will occur, given the knowledge that an event A has already occurred. This probability is written $P(B A)$ ('the probability of B given A '). In the case where events A and B are independent, $P(B A) = P(B)$. If events A and B are dependent, then the probability that both events occur is $P(A \text{ and } B) = P(A) \times P(B A)$. |
| dependent and independent events | Two events are said to be dependent when the outcome of one has an influence on the outcome of the other. |

| | |
|---------------------------|---|
| | <p>Example: Picking two balls from a bag of balls when the first one is not replaced; the probability of choosing the second ball will be influenced by which ball is chosen first.</p> <p>When there is no such influence, the events are said to be independent.</p> <p>Example: Picking two balls from a bag of balls when the first ball is replaced before picking the second.</p> |
| mutually exclusive events | In probability, events that cannot both occur in one experiment. When the mutually exclusive events cover all possible outcomes, the sum of their probabilities is one. |
| probability | <p>The likelihood of an event happening. Probability is expressed on a scale from zero to one. Where an event cannot happen, its probability is zero and where it is certain its probability is one.</p> <p>Example: The probability of scoring one with a fair dice is $\frac{1}{6}$.</p> <p>The denominator of the fraction expresses the total number of equally likely outcomes. The numerator expresses the number of outcomes that represent a 'successful' occurrence.</p> <p>Where events are mutually exclusive and exhaustive the total of their probabilities is one.</p> |
| sample space | The sample space is the set of all possible outcomes of a trial. The sum of all the probabilities for all the events in a sample space is one. |

Collaborative planning

Below we break down each of the three statements within *Probability* into a set of key ideas to support more detailed discussion and planning within your department. You may choose to break them down differently depending on the needs of your students and timetabling; however, we hope that our suggestions help you and your colleagues to focus your teaching on the key points and avoid conflating too many ideas.

Please note: We make no suggestion that each key idea represents a lesson. Rather, the 'fine-grained' distinctions we offer are intended to help you think about the learning journey irrespective of the number of lessons taught. Not all key ideas are equal in length and the amount of classroom time required for them to be mastered will vary, but each is a noteworthy contribution to the statement of knowledge, skills and understanding with which it is associated.

The following letters draw attention to particular features:

- D** Suggested opportunities for **deepening** students' understanding through encouraging mathematical thinking.
- L** Examples of shared use of **language** that can help students to understand the structure of the mathematics. For example, sentences that all students might say together and be encouraged to use individually in their talk and their thinking to support their understanding (for example, *'The smaller the denominator, the bigger the fraction.'*).
- R** Suggestions for use of **representations** that support students in developing conceptual understanding as well as procedural fluency.

- V** Examples of the use of **variation** to draw students' attention to the important points and help them to see the mathematical structures and relationships.
- PD** Suggestions of questions and prompts that you can use to support a **professional development** session.

For selected key ideas, marked with an asterisk (*), we exemplify the common difficulties and misconceptions that students might have and include elements of what teaching for mastery may look like. We provide examples of possible student tasks and teaching approaches, together with suggestions and prompts to support professional development and collaborative planning. You can find these at the end of the set of key ideas.

Key ideas

5.3.1 Explore, describe and analyse the frequency of outcomes in a range of situations

Before they quantify probabilities, students need to appreciate that, where an event has different possible outcomes, some of these outcomes may be more or less likely than others for different possible reasons.

One factor that underpins uncertainty is that of randomness. A key awareness for students is to understand that although an individual event might be random, reasoning about uncertain events can be fruitful when they are repeated many times. Given enough time, trends in apparently random behaviour can become predictable by analysing the frequency of outcomes.

Research suggests that students view randomness in a number of different and, sometimes, contradictory ways. Pratt and Noss (2002)[†] identified four 'naïve' ways that 10- and 11-year-old students described randomness. These were unpredictability (can the outcome of an event be predicted?), unsteerability (can the outcome of an event be controlled?), irregularity (can a pattern be seen in the outcomes of an event?) and fairness.

To overcome this, students should be exposed to practical experimentation, both engaging in experimentation themselves and collecting collaborative data to enable larger sets of data to be reflected upon. Technology can be used here to generate large data sets for students to work with.

5.3.1.1 Understand that some outcomes are equally likely, and some are not

5.3.1.2 Understand that the likelihood of events happening can be ordered on a scale from impossible to certain

5.3.1.3* Understand that the likelihood of outcomes can be determined by designing and carrying out a probability experiment

5.3.2 Systematically record outcomes to find theoretical probabilities

Identifying the range of possible outcomes (the sample space) for an event is key for students to be able to reason about the likelihood of one of those outcomes occurring. Students should understand that for situations with equally likely outcomes, the greater the possible number of different outcomes, the less likely each individual outcome becomes. Bryant and Nunes (2012: 4)[‡] state that '*working out the sample space is not just a necessary part of the calculation of the*

[†] D. Pratt & R. Noss, 2002, The Microevolution of Mathematical Knowledge: The Case of Randomness, *Journal of the Learning Sciences*, 11(4), 453–488, Philadelphia, Taylor & Francis

[‡] P. Bryant & T. Nunes, 2012, *Children's understanding of probability: A literature review (summary report)*, London, Nuffield Foundation

probabilities of particular event[s], but also an essential element in understanding the nature of probability.'

Students should experience different ways to record and represent outcomes, including lists, tables, grids and Venn diagrams. Tree diagrams are introduced in the national curriculum Key Stage 4 programme of study, but you might consider introducing them here, as an additional representation.

Working systematically to construct and record the outcomes of an event efficiently is not trivial. Systematic listing of outcomes should be made explicit when working with students.

As the number of equally likely events is increased, consideration of the sample space becomes more crucial. For example, when flipping two coins, many students may say that an outcome of two heads, two tails or a head and a tail are all equally likely. The use of a probability space diagram, where outcomes are assigned probabilities, can help make sense of this misconception. For example, Shaughnessy and Ciancetta (2002)[§] asked a sample of American students whether they agreed that, in a game where players spin two fair spinners like these:



and win a prize when *both* arrows land on black, that they had a 50–50 chance of winning. They found that only around 20% of students aged 11 to 13 answered correctly, with most students believing that it was a 50–50 chance.

- 5.3.2.1 Systematically find all the possible outcomes for two events using a range of appropriate diagrams
- 5.3.2.2 Systematically identify all possible outcomes for more than two events using appropriate diagrams, e.g. lists
- 5.3.2.3 Find theoretical probabilities from sets of outcomes organised in a systematic way from a range of appropriate representations

5.3.3 Calculate and use probabilities of single and combined events

Probability is quantified using proportion, and this proportion is usually represented as a fraction, although a decimal or percentage can also be used. Students can find reasoning about proportion challenging, and reasoning about proportion in probability adds an extra layer of complexity.

Probability is also frequently quantified using a ratio, which implies a slightly different perspective on probability.

Consider a situation in which two blue counters and three red counters are in a bag, and a counter is repeatedly taken out of the bag and then replaced. The probability that a blue counter is drawn can be quantified as $\frac{2}{5}$; that is, for every five counters selected, two of them can be expected to be blue. When represented as a ratio, this becomes 2 : 3, with the implicit interpretation that, for every two blue counters drawn out, three red counters remain.

[§] M. Shaughnessy & M. Ciancetta, 2002, Students' understanding of variability in a probability environment, in B. Phillips (ed.), *CD of the Proceedings of the Sixth International Conference on Teaching Statistics: Developing a statistically literate society*, Cape Town, South Africa, Netherlands, International Statistics Institute

Watson, Jones and Pratt (2013: 162)** state that *'the danger inherent in the UK curriculum is that probability teaching focuses on the algebra of probability, which is meaningless if probability is not seen as useful for making judgements in real life and for solving problems.'* Consequently, it is important that students are given a range of opportunities to interpret, compare and make decisions on, based around quantifiable probabilities related to real life.

Students will learn that the total chance of all the outcomes of an event will sum to one and should understand how this can be illustrated on a number line, and subsequently link this knowledge to the relationship:

[the chance of an outcome not happening] = 100% – [the chance of it happening]

- 5.3.3.1* Understand that probability is a measure of the likelihood of an event happening and that it can be assigned a numerical value
- 5.3.3.2 Calculate and use theoretical probabilities for single events
- 5.3.3.3 Understand that the probabilities of all possible outcomes sum to one
- 5.3.3.4 Calculate and use theoretical probabilities for combined events using a variety of appropriate representations, including Venn diagrams

** A. Watson, K. Jones & D. Pratt, 2013, *Key Ideas in Teaching Mathematics: Research-based guidance for ages 9–19*, Oxford, Oxford University Press

Exemplified key ideas

5.3.1.3 Understand that the likelihood of outcomes can be determined by designing and carrying out a probability experiment

Common difficulties and misconceptions

In order to understand a probability experiment, students must understand that outcomes should be the result of a random process, but randomness is not necessarily a well-understood concept.

A study from Pratt and Noss (2002)^{††} showed that students might, for example, view a non-uniform spinner as not being random because it was 'not fair', even though it was seen to fit other features of randomness, while a uniform spinner was perceived to be random.

Giving students an opportunity to explore the same mathematical ideas in different contexts can draw attention to the key structures that underpin the mathematics, and so support students in developing a deeper understanding of randomness.

Within the context of independent events, a common misconception is that having flipped a coin and obtained a head, the next flip 'should' give tails because it is an equal probability. Allowing students the opportunity to experience such situations and analyse the possible outcomes will help address the misconception.

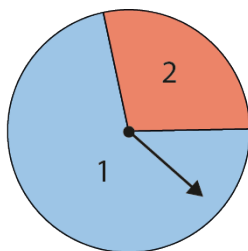
What students need to understand

Recognise events in which outcomes are random.

Example 1:

a) Which of these events do you think will select a number at random?

- (i) Rolling a six-sided dice numbered 1 to 6.
- (ii) Rolling a nine-sided dice numbered 1 to 9.
- (iii) Rolling a nine-sided dice numbered with four 1s and five 2s.
- (iv) Spinning this spinner.



Guidance, discussion points and prompts

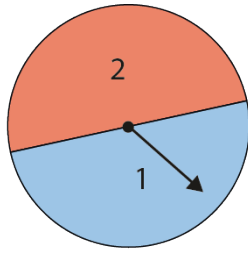
V *Example 1* provides an opportunity to explore students' understanding of randomness and to support them in developing a deeper understanding of the ideas.

Increasing the number of options, moving from part (i) to (ii), allows students to discuss whether this increases the randomness of the situation. Similarly, in parts (iv), (v) and (vi), students may identify spinner (vi) as generating more random results because there are more options than in spinner (v) and they are arranged differently.

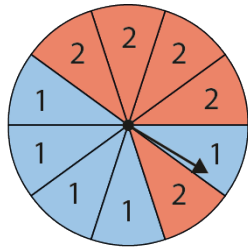
Parts b) and c) are prompts to give students an opportunity to think in a different way about their answers to part a). Research suggests that students can hold contradictory opinions about randomness and fairness, which these prompts may expose.

^{††} D. Pratt & R. Noss, 2002, The Microevolution of Mathematical Knowledge: The Case of Randomness, *Journal of the Learning Sciences*, 11(4), 453–488, Philadelphia, Taylor & Francis

(v) Spinning this spinner.



(vi) Spinning this spinner.



(vii) Asking the person sitting next to you to pick a whole number between 1 and 10.

- b) Do you think that any of the above are 'more random' than the others? Explain why.
- c) Which of the above do you think is a 'fair' way to select a random number? Explain why the other ones are not fair.

D You might like to ask students to consider, if they wanted to score a 2, which of the methods in *Example 1* they would choose. At this point, students have not yet formally met the idea of quantifying probability, but this situation presents the opportunity to consider the number of ways of scoring 2 and to compare it with the number of other outcomes. This may raise issues around the nature of a random selection.

L Most of the contexts used here are familiar to students and are ones in which they will have an intuitive grasp of the likely outcomes. This is an opportunity for students to discuss their ideas using the terms 'likely', 'unlikely', 'even', 'impossible' and 'certain', rather than attempting to quantify the probabilities.

Example 2:

The weather forecast says that there is a 40% chance of rain.

Ella says, 'That's quite likely – I'll take an umbrella.'

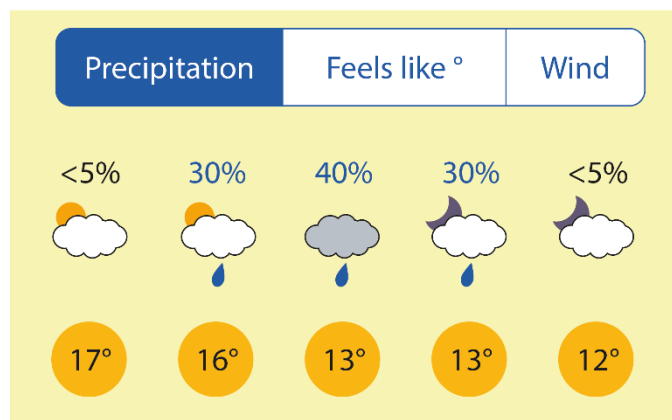
Gavin says, 'I don't think it's going to rain.'

Raj says, 'It's just random – it might rain, it might not.'

Who do you most agree with?

Explain why you think the others are wrong.

Example 2 again allows for discussion of randomness and what it means to assign a probability to an outcome of an event. This example gives students an opportunity to consider what it means to make a prediction about the likelihood of a future event, based on information gathered from previous occurrences.

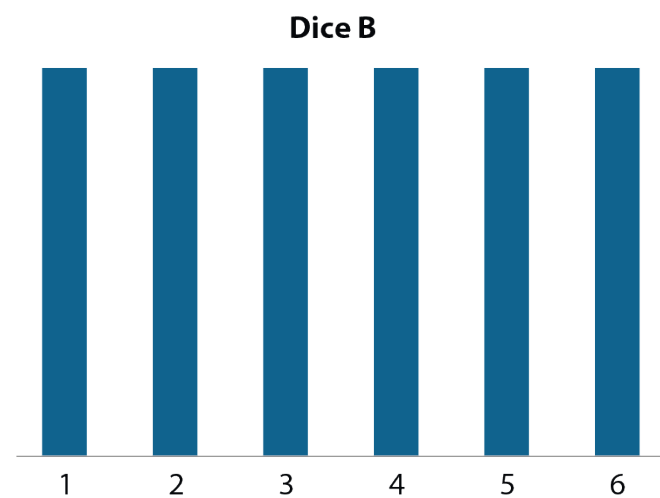
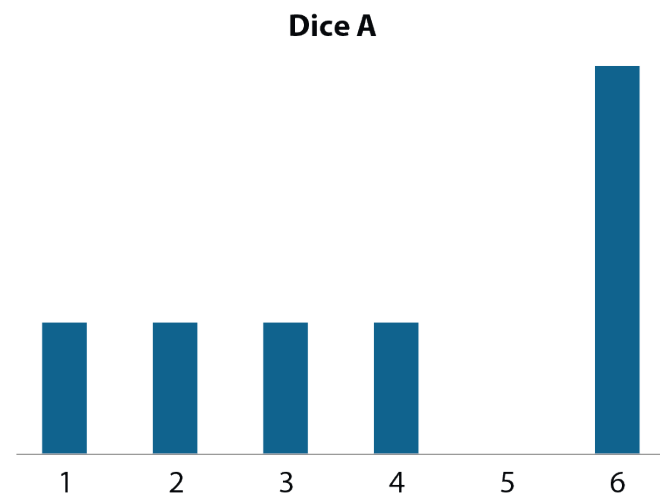


Understand that previous outcomes can be used to make predictions about behaviour over a large number of trials.

Example 3:

Libby rolled each of three dice over 200 times.

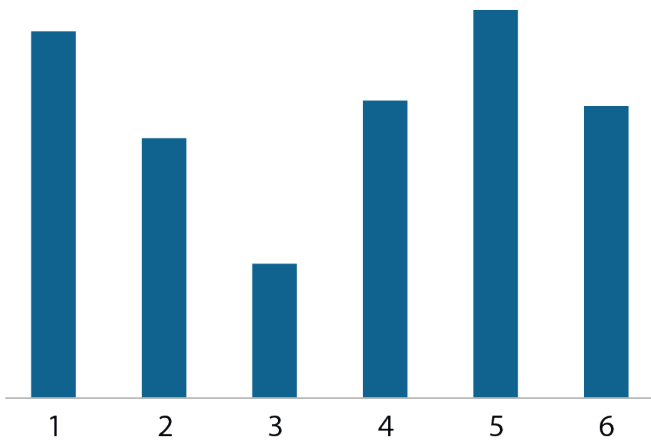
These bar charts show the frequency of each score.



V In *Example 3*, care has been taken to not offer specific values to the number of outcomes, in order to focus students' attention on the way in which the frequency of events is distributed.

The intention here is for students to understand that the frequency of each outcome over time can be used to make a prediction about future outcomes. They may conclude that, while it is not certain that any one of these dice will roll a 5, dice C appears to be disproportionately favouring that outcome. The variation in the representations is designed to draw attention to this, along with notions of 'fairness' and 'impossibility'.

Dice C



Which dice would you use if you were one step away from finishing a game and needed to roll a 5 to win? Explain why.

Example 4:

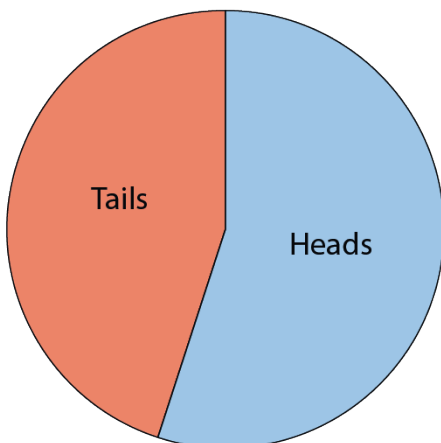
Matilda flipped a fair coin 200 times.

She drew a pie chart to show the results after 10 flips, after 50 flips and after 200 flips.

Which of these pie charts do you think shows the results after each number of flips?

Explain how you know.

Pie Chart A



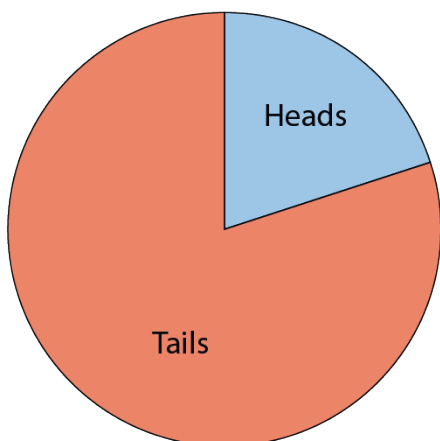
Example 4 is designed to explore students' understanding of the way that outcomes of an experiment will tend towards expectations as the number of trials increases.

Students are likely to reason that pie chart B is the snapshot after 10 flips, but the other two pie charts are less easy to decide.

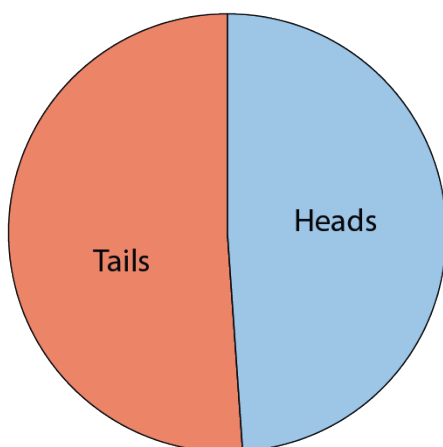
Pie chart A has a greater proportion of heads, and some students may raise the misconception that this must represent the outcome after 200 flips as the proportion of heads is growing.

The intention is that students appreciate the greater the number of trials, the greater the accuracy of the prediction (i.e. the closer to a 50:50 split of heads and tails).

Pie Chart B



Pie Chart C



Example 5:

Which of these tables do you think might be the results of flipping a coin that has been tampered with so that it is weighted to one side?

Table 1

| | |
|--------------|---|
| Heads | 3 |
| Tails | 5 |

Table 3

| | |
|--------------|-----|
| Heads | 203 |
| Tails | 189 |

Table 2

| | |
|--------------|----|
| Heads | 21 |
| Tails | 34 |

Table 4

| | |
|--------------|----|
| Heads | 18 |
| Tails | 11 |

As with *Example 4*, *Example 5* offers the results of flipping a coin after a different number of trials, with a focus on developing understanding that the greater the number of trials, the more accurate the prediction.

- D** The addition of the biased coin in this example is intended to give the opportunity to discuss the outcomes that may be expected with a fair coin and those that exceed those expectations as the number of trials increases.

Recognise when prior outcomes can be used to make predictions.

Example 6:

- a) Gianmarco flipped a fair coin 10 times.

His results were:

T H H H H H H H H H

Gabriel says, 'You've got to get tails next time! It's the law of averages.'

Gianmarco says, 'No, it could be either heads or tails. It's an equal chance.'

Who do you agree with? Explain your answer.

- b) Jack runs a café – for lunch he sells two different types of soup.

In half an hour he sells 10 cups of soup:

*Tomato Chicken Chicken Chicken Chicken
Chicken Chicken Chicken Chicken Chicken*

Willow says, 'You'd better make more chicken than tomato soup tomorrow! People love it!'

Jack says, 'No, they can choose either tomato or chicken. I'll make the same amount of both.'

Who do you agree with? Explain your answer.

- c) A rugby team's results over 10 weeks are:

L W W W W W W W W W

Louis says, 'We're bound to win the next match! We're on a winning streak!'

Sasha says, 'We could win or lose. It can go either way.'

Who do you agree with? Explain your answer.

V *Example 6* offers situations with the same mathematical structure presented in three different contexts. Here, students need to focus on the difference that the context makes, considering issues around randomness, independent events and the number of trials that are being considered.

PD Consider the examples offered in this key idea – it is likely that students will have different viewpoints when discussing them. What structures and strategies might be successful in allowing you to hear students' current understanding as they express it, and then move them forward?

5.3.3.1 Understand that probability is a measure of the likelihood of an event happening and that it can be assigned a numerical value

Common difficulties and misconceptions

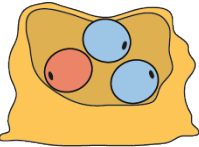
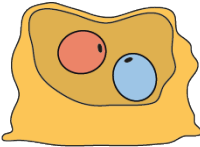


Students tend to have an innate understanding of 'more likely' and 'less likely' from their own experiences and can justify these using numerical values given a basic understanding of fractions.

However, difficulties can occur when students focus on the absolute number of possible successful outcomes, irrespective of the total number of all possible outcomes. For example, *'It's more likely because there are more ways to win.'* To overcome this, students should have experience of a variety of situations where the total number of successful outcomes might be the same, but they represent a different proportion of all the possible outcomes. For example, the probability of choosing a black ace from a pack of cards changes when:

- the whole pack is used
- the red aces are removed
- all the diamonds are removed, etc.

It is important for students to develop an understanding of the number of trials needed to be able to make reliable predictions, but it is often larger and more time consuming than is pragmatic for students to do individually within a lesson. Collecting different groups' results and putting them together to achieve a larger class set is one way of addressing this. Another way is students doing a few trials each so that they understand the process, before switching to IT-based probability simulation programs.

L Initially, students may use phrases such as *'Three ways of winning out of five ways in total.'* It will be important to introduce the language of *'The probability of winning is...'* Probability has a number of unfamiliar terms that can easily get confused by students. You should make every effort to use precise language and appropriate terms when referring to, for instance, outcomes, events, occurrences, probability and possibility, which can easily sound similar, but can mean very different things.

| What students need to understand | Guidance, discussion points and prompts |
|---|---|
| <p>Distinguish between events that have different likelihoods and start to explain numerically why one event is more likely than another.</p> <p><i>Example 1:</i></p> <p>Play this game with a partner.</p> <p>Each player has a bag from which they take turns to randomly select a bead and then replace it.</p> <p>Each time a player selects a red bead they score a point.</p> <p>The first player to score 5 points wins the game.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>Player 1</p> </div> <div style="text-align: center;">  <p>Player 2</p> </div> </div> <p>Explain if the game you have played is fair and, if not, why not.</p> | <p>V The game in <i>Example 1</i> has been designed so that, while the number of red beads is the same in each bag, the proportion of red beads to blue beads is different. This will be a key point to draw out during discussion after students have played the game.</p> <p>L In supporting students to explain their thinking clearly, encourage them to say: 'The probability of choosing a red is 1 out of 3 (or 1 out of 2)' and later, 'The probability of choosing a red is $\frac{1}{3}$ (or $\frac{1}{2}$).'</p> |
| <p><i>Example 2:</i></p> <p>Play this game with a partner.</p> <p>Each player has a counter to move along the track.</p> <p>Players take turns to roll a 6-sided dice.</p> <p>Player 1 can move their counter one space along the track if a number greater than 4 is rolled.</p> <p>Player 2 can move their counter one space if a number less than 4 is rolled.</p> <p>If 4 is rolled, the player rolls the dice again.</p> <p>The first player to reach the end of the track is the winner.</p> <div style="display: flex; align-items: center; margin-bottom: 10px;"> Start <div style="border: 1px solid black; width: 300px; height: 40px; display: flex; flex-direction: column;"> <div style="display: flex; justify-content: space-between; padding: 0 10px;"> Start Finish </div> <div style="display: flex; justify-content: space-between; padding: 5px;"> <div style="width: 10%; text-align: center;">  </div> <div style="width: 80%; border-bottom: 1px solid black; position: relative;"> <div style="position: absolute; left: 0; bottom: 0; width: 100%; height: 100%; background-color: #e0e0e0;"></div> </div> <div style="width: 10%; text-align: center;">  </div> </div> </div> </div> <p>Explain if the game you have played is fair and, if not, why not.</p> | <p>The rules of the game in <i>Example 2</i> have been chosen to draw attention to the number of possible outcomes (out of a total of six equally likely ones) that allow player 1 or player 2 to move along the track. Discussion following the playing of the game should attempt to draw out this point.</p> <p>D It is also important to draw students' attention to the fact that, while games such as those in <i>Examples 1</i> and <i>2</i> are biased towards one player, this does not mean they would automatically win. It may be worth discussing with students what effect they think it might have on the results of a series of games if the track was longer or shorter, or, in the case of the first game, there was a different number of points required to win.</p> |

Understand that the probability of an event with only two equally likely outcomes is a half, and that each possible outcome will occur approximately half of the time.

Example 3:

Flip a coin and record your results on a tally chart.

Every 5 flips, calculate the percentage number of heads that have been flipped and record this on a graph.

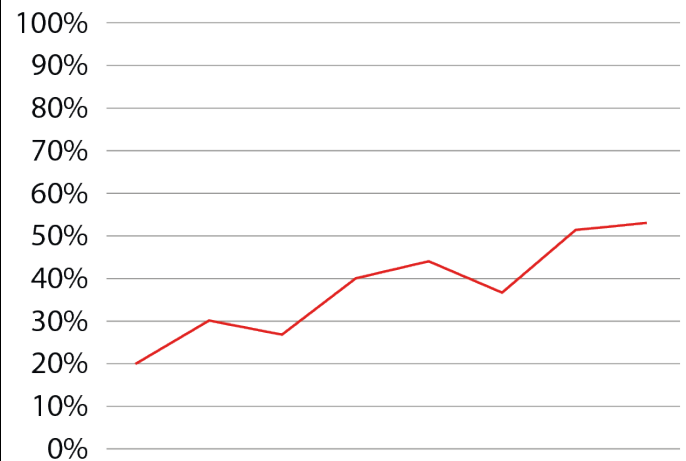
Continue for 50 flips, recording each percentage every 5 flips and marking this on the graph to show the progression of the percentage results as a line.

This is a classic mathematics experiment and can be repeated for scenarios where the probabilities are $\frac{1}{3}$ and $\frac{2}{3}$ or $\frac{1}{4}$ and $\frac{3}{4}$.

Students often need help with collating and calculating the results to begin with, so you could encourage them to create a table, like the one below, and complete it until they reach 50 flips.

| | | | | |
|---------------------------------------|-----|-----|-----|-----|
| Number of heads this set of 5 | 1 | 2 | 1 | ... |
| Total number of heads obtained | 1 | 3 | 4 | ... |
| Number of flips so far | 5 | 10 | 15 | ... |
| Percentage number of heads | 20% | 30% | 27% | ... |

The key issue is that the resulting graphs tend to, but not necessarily, arrive at the numerical probability of an occurrence.



Different resulting graphs can be compared, and a simple spreadsheet designed to model the results so that multiple examples can be quickly created to illustrate the general nature of tending to (in this example) 50%.

Use the theoretical probability of a particular outcome to predict an expected number of occurrences of that outcome in an experiment.

Example 4:

Am I lucky today?

For each scenario, roll a dice 30 times and record the resulting number of points.

- You score a point every time you roll an even number.*
- You score a point every time you roll a 6.*
- You score a point every time you roll a number less than 6.*
- You score a point every time you roll a number greater than 2.*
- You score a point every time you roll a square number.*
- You score a point every time you roll a number less than 10.*

In each case, decide whether you were luckier or not than expected. Explain your reasoning by comparing your results with the expected number of occurrences.

In *Example 4*, it is important for students to get a sense of how likely they think the outcomes will be while playing the game, and it may help to ask students to predict what they think the outcome will be after 30 rolls of the dice.

- L** You can help formalise students' ideas by offering the language of:
'The total number of possible outcomes is ...'
'The total number of successful outcomes is ...'
'The probability of ... is ...'
- D** The phrase 'Am I lucky today?' draws attention to the difference between expected results and actual results. Class discussion of this could again raise the idea that the results of such experiments settle down and tend to the expected value if the dice is rolled enough times.

Identify the numerical values of outcomes in different single events and use these values to order the outcomes according to their likelihood.

Example 5:

Which is more likely?

Play these games in pairs. Each time, decide which player is more likely to win.

Game 1

Player 1 has the cards spelling the word 'red'.

Player 2 has the cards spelling the word 'blue'.

Each player shuffles their cards and picks a card – they do this 30 times and record what they get.

You win if you pick the letter E more times than your opponent.

| | | |
|---|---|---|
| R | E | D |
|---|---|---|

| | | | |
|---|---|---|---|
| B | L | U | E |
|---|---|---|---|

The games in *Example 5* offer students the opportunity to focus on the number of possible outcomes and the number of successful outcomes.

- V** The questions have been selected to include the following aspects:
 In Game 1, the number of successful outcomes is the same, while the total number of outcomes is different.
 Game 2 involves only one scenario, and so the number of total outcomes is the same, but the number of successful outcomes is different for each player.
 Game 3 requires more in terms of comparing the resulting fractions, and so conversion to either a decimal or percentage could be a useful strategy.
 Game 4 introduces examples of outcomes that are not mutually exclusive, although at

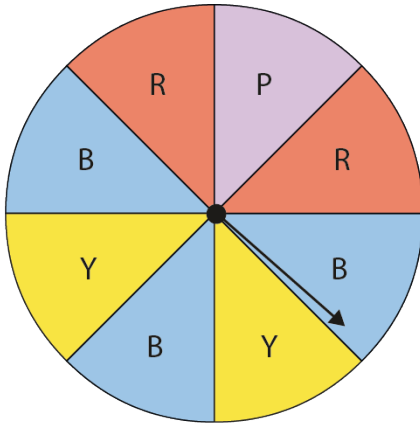
Game 2

Each player spins the spinner.

Player 1 gets a point when the spinner lands on red.

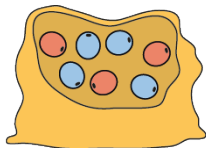
Player 2 gets a point when the spinner lands on blue.

Each player spins the spinner 30 times.

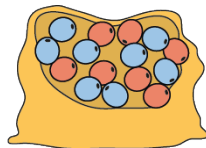
**Game 3**

Player 1 picks a bead from bag 1.

Player 2 picks a bead from bag 2.



Bag 1



Bag 2

Each player gets a point when they pick a green bead. They each have 30 goes.

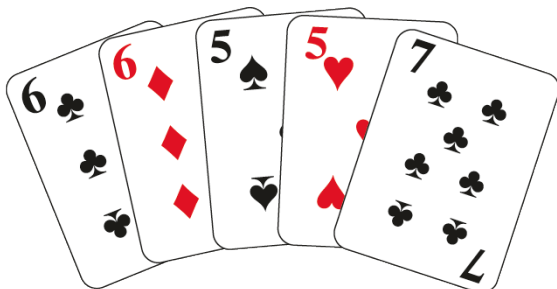
Game 4

Players 1 and 2 each have an identical hand of five cards, as shown. They shuffle their cards and choose one.

Player 1 gets a point if they choose a red card.

Player 2 gets a point if they choose a six.

They each choose 30 times.



this stage this is not important, because the comparison can still be made and the probabilities are equal.

- D** A similar example to this final game can easily be devised where the two probabilities are greater than a half. In this case, the question could be posed how it is possible to have a greater number of possible outcomes than there are total outcomes, highlighting the non-mutually exclusive nature of the question.

Weblinks

- ¹ NCETM primary mastery professional development materials
<https://www.ncetm.org.uk/resources/50639>
- ² NCETM primary assessment materials
<https://www.ncetm.org.uk/resources/46689>