NCETM
NATIONAL CENTRE FOR EXCELLENCE IN THE TEACHING OF MATHEMATICS

Core concept 6.2: Perimeter, area and volume

This document is part of a set that forms the subject knowledge content audit for Key Stage 3 maths. The audit is based on the NCETM Secondary Professional Development materials and there is one document for each of the 17 core concepts. Each document contains audit questions with check boxes you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications and explanations, and further support links. At the end of each document there is space to type reflections, targets and notes. The document can then be saved for your records.

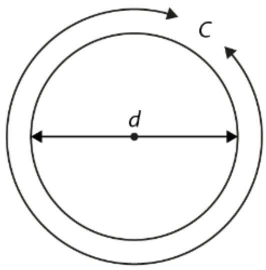
6.2.1 Understand the concept of perimeter and use it in a range of problem-solving situations

How confident are you that you could deduce the perimeters of a range of polygons and calculate unknown lengths in contexts involving the circumference of circles?

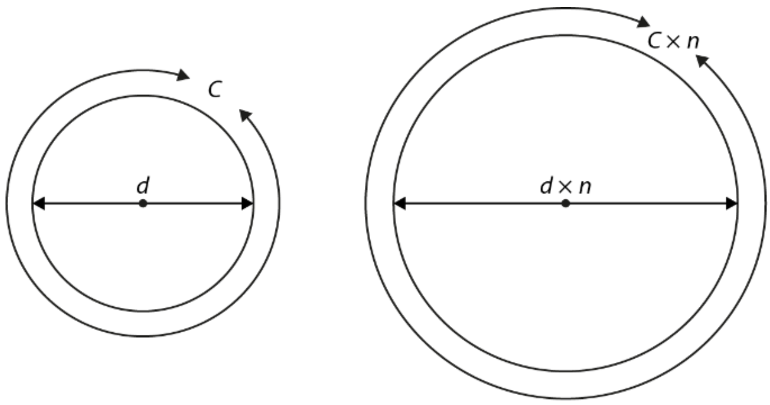
1 ☐ 2 ☐ 3 ☐ 4 ☐

Students should be exposed to a range of problems involving the perimeter of rectilinear shapes and circles. These problems should require students to choose which lengths to include, which lengths not to include and which lengths must be found by reasoning.

When circles and the ratio π are introduced, a key awareness is that no matter how large or small the circle, the ratio between its circumference and its diameter is always the same. This is the classic multiplicative relationship *within* every circle, which is encapsulated by the formula $C = \pi d$ or $\pi = \frac{C}{d}$.



Students should also be aware of the corresponding multiplicative relationship between any two circles – i.e., if one circle has a diameter n times the length of another, then its circumference will be n times the circumference of the other.



Further support links

- NCETM Secondary Professional Development materials: 6.2 Perimeter, area and volume, pages 9-10

6.2.2 Understand the concept of area and use it in a range of problem-solving situations

How confident are you that you can derive the area formulae for a triangle, a parallelogram a trapezium and a circle?

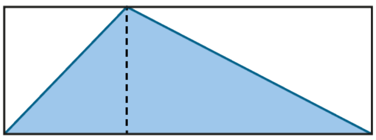
1 ☐ 2 ☐ 3 ☐ 4 ☐

How confident are you that you can find the surface area of a 3D shape efficiently?

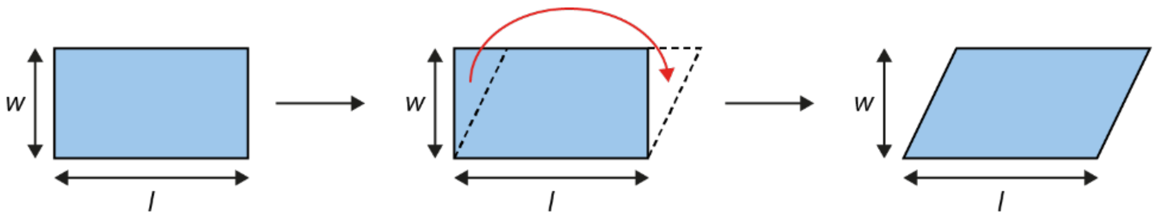
1 ☐ 2 ☐ 3 ☐ 4 ☐

Students should be aware of differing ways that shapes can be partitioned to enable their areas to be found.

For example, the knowledge of the area of a rectangle can be used to derive the areas of triangles, parallelograms and then trapezia.



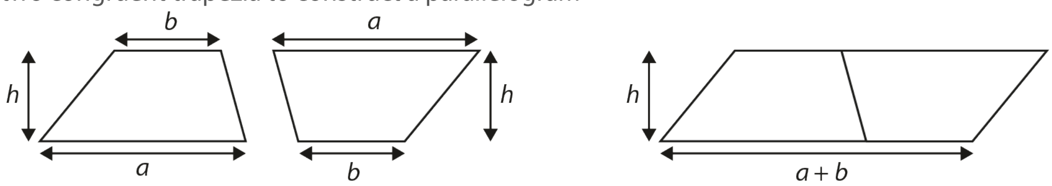
Area of a triangle = $\frac{1}{2}$ × base × perpendicular height



Area of a parallelogram = base × perpendicular height

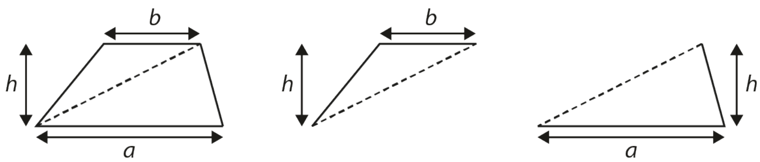
There are many ways to explore the formula for the area of a trapezium using representations and making connections with the area of a parallelogram, triangle and rectangle.

Use two congruent trapezia to construct a parallelogram



This is a useful representation to support students' understanding of the significance of the $\frac{1}{2}$ and the 'sum of the parallel sides' parts of the formula for the area of a trapezium. Students should be able to see that, since the area of the parallelogram = (a + b)h, the area of the trapezium must be half, or $\frac{1}{2}(a + b)h$.

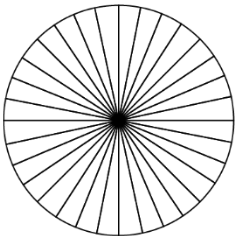
Partitioning a trapezium into two triangles



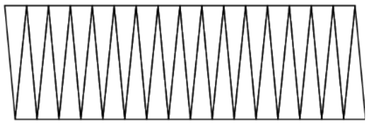
This derivation helps to support students' understanding of the significance of the 'perpendicular height' part of the formula as they realise that the area of a trapezium is the sum of the area of two triangles, or $\frac{1}{2}b \times h + \frac{1}{2}a \times h = \frac{1}{2}(a + b)h$.

Partitioning into shapes whose area formula is known can also support the derivation of the area of a circle, πr^2.

The diagram shows a circle split into equal sectors.



The sectors can be rearranged.

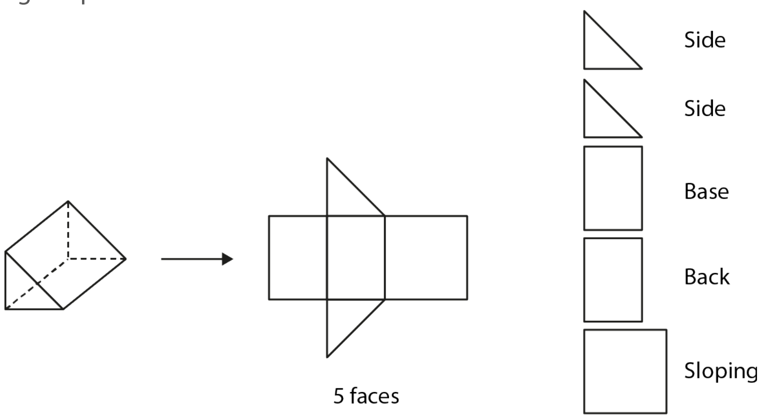


When splitting the circle up into more and more sectors, the new shape becomes very nearly a rectangle. The length of the rectangle is half the circumference of the circle, and the width is equal to the radius of the circle.

The area of the rectangle is equal to the area of the circle.

$$\begin{aligned}\text{Area of circle} &= \tfrac{1}{2} \times \text{circumference} \times \text{radius}\\ &= 0.5 \times 2\pi r \times r\\ &= \pi r^2\end{aligned}$$

For surface area, sketching 2D representations of a prism will help students identify the number of faces, the shapes of the faces and their dimensions. This should consolidate the idea that the surface area of a prism is the sum of the areas of all its faces. If students find that sketching the net is difficult, identifying and sketching the faces independently, possibly by using a 3D model as support, will help them make this connection. This is particularly helpful when students are trying to identify the 'sloping' rectangular faces of, for example, triangular prisms.



Giving students a range of prisms and asking them to calculate the surface area should help them realise the key idea that the surface area of a prism is equal to the sum of the areas of all its faces.

Further support links

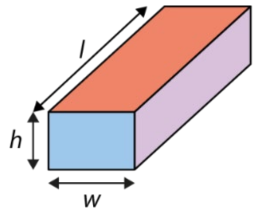
- NCETM Secondary Professional Development materials: 6.2 Perimeter, area and volume, pages 11, 14 and 26

6.2.3 Understand the concept of volume and use it in a range of problem-solving situations

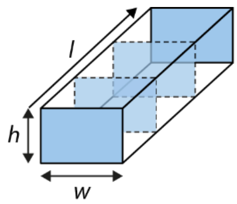
How confident are you that you can explain the use of the cross-sectional properties to find the volume of a prism?

1 ☐ 2 ☐ 3 ☐ 4 ☐

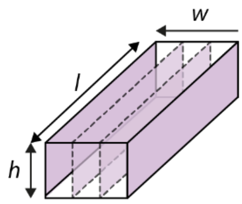
Students will be familiar with finding the volume of cubes and cuboids from Key Stage 2 and will have used the formula Volume = width × height × length (or similar) to calculate volumes. At Key Stage 3, these ideas are developed to include the volume of prisms more generally.For example, when considering a cuboid, such as this there are various ways of calculating the volume.



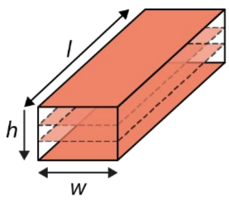
Find the area of the blue face ($h \times w$) and multiply by the length (l).



Find the area of the purple face ($h \times l$) and multiply by the width (w).



Find the area of the red face ($w \times l$) and multiply by the height (h).



Through this sort of analysis, students will realise that the volume of a cuboid is actually the area of one of the faces multiplied by the other dimension. This can then be generalised in Key Stage 3 to other prisms and to the formula

Volume of a prism = area of cross-section × length.

Students will use and apply their knowledge of the area of 2D shapes to calculate the cross-sectional area of a variety of prisms.

Although a cylinder is not strictly a prism (a prism has a polygonal uniform cross-section), it is important for students to appreciate that it has the same structure as a prism (with the uniform cross-section being a circle) and its volume can be calculated in a similar way. Thereby, students will see the formula $V = \pi r^2 h$ as an example of a general geometrical property of cylinders that has meaning, and not just a collection of symbols to be memorised.

Further support links

- NCETM Secondary Professional Development materials: 6.2 Perimeter, area and volume, pages 12 and 13

Notes