Mastery Professional Development

Multiplication and Division

2.13 Calculation: multiplying and dividing by 10 or 100

Teacher guide | Year 4

Teaching point 1:

Finding 10 times as many is the same as multiplying by 10 (for positive numbers); to multiply a whole number by 10, place a zero after the final digit of that number.

Teaching point 2:

To divide a multiple of 10 by 10, remove the final zero digit (in the ones place) from that number.

Teaching point 3:

Finding 100 times as many is the same as multiplying by 100 (for positive numbers); to multiply a whole number by 100, place two zeros after the final digit of that number.

Teaching point 4:

To divide a multiple of 100 by 100, remove the final two zero digits (in the tens and ones places) from that number.

Teaching point 5:

Multiplying a number by 100 is equivalent to multiplying by 10, and then multiplying the product by 10. Dividing a multiple of 100 by 100 is equivalent to dividing by 10, and then dividing the quotient by 10.

Teaching point 6:

If one factor is made 10 times the size, the product will be 10 times the size. If the dividend is made 10 times the size, the quotient will be 10 times the size.

Teaching point 7:

If one factor is made 100 times the size, the product will be 100 times the size. If the dividend is made 100 times the size, the quotient will be 100 times the size.

Overview of learning

In this segment, children will begin to be exposed to the idea of multiplication and division as scaling, in the context of learning strategies for multiplying and dividing by 10 and 100. In *Teaching points 1* and *3*, children will learn how to multiply any whole number by 10 or by 100, using the context of finding 10/100 times as many of a quantity of countable items; for example, *'Emily has two pencils; Jamie has ten times as many. How many pencils does Jamie have?'*

In *Teaching points 2* and *4*, children will learn how to divide multiples of 10 by 10, and multiples of 100 by 100, using the inverse of the contextual problems used in *Teaching points 1 and 3*; for example, *'Jamie has twenty pencils; he has ten times as many pencils as Emily. How many pencils does Emily have?'*

An important linguistic distinction is made throughout *Teaching points 1-4*:

- When describing contextual problems inolving countable items, the phrases 'ten times as many' and 'one hundred times as many' are used (as in the examples above).
- When describing relationships between the abstract numbers, the phrases 'ten times the size' and 'one hundred times the size' are used; for example '*Twenty is ten times the size of two.'*

It is important to avoid language such as *'Emily has ten times more/fewer pencils than Jamie'*, '*Twenty is ten times bigger than two.'* and '*Two is ten times smaller than twenty.*' Such language is mathematically imprecise; the language is also misleading when dividing by 10 or 100, since 'ten times' implies multiplication, and it is not possible to multiply by a whole number and get a product that is less than the multiplicand.

As noted, for now children will only consider division by 10 (or 100), in scaling contexts, as the inverse of multiplication; in segment *2.17 Structures: using measures and comparison to understand scaling*, children will explore scaling in more detail (including measures contexts), and at that stage they will use the link between division by a whole number and multiplication by a unit fraction, connecting division to phrases such as *'one-third times the length/mass…'* and *'one third times the size'* (not '*three times as short/heavy'* and not '*three times smaller*').

So far in Spine 2, the language of multiplication has consisted of factor–factor–product, where a factor was interpreted contextually as either the size of the equal groups or the number of groups. In this segment, for teachers only, the language of 'multiplicand' (the number that gets multiplied) and 'multiplier' (the number you are multiplying by) is used. This provides clarity when connecting to the scaling structure; in the example problem: *'Emily has two pencils; Jamie has ten times as many. How many pencils does Jamie have?'*, '2' is the multiplicand and '10' is the multiplier. Although the muliplicand and multiplier can be written in either order (due to the commutativity of multiplication), throughout this segment the multiplicand is presented first and the multiplier second (in the example problem here, $2 \times 10 = 20$). Although the terms 'multiplicand' and multiplier' are not used with children, the idea is implicit in the emerging use of the terms *'times by'* and *'multiplied by'* which, until now, have been avoided. These phrases should only be used when describing abstract equations, or when connected to the scaling structure of multiplication (not the grouping structure).

Teaching point 6 (and *7*) explores:

- the effect on the product of making one factor 10 (or 100) times the size.
- the effect on the quotient of making the dividend 10 (or 100) times the size.

The generalisations reached are special cases of a wider exploration of the effect of scaling a factor or the dividend, which will be conducted in segment *2.25 Using compensation to calculate*.

Throughout *Teaching points 1–4*, the Gattegno Chart, ratio tables and place-value charts are used to support children in generalising about the process of multiplying and dividing by 10 and 100.
In *Teaching point 5*, these representations support the understanding that multiplying by 100 is equivalent to multiplying by 10, and then by 10 again, and that dividing by 100 is equivalent to dividing by 10, and then by 10 again. The generalisations reached take the following form (exemplified here for multiplication/division by ten):

- ***'To multiply a whole number by ten, place a zero after the final digit of that number.'***
- ***'To divide a multiple of ten by ten, remove the zero from the ones place.'***

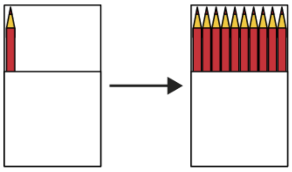
It is important to use the phrase *'place a zero'* rather than *'add a zero'*. Teachers should draw attention to the fact that when a number is, for example, multiplied by ten, each digit in that number is moved one column to the left on the place-value chart, and that the '0' is placed in the vacated ones position as a place-value holder; the '0' is not 'added', either in the sense of addition (+ 0) or in the sense of being appended to a new place-value position to the right of the existing digits. Similarly, for division by ten, the '0' that was in the ones place is removed only because it is no longer a significant digit (it is no longer required as a place-value holder once it has 'moved' to the right of the decimal point, in this context).

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Finding 10 times as many is the same as multiplying by 10 (for positive numbers); to multiply a whole number by 10, place a zero after the final digit of that number.

Steps in learning

	Guidance	**Representations**
1:1	This teaching point builds on children's knowledge of the ten times table. The main focus is development of the strategy of placing a zero as the final digit to multiply by ten and removing the final zero to divide by ten, allowing children to move beyond known ten times table facts. New language, connected to the idea of making a value 'ten times the size' and 'multiplying by ten', is developed (in contrast to 'groups of ten' or 'ten equal groups'), and is connected to understanding of place value. Begin by briefly reviewing the ten times table, including: • skip counting in tens (*'zero, ten, twenty…'*) • reciting the ten times table (*'zero tens are zero, one ten is ten, two tens are twenty…'* and *'ten, zero times is zero; ten, one time is ten; ten, two times is twenty…'*) • checking isolated multiplication facts (for example, *'What is six times ten?'*). For supporting representations, see segment *2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1, Teaching point 1*. Then, present a cardinal problem, corresponding to a multiplicand of '1' and a multiplier of '10'; i.e. making a value of '1' ten times the size. For example, *'Emily has one pencil; Jamie has ten times as many. How many pencils does Jamie have?'* Working with concrete resources, or	Multiplicand = 1; multiplier = 10: *'Emily has <u>one</u> pencil; Jamie has ten times as many. How many pencils does Jamie have?'*  • *'For every one pencil of Emily's, Jamie has ten.'* • *'Think of "1" and make it ten times the size.'* • *'Think of "1" and multiply by ten.'* 1×10 • *'One multiplied by ten is equal to ten.'* $1 \times 10 = 10$ • *'Ten is ten times the size of one.'* • *'Ten pencils is ten times as many as one pencil. Jamie has ten pencils.'*

pictorially, model gathering one pencil (a copy of Emily's quantity) 10 times (to make Jamie's quantity). This will support children in understanding why the multiplication calculation *1 × 10 = ?* represents the problem. Use the following sentences emphasising the equivalence between *'ten times the size'* (applied to the values, not the items) and *'multiply by ten'*:

- *'For every one pencil of Emily's, Jamie has ten.'*
- *'Think of "1" and make it ten times the size.'*
- *'Think of "1" and multiply by ten.'*

Prompt children to use their known facts to find the product, completing the multiplication equation. Then use the following sentences to describe the equation:

- *'One multiplied by ten is equal to ten.'*
- *'Ten is ten times the size of one.'*

Finally connect back to the context:

- *'Ten pencils is ten times as many as one pencil. Jamie has ten pencils.'*

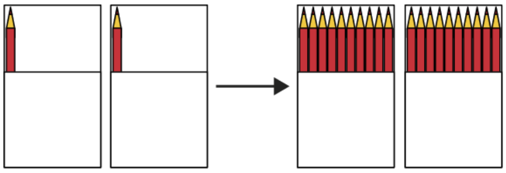
Then repeat the problem, this time with Emily having two pencils, and Jamie having ten times that quantity. Use the following stem sentences to describe the calculation, equation and context respectively:

- ***'For every one pencil of Emily's, Jamie has ten.'***
- ***'Think of ___ and make it ten times the size.'***
- ***'Think of ___ and multiply by ten.'***
- ***'___ multiplied by ten is equal to ___.'***
- ***'___ is ten times the size of ___.'***
- ***'___ pencils is ten times as many as ___ pencils. Jamie has ___ pencils.'***

Continue, systematically increasing the multiplicand (the number of pencils that Emily has) until children become

Multiplicand = 2; multiplier = 10:

'Emily has <u>two</u> pencils; Jamie has ten times as many. How many pencils does Jamie have?'



- *'For every one pencil of Emily's, Jamie has ten.'*

- *'Think of "2" and make it ten times the size.'*
- *'Think of "2" and multiply by ten.'*

  2 × 10

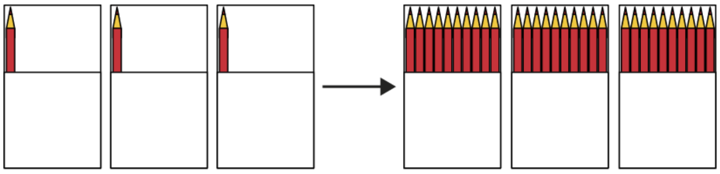
- *'Two multiplied by ten is equal to twenty.'*

  2 × 10 = 20

- *'Twenty is ten times the size of two.'*
- *'Twenty pencils is ten times as many as two pencils. Jamie has twenty pencils.'*

Multiplicand = 3; multiplier = 10:

'Emily has <u>three</u> pencils; Jamie has ten times as many. How many pencils does Jamie have?'



- *'For every one pencil of Emily's, Jamie has ten.'*

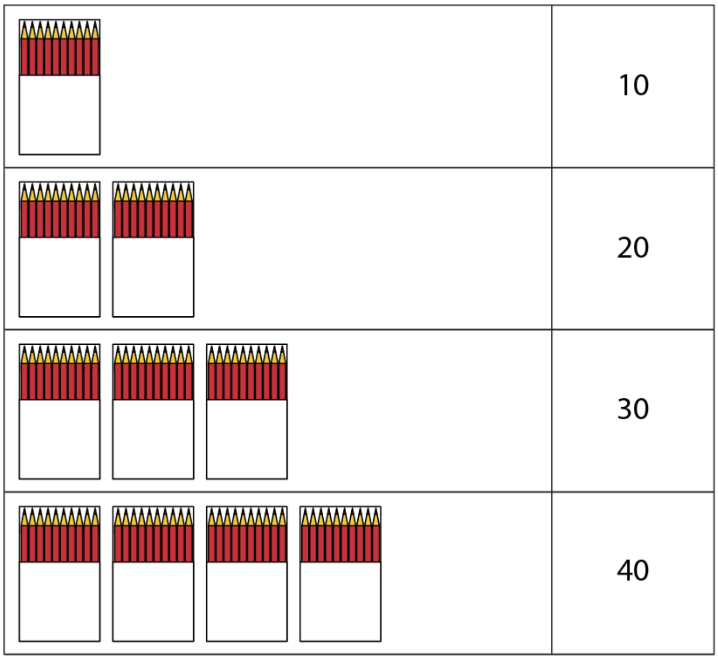
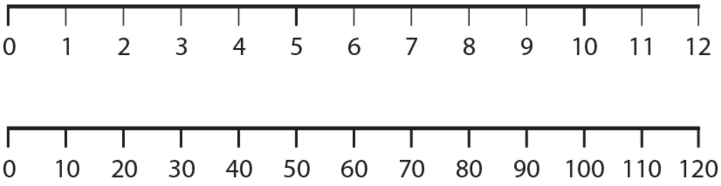
- *'Think of "3" and make it ten times the size.'*
- *Think of "3" and multiply by ten.'*

  3 × 10

- *'Three multiplied by ten is equal to thirty.'*

  3 × 10 = 30

- *'Thirty is ten times the size of three.'*
- *'Thirty pencils is ten times as many as three pencils. Jamie has thirty pencils.'*

	confident with the language and patterns. Work towards the following generalisation: ***'To find ten times as many, multiply by ten.'*** Teachers should note that, since the language applies to countable items, this does not apply to negative numbers.	
1:2	Children should already be aware that all multiples of 10 have a ones digit of zero, from *Spine 1: Number, Addition and Subtraction*, segment *1.8*, *Teaching point 2*. This was also reinforced when they learnt the ten times table and associated divisibility rule earlier in this spine. Briefly review this now, using: - the context from step *1:1*; draw attention to the products, and identify them as multiples of ten - stacked number lines, as shown opposite. Generalise: ***'All multiples of ten have a ones digit of zero.'*** Practise, as a class, sorting numbers according to whether they are multiples of ten or not, encouraging children to explain their reasoning. The follow-up dòng nǎo jīn question, shown opposite, begins to look at the inverse of multiplying by ten, which will be considered in *Teaching point 2*. You can provide children with further practice, such as true/false questions: *'True or false?'* - *'508 is a multiple of ten because it has a tens digit of zero.'* - *'4000 is not a multiple of ten because it has a tens digit of zero.'* - *'3040 is a multiple of ten because it has a ones digit of zero.'* - *'2130 is not a multiple of ten because it does not have a tens digit of zero.'*	Multiples of ten – contextual:  Multiples of ten – stacked number lines: 

Sorting activity:

- *'Put each number into the correct column according to whether it is a multiple of ten or not.'*

1 4 15 200 42 208 30

Multiple of 10	**Not a multiple of 10**

- Dòng nǎo jīn:
  'Work out what number was multiplied by 10 to get each of the numbers in the 'multiple of 10' column in the table above.'

True/false practice:

	Multiple of 10: true (✓) or false (×)?
20	
120	
102	
1000	
1020	

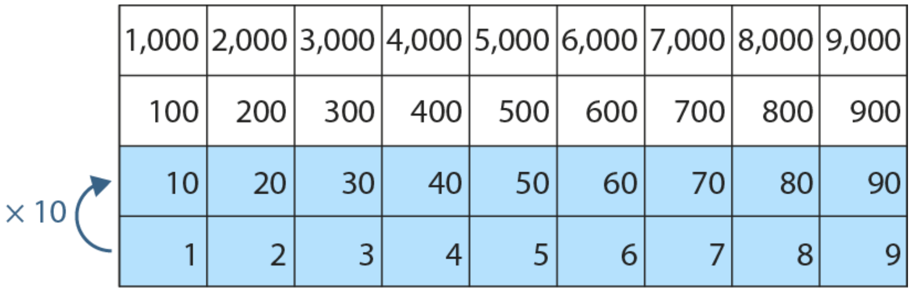
1:3 Now use the Gattegno chart to review, systematically, multiplying each of the numbers one to nine by ten, applying the language introduced in step *1:1*:

- ***'___ multiplied by ten is equal to ___.'***
- ***'___ is ten times the size of ___.'***

You can indicate the relationship between the single-digit row and two-digit row of the Gattegno chart as shown on the next page. For each calculation, write out the accompanying multiplication equation.

Bring together learning from the previous steps to generalise: ***'When a number is multiplied by ten, the product is a multiple of ten.'***

Gattegno chart:



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1:4

By now, children will probably be picking up on the idea that to multiply a whole number by 10, we can place a zero after the final digit of that number. However, it is important that children undertand *why* this works, rather than just seeing it as a 'rule' to be applied. Explore this explicitly now, using a place-value chart.

Begin by focusing on the 'movement is magnitude' principle: if we move a counter, or digit, one place to the left, the value of that digit becomes ten times the size, so a digit in the tens column has a value ten times the size of the same digit when in the ones column.

Now take a positive integer, such as '6', and multiply it by ten, writing an equation and describing it as shown below (see 'using known facts'). Then show how we can record the same calculation on the place-value chart:

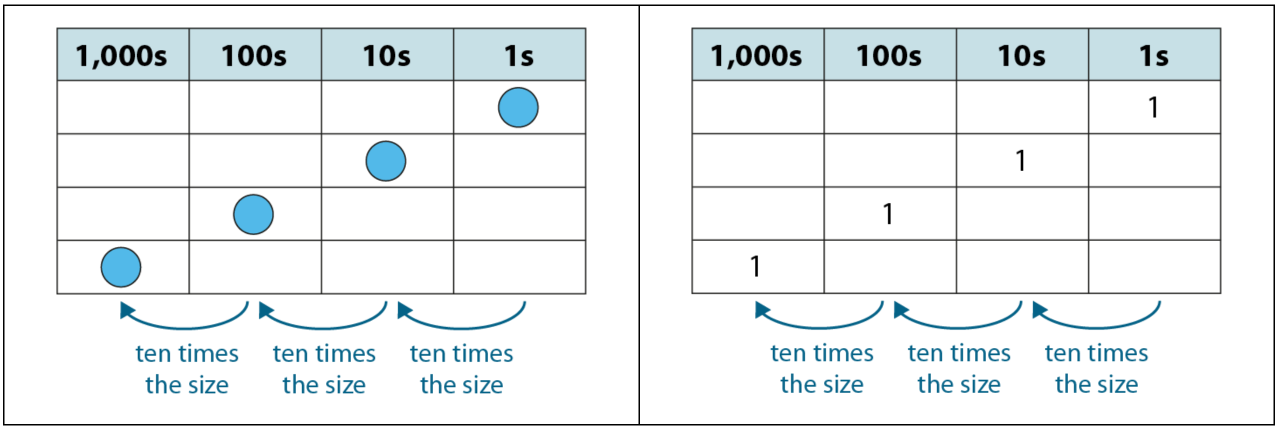
- Record '6' in the ones column.
- Say that, to multiply '6' by ten, we want to make it ten times the size, and move the digit into the tens column.
- Then ask children what would happen if we took away the place-value headings. Model doing this, and encourage children to notice that now the number is just '6'; we need to place a '0' in the column to the right.
- Finally, replace the place-value headings and emphasise that the '6' is now in the tens column and we have a '0' in the ones column.

Ask children to compare the value of digits before and after multiplying by ten, as shown on the next page.

Repeat for several different single-digit starting values, each time emphasising that when we've multiplied it by ten, the digit is now in the tens place, and there is a 'new' ones digit of zero (after a few iterations, you should no longer need to include the process of removing the place-value headings to explain why we need a zero in the ones place).

Generalise: '***To multiply a whole number by ten, place a zero after the final digit of that number***.'

Place-value chart – movement is magnitude (ten times the size):



1,000s	100s	10s	1s
			●
		●	
	●		
●			

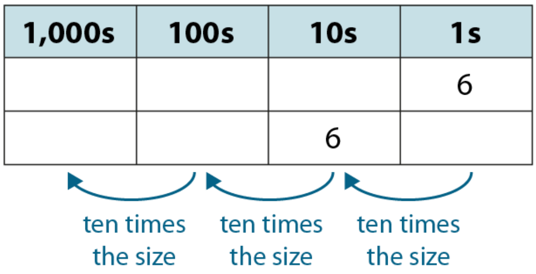
1,000s	100s	10s	1s
			1
		1	
	1		
1			

Multiplying '6' by ten:

- Using known facts:
  - *'Think of "6" and make it ten times the size.'*
  - *'Think of "6" and multiply by ten.'*
    $6 \times 10 = 60$
  - *'Six multiplied by ten is equal to sixty.'*

- On the place-value chart:

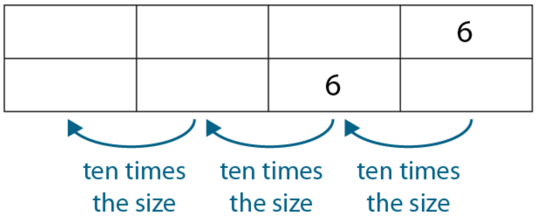
Step 1 – move '6' one place to the left



1,000s	100s	10s	1s
			6
		6	

↓ × 10 *Think of '6' and make it ten times the size.*

Step 2 – remove the place-value headings



			6
		6	

↓ × 10 *Think of '6' and make it ten times the size.*

Step 3 – write a '0' in the ones place

			6
		6	0

↓ × 10 *Think of '6' and make it ten times the size.*

Step 4 – reintroduce the place-value headings

1,000s	**100s**	**10s**	**1s**
			6
		6	0

↓ × 10 *Think of '6' and make it ten times the size.*

Step 5 – summarise and compare the value of the digits

6 × 10 = **60**

'What is the value of the "6" in six?'

- *'six'*
- 6

'What is the value of the "6" in sixty?'

- *'sixty'*
- 60

'We had six ones. We now have six tens.'

1:5 Briefly review how we can visualise the generalisation from step *1:4* on the Gattegno chart and on a ratio chart.

Gattegno chart:

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	**2**0	**3**0	**4**0	**5**0	**6**0	**7**0	**8**0	**9**0
1	2	3	4	5	6	7	8	9

Ratio chart:

× 10 ↓	0	1	2	3	4	5	6	7	8	9
	0	10	20	30	40	50	60	70	80	90

1:6

Now explore how the generalisation reached in step *1:4* can be applied to multiplying two-digit whole numbers by ten.

Begin by using place-value counters to represent a two-digit number. Then:

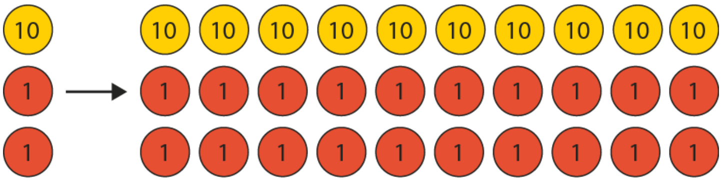
- describe the problem (here, we have 12, which is one ten and two ones, and we want to multiply by ten)
- model gathering one set of counters ten times to represent multiplying by ten, and write a multiplication expression (here, *12 × 10*)
- use the familiar stem sentences to describe the expression:
  - ***'Think of ___ and make it ten times the size.'***
  - ***'Think of ___ and multiply by ten.'***
- complete the multiplication equation using place-value knowledge to describe the counters (here we have ten 10s, which is 100, and twenty 1s, which is 20)
- describe the multiplication *equation* using the stem sentences:
  - ***'___ multiplied by ten is equal to ___.'***
  - ***'___ is ten times the size of ___.'***

Finally, remind children of the generalisation: ***'To multiply a whole number by ten, place a zero after the final digit of that number.'***

Ask children if this generalisation works here for the two-digit number, and conclude that it does.

Multiplicand = 12; multiplier = 10; place-value counters:

'I have twelve. This is one ten and two ones. How much is ten times this amount?'



- *'Think of "12" and make it ten times the size.'*
- *Think of "12" and multiply by ten.'*

12 × 10

- *'We have:'*
  - *'ten tens; that's one hundred'*

  and
  - *'twenty ones; that's twenty'*
  - *'Twelve multiplied by ten is equal to one hundred and twenty'*

  12 × 10 = 120
- *'One hundred and twenty is ten times the size of twelve.'*

1:7

Using the same two-digit number as in step *1:6*, now examine multiplying it by ten on the place-value chart (in the same way as in step *1:4*). Ask children to compare the value of digits before and after multiplying by ten, as shown below.

Repeat for several different two-digit starting values, each time emphasizing the change in value of the digits after multiplying by ten, and the 'new' ones digit of zero.

Multiplicand = 12; multiplier = 10; place-value chart:
Step 1 – move each of the digits one place to the left

1,000s	100s	10s	1s
		1	2
	1	2	

↓ × 10 *Think of '12' and make it ten times the size.*

Step 2 – write a '0' in the ones place

1,000s	100s	10s	1s
		1	2
	1	2	0

↓ × 10 *Think of '12' and make it ten times the size.*

Step 3 – summarise and compare the value of the digits

12	×	10	=	**120**

- *'What is the value of the "2" in twelve?'*
  - *'two'*
  - 2
- *'What is the value of the "1" in twelve?'*
  - *'ten'*
  - 10

'What is the value of the "2" in one hundred and twenty?'
- *'twenty'*
- 20
- *'What is the value of the "1" in one hundred and twenty?'*
  - *'One hundred'*
  - 100

'We had twelve <u>ones</u>. We now have twelve <u>tens</u>.'

1:8

Now work systematically through the numbers 10–20, extending the ratio chart from step *1:5*.

If children make errors such as

13 × 10 = 103 ✗

return to use of the place-value chart, comparison of the value of the digits, and the generalisation. Draw attention to the fact that the digits stay in the same order.

× 10 ↓	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200

1:9

Provide children with practice multiplying by ten/making quantities ten times the size, including:

- missing-number problems (including missing multiplicands, to prepare children for the next teaching point)
- word problems, for example:
  - *'Bethany has twenty-five crayons; Nasir has ten times as many. How many crayons does Nasir have?'*
  - *'Ian has fifteen pence. Tom has ten times as much. How much money does Tom have?'*

Include multiplicands up to and including 25.

Also support children to realise that they can use the *strategy* they have learnt for multiplying by ten to solve grouping problems as well as scaling problems; for example:

- *'If one art-set costs £10. How much do fourteen art-sets cost?'*
  (groups of ten)
- *'There are ten football teams taking part in a tournament. Each team has seventeen players, including substitutes. How many players are there altogether?'*
  (ten equal groups)

Missing-number problems:

'Fill in the missing numbers.'

$\boxed{14} \xrightarrow{\times 10} \boxed{}$ $\qquad$ $\boxed{} \xrightarrow{\times 10} \boxed{170}$

$\boxed{20} \xrightarrow{\text{10 times the size}} \boxed{}$ $\qquad$ $\boxed{} \xrightarrow{\text{10 times the size}} \boxed{250}$

$5 \times 10 = \boxed{}$ $\qquad$ $\boxed{} = 19 \times 10$

$\boxed{} \times 10 = 60$ $\qquad$ $150 = \boxed{} \times 10$

$7 \times \boxed{} = 70$ $\qquad$ $210 = \boxed{} \times 10$

Dòng nǎo jīn:

'Fill in the missing numbers.'

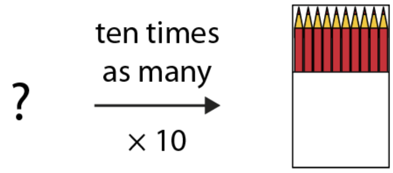
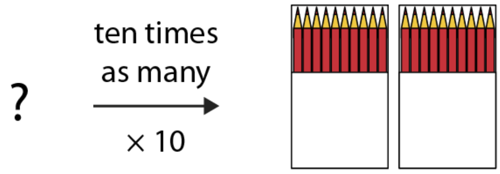
100 × 10 = 1,000
101 × 10 = 1,010
102 × 10 = 1,020
103 × 10 = ☐
104 × 10 = ☐
105 × 10 = ☐
☐ × 10 = 1,060
☐ × 10 = 1,070
☐ × 10 = 1,080
109 × 10 = ☐
110 × 10 = ☐

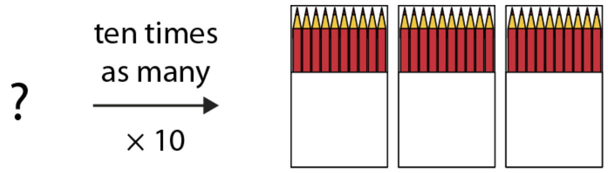
400 × 10 =4,000
410 × 10 = 4,100
420 × 10 = 4,200
430 × 10 = ☐
440 × 10 = ☐
450 × 10 = ☐
☐ × 10 = 4,600
☐ × 10 = 4,700
☐ × 10 = 4,800
490 × 10 = ☐
500 × 10 = ☐

Teaching point 2:

To divide a multiple of 10 by 10, remove the final zero digit (in the ones place) from that number.

Steps in learning

	Guidance	**Representations**
2:1	Now, building on the learning about *multiplying* by ten, work to develop children's understanding of *dividing* by ten. For now, the understanding of division as scaling will be based on the inverse of *'ten times as many'/'ten times the size'*. For example, we will consider contextual problems such as *'Jamie has ten pencils; he has ten times as many pencils as Emily. How many pencils does Emily have?'* In segment *2.17 Structures: multiplication and division as scaling*, fractional language will be used to describe such problems as *'Jamie has ten pencils. Emily has one-tenth as many pencils as Jamie. How many pencils does Emily have?'* It is important to avoid language such as *'Emily has ten times fewer pencils than Jamie'* or *'one is ten times smaller than ten'*, since 'ten times' implies multiplication, and it is not possible to multiply by a whole number and get a product that is less than the multiplicand. Such language is misleading, which is why, until children have covered tenths and hundredths, we only consider division by ten, within scaling, as the inverse of multiplication by ten. Begin by presenting a cardinal problem similar to that in step *1:1*, but with the multiplicand unknown, for example: *'Jamie has ten pencils; he has ten times as many pencils as Emily. How many pencils does Emily have?'* Represent the problem with a multiplication calculation, with missing	Multiplicand = ?; multiplier = 10; product = 10: *'Jamie has ten pencils; he has ten times as many pencils as Emily. How many pencils does Emily have?'*  ☐ × 10 = 10 10 ÷ 10 = ☐ - *'One multiplied by ten is equal to ten.'* **1** × 10 = 10 - *'Ten divided by ten is equal to one.'* 10 ÷ 10 = **1** - *'Ten is ten times the size of one.'* - *'Emily has one pencil.'* Multiplicand = ?; multiplier = 10; product = 20: *'Jamie has twenty pencils; he has ten times as many pencils as Emily. How many pencils does Emily have?'*  ☐ × 10 = 20 20 ÷ 10 = ☐ - *'Two multiplied by ten is equal to twenty.'* **2** × 10 = 20 - *'Twenty divided by ten is equal to two.'* 20 ÷ 10 = **2** - *'Twenty is ten times the size of two.'* - *'Emily has two pencils.'*

	multiplicand (? × 10 = 10), and use the known fact to complete the equation. Describe it using 'multiplied by' language as shown opposite. Then ask children how the missing-multiplicand equation could be represented by a division equation (*10 ÷ 10 = ?*), complete that equation, and describe it using 'divided by' language as shown opposite. Then repeat the problem, this time with Jamie having twenty pencils. Use the following stem sentences from step *1:1*, this time emphasising the quantity that is made ten times the size (as exemplified opposite): - ***'___ multiplied by ten is equal to ___.'*** - ***'___ is ten times the size of ___.'*** Use the following stem sentences to describe the division equation and the solution to the problem: - ***'___ divided by ten is equal to ___.'*** - ***'Emily has ___ pencils.'*** Continue, systematically, increasing the number of pencils that Jamie has (in multiples of ten) until children become confident with the language and patterns. Work towards the following generalisation: ***'To find the inverse of ten times as many, divide by ten.'*** Teachers should note that this does not apply to negative numbers. This will be developed further in segment *2.17*, where children will use the language of fractions, for example, *'To find one tenth as many, divide by ten.'*	Multiplicand = ?; multiplier = 10; product = 30: *'Jamie has thirty pencils; he has ten times as many pencils as Emily. How many pencils does Emily have?'*  $\boxed{} \times 10 = 30$ $30 \div 10 = \boxed{}$ - *'Three multiplied by ten is equal to thirty.'* $\mathbf{3} \times 10 = 10$ - *'Thirty divided by ten is equal to three.'* $10 \div 10 = \mathbf{3}$ - *'Thirty is ten times the size of three.'* - *'Emily has three pencils.'*

2:2

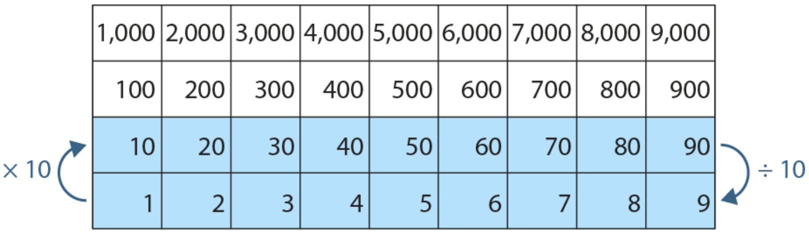
Now use the Gattegno chart to review, systematically, dividing each of the multiples of ten from 10–90 by ten. Use the following stem sentences to link division by ten to the inverse of multiplication by ten:

- ***'___ multiplied by ten is equal to ___.'***
- ***'___ is ten times the size of ___.'***
- ***'___ divided by ten is equal to ___.'***

For each calculation, write out the accompanying multiplication and division equations.

Draw attention to the fact that all of the numbers in the 'tens' row are multiples of ten, and that all of the dividends in the division equations are multiples of ten.

Gattegno chart:



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

2:3

By now, children will probably be picking up on the idea that to divide a multiple of ten by ten, we can remove the zero from the ones place. However, as in step *1:4*, it is important that children undertand *why* this works. Explore this now, using a place-value chart.

Take a multiple of ten, such as 90, and divide it by ten, writing an equation and describing it as shown below (see 'using known facts'). Then show how we can record the same calculation on the place-value chart:

- record '90' on the place-value chart, describing how we have '9' in the tens column and '0' in the ones column
- ask children what direction we would move the digits in if we were *multiplying* by ten (to the left) and then ask them to reason what direction we need to move the digits in to *divide* by ten (to the right); move both digits to the right, emphasising the need to keep them in the same order (because the smallest value column in the place-value chart is the ones, the '0' will end up outside the chart; since children have not yet learnt about the decimal point (see *Spine 1: Number, Addition and Subtraction*, segment *1.23*) simply cross out the zero)
- finally, emphasise that the '9' is now in the ones column and we no longer have the '0', by asking children to compare the value of the digit before and after dividing by ten, as shown on the next page.

Repeat for several different multiple-of-ten starting values, each time emphasising that when we've divided by ten, the tens digit is now in the ones place, and the zero has been removed.

Generalise: ***'To divide a multiple of ten by ten, remove the zero from the ones place.'***

Dividing by ten – using known facts:

90 ÷ 10 = ?

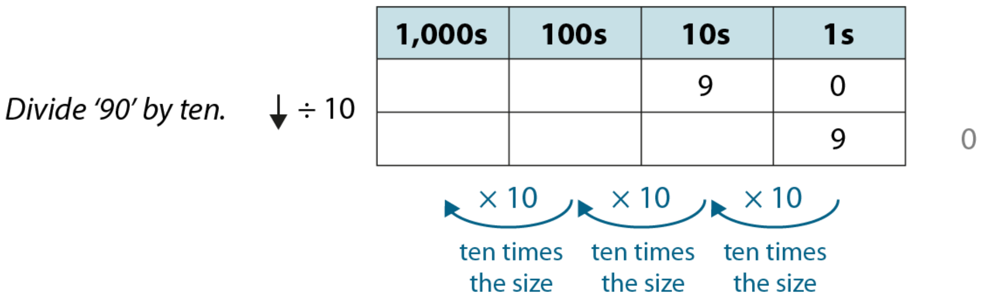
- *'I know that nine times ten is equal to ninety.'*

9 × 10 = 90

- *'So, ninety divided by ten is equal to nine.'*

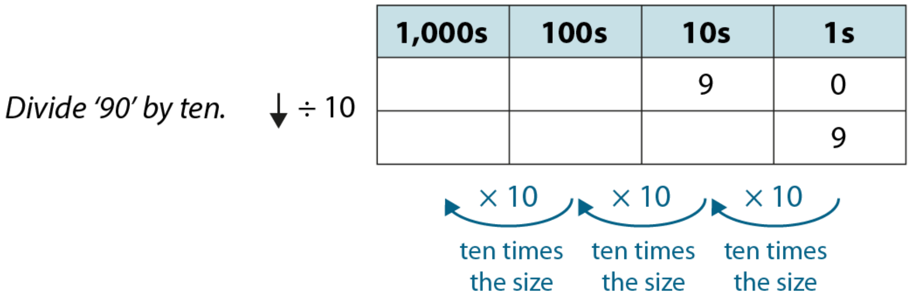
90 ÷ 10 = 9

Dividing by 10 – place-value chart:
Step 1 – move each of the digits one place to the right



1,000s	100s	10s	1s
		9	0
			9

Step 2 – remove zero



1,000s	100s	10s	1s
		9	0
			9

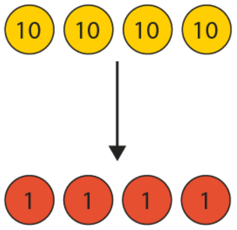
Step 3 – summarise and compare the value of the digits

<table>
<tr><th>90</th><th>÷</th><th>10</th><th>=</th><th>9</th></tr>
<tr><td><i>'What is the value of the "9" in ninety?'</i>
• <i>'ninety'</i>
• 90</td><td></td><td></td><td></td><td><i>'What is the value of the "9" in nine?'</i>
• <i>'nine'</i>
• 9</td></tr>
<tr><td colspan="5"><i>'We had nine <u>tens</u>. We now have nine <u>ones</u>.'</i></td></tr>
</table>

2:4

You can further emphasise the removal of the zero by using place-value counters and unitising language, as shown opposite.

'I have forty. This is four tens. How much will I have if I divide by ten?'



4 tens ÷ 10 = 4 ones

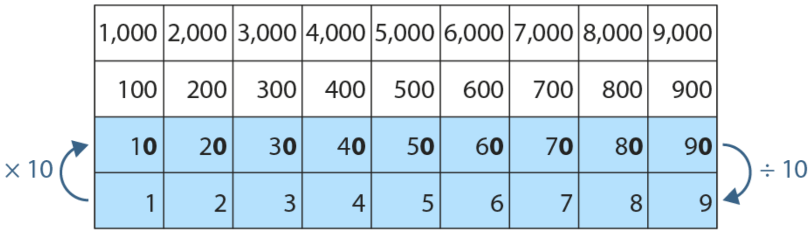
40 ÷ 10 = 4

- *'We had four tens. We now have four ones.'*
- *'Four tens divided by ten is equal to four ones.'*
- *'Forty divided by ten is equal to four.'*

2:5

Briefly review how we can visualise the generalisation from step *2:3* on the Gattegno chart and on a ratio chart.

Gattegno chart:



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Ratio chart:

÷ 10 ↑	0	1	2	3	4	5	6	7	8	9
	0	10	20	30	40	50	60	70	80	90

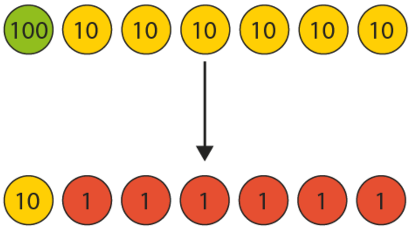
2:6

Now explore how the generalisation reached in step *2:3* can be applied to dividing three-digit multiples of ten by ten. Remind children that 100 is equal to ten 10s (*Spine 1: Number, Addition and Subtraction*, segment *1.17* and above). Then use place-value counters and unitising language, and a place-value chart in the same way as in steps *2:3* and *2:4*.

Repeat for several different three-digit multiples of ten, each time emphasising the change in value of the digits after dividing by ten, and removal of the zero from the ones place.

Dividing by ten – place-value counters:

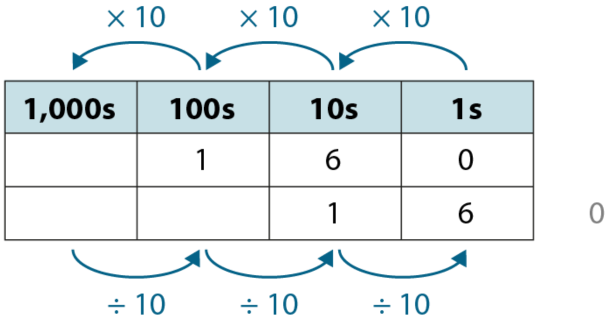
'I have one hundred and sixty. This is one hundred, and six tens. How much will I have if I divide by ten?'



160 ÷ 10 = 16

Dividing by ten – place-value chart:
Step 1 – move each of the digits one place to the right

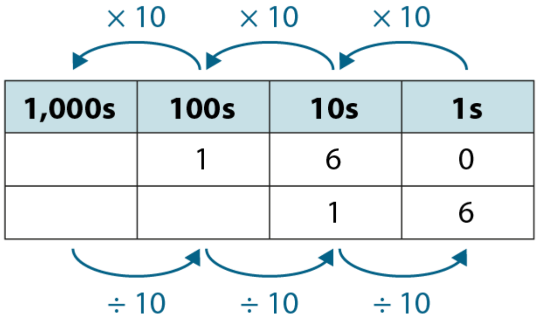
Divide '160' by ten. ↓ ÷ 10



1,000s	100s	10s	1s
	1	6	0
		1	6

Step 2 – remove zero

Divide '160' by ten. ↓ ÷ 10



1,000s	100s	10s	1s
	1	6	0
		1	6

Step 3 – summarise and compare the value of the digits

160 ÷ 10 = **16**

- *'What is the value of the "6" in one hundred and sixty?'*
  - *'sixty'*
  - 60
- *'What is the value of the "1" in one hundred and sixty?'*
  - *'one hundred'*
  - 100

- *'What is the value of the "6" in sixteen?'*
  - *'six'*
  - 6
- *'What is the value of the "1" in sixteen?'*
  - *'Ten'*
  - 10

'We had sixteen tens. We now have sixteen ones.'

2:7

Now work systematically to extend the ratio chart from step *2:5*, dividing each of the multiples of ten from 100–200 by ten.

÷ 10 ↑	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200

2:8

Provide children with practice dividing by ten, including:

- missing-number problems (including those that support the link between multiplying and dividing by ten)
- true/false style problems
- word problems, for example:
  - *'Nasir has one hundred and forty crayons; he has ten times as many crayons as Bethany. How many crayons does Bethany have?'*
  - *'Tom has saved £150; he has saved ten times as much as Ian. How much money has Ian saved?'*

Include dividends up to and including 250.

Also support children to realise that they can use the *strategy* they have learnt for dividing by ten to solve quotitive and partitive division problems, for example:

- *'Ten people can fit in a minibus. How many minibuses are needed for one hundred and fifty people?'*
  (quotitive)
- *'One hundred and forty tins of beans are shared equally between ten boxes. How many tins will there be in each box?'*
  (partitive)

Missing-number problems:

'Fill in the missing numbers.'

$$\begin{tikzcd} \boxed{14} \arrow[r, shift left, "\times 10"] & \boxed{} \arrow[l, shift left, "\div 10"] \end{tikzcd} \qquad \begin{tikzcd} \boxed{} \arrow[r, shift left, "\times 10"] & \boxed{250} \arrow[l, shift left, "\div 10"] \end{tikzcd}$$

$$\begin{tikzcd} \boxed{60} \arrow[r, "\div 10"] & \boxed{} \end{tikzcd} \qquad \begin{tikzcd} \boxed{} \arrow[r, "\div 10"] & \boxed{13} \end{tikzcd}$$

$$\begin{tikzcd} \boxed{190} & \boxed{} \arrow[l, "\text{10 times the size}"'] \end{tikzcd} \qquad \begin{tikzcd} \boxed{180} & \boxed{} \arrow[l, "\text{10 times the size}"'] \end{tikzcd}$$

$14 \times 10 = \boxed{}$

$140 \div 10 = \boxed{}$

$27 \times 10 = \boxed{}$

$\boxed{} \div 10 = 27$

$40 \div 10 = \boxed{}$

$\boxed{}$

$\boxed{} \div 10 = 7$

$\boxed{} = 17 \div 10$

$12 = \boxed{} \div 10$

$20 = \boxed{} \div 10$

True/false problem:

'Decide whether each calculation will result in a multiple of ten or not.'

	Answer is a multiple of 10: true (✓) or false (×)?
25 × 10	
250 ÷ 10	
50 ÷ 10	
12 × 10	
9 × 10	
200 ÷ 10	
6 × 10	

Dòng nǎo jīn:

'Fill in the missing numbers.'

1,000 ÷ 10 = 100

1,010 ÷ 10 = 101

1,020 ÷ 10 = 102

1,030 ÷ 10 = ☐

1,040 ÷ 10 = ☐

☐ ÷ 10 = 105

☐ ÷ 10 = 106

1,070 ÷ ☐ = 107

1,080 ÷ ☐ = 108

1,090 ÷ 10 = ☐

1,040 ÷ 10 = ☐

4,000 ÷ 10 = 400

4,100 ÷ 10 = 410

4,200 ÷ 10 = 420

4,300 ÷ 10 = ☐

4,400 ÷10 = ☐

☐ ÷ 10 = 450

☐ ÷ 10 = 460

4,700 ÷ ☐ = 470

4,800 ÷ ☐ = 480

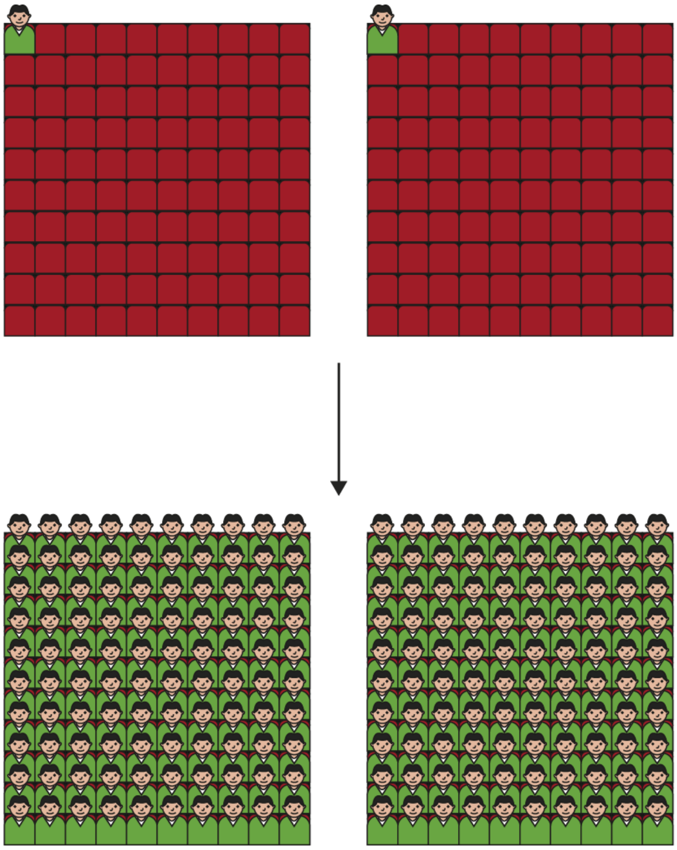
4,900 ÷ 10 = ☐

5,000 ÷ 10 = ☐

Teaching point 3:

Finding 100 times as many is the same as multiplying by 100 (for positive numbers); to multiply a whole number by 100, place two zeros after the final digit of that number.

Steps in learning

	Guidance	**Representations**
3:1	This teaching point follows a similar progression to *Teaching point 1*, but now we are multiplying by 100, rather than by ten. A selection of representations are provided here, alongside brief guidance; for more detail see *Teaching point 1*. Begin by briefly reviewing skip counting in multiples of 100 up to 1,000 (see *Spine 1: Number, Addition and Subtraction*, segment *1.18*), using a number line or the Gattegno chart for support. Count in two ways: • *'One one hundred, two one hundreds, three one hundreds… ten one hundreds.'* • *'One hundred, two hundred, three hundred… one thousand.'* Then, present a cardinal problem corresponding to a multiplicand of '1' and a multiplier of '100'; i.e. making a value of '1' 100 times larger; for example: *'This afternoon there was one person in the cinema. This evening there are one hundred times as many people in the cinema. How many people are in the cinema this evening?'* Model gathering one item, 100 times, to help children understand why the multiplication calculation $1 \times 100 = ?$ represents the problem; this will be a fairly lengthy process, but it is worth working through it at least once. Use the following sentences emphasising the equivalence between *'one hundred times the size'* and *'multiply by one hundred'*:	Example – multiplicand = 2; multiplier = 100: *'This afternoon there were two people in the cinema. This evening there are one hundred times as many people in the cinema. How many people are in the cinema this evening?'*  • *'Think of "2" and make it one hundred times the size.'* • *'Think of "2" and multiply by one hundred.'* 2×100 • *'Two multiplied by one hundred is equal to two hundred.'* $2 \times 100 = 200$ • *'Two hundred is one hundred times the size of two.'* • *'Two hundred people is one hundred times as many as two people. There are two hundred people in the cinema this evening.'*

- *'Think of "1" and make it one hundred times the size.'*
- *'Think of "1" and multiply by one hundred.'*

Completing the multiplication equation, then use the following sentences:

- *'One multiplied by one hundred is equal to one hundred.'*
- *'One hundred is one hundred times the size of one.'*

Finally connect back to the context:

- *'One hundred people is one hundred times as many as one person. There are one hundred people in the cinema this evening.'*

Repeat the problem, this time beginning with two people in the cinema. Use the following stem sentences to describe the calculation, equation and context respectively:

- ***'Think of ___ and make it one hundred times the size.'***
  'Think of ___ and multiply by one hundred.'
- ***'___ multiplied by one hundred is equal to ___.'***
  '___ is one hundred times the size of ___.'
- ***'___ people is one hundred times as many as ___ people. There are ___ people in the cinema this evening.'***

Continue, systematically increasing the multiplicand (the initial number of people in the cinema) until children become confident with the language and patterns. Work towards the following generalisation: *'**To find one hundred times as many, multiply by one hundred.'***

As for step *1:1*, teachers should note that this language does not apply to negative numbers.

3:2

Using the examples from the previous step, draw children’s attention to the fact that the products are all multiples of 100. Ask children:

- what the multiples of 100 have in common (they all have a zero in both the tens place and the ones place)
- to identify what digit is in each place-value position.

Generalise: ‘***All multiples of one hundred have both a tens and ones digit of zero.’***

Practise, as a class, sorting numbers according to whether they are multiples of 100 or not, encouraging children to explain their reasoning.

Comparing and describing multiples of 100:

- 1 × 100 = **100**
- 2 × 100 = **200**
- 3 × 100 = **300**

- *‘What digit is in the hundreds place in “300”?’*
  3
- *‘What digit is in the tens place in ‘“300”?’*
  0
- *‘What digit is in the ones place in ‘“300”?’*
  0

Sorting activity:

- *‘Put each number into the correct column according to whether it is a multiple of one hundred or not.’*

0 300 150 400 610

700 601 4000 4001

Multiple of 100	Not a multiple of 100

- Dòng nǎo jīn:
  ‘Work out what number was multiplied by one hundred to get each of the numbers in the “multiple of 100” column in the table above.’

3:3

Now use the Gattegno chart to review, systematically, multiplying each of the numbers one to nine by 100, applying the language introduced in step *3:1*:

- ***‘___ multiplied by one hundred is equal to ___.’***
- ***‘___ is one hundred times the size of ___.’***

For each calculation, write out the accompanying multiplication equation.

Bring together learning from the previous steps to generalise: ***‘When a number is multiplied by one hundred, the product is a multiple of one hundred.’***

Gattegno chart:

× 100

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

3:4

Most children will likely see the emerging pattern: namely that to multiply a whole number by 100, we can place two zeros after the final digit of that number. Explore this now using a place-value chart. Begin, as in step *1:4*, by focusing on the 'movement is magnitude' principle: if we move a counter, or digit, *two* places to the left, the value of that digit becomes *100* times the size, so a digit in the hundreds column has a value 100 times the size of the same digit when in the ones column.

Then multiply a positive integer (such as '6') by 100, using the place-value chart, as shown below. Work through several different single-digit starting values. Then generalise: ***'To multiply a whole number by one hundred, place two zeros after the final digit of that number.'***

Place-value chart – movement is magnitude (one hundred times the size):

1,000s	**100s**	**10s**	**1s**
			●
	●		

100 times the size

1,000s	**100s**	**10s**	**1s**
			1
	1		

100 times the size

Multiplying by 100 using a place-value chart:
Step 1 – move '6' <u>two</u> places to the left

1,000s	**100s**	**10s**	**1s**
			6
	6		

↓ × 100 *Think of '6' and make it 100 times the size.*

100 times the size

Step 2 – write zeros in the tens and ones places

1,000s	**100s**	**10s**	**1s**
			6
	6	0	0

↓ × 100 *Think of '6' and make it 100 times the size.*

100 times the size

Step 3 – summarise and compare the value of the digits

6	×	100	=	**600**

'What is the value of the "6" in six?'

- *'six'*
- 6

'What is the value of the "6" in six hundred?'

- *'six hundred'*
- 600

'We had six ones. We now have six hundreds.'

3:5 Briefly review how we can visualise the generalisation from step *3:4* on the Gattegno chart and on a ratio chart.

Gattegno chart:

× 100

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	**200**	**300**	**400**	**500**	**600**	**700**	**800**	**900**
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Ratio chart:

× 100 ↓	0	1	2	3	4	5	6	7	8	9
	0	100	200	300	400	500	600	700	800	900

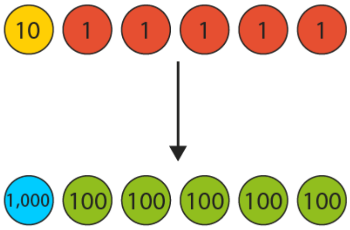
3:6 Now explore how the generalisation reached in step *3:4* can be applied to multiplying two-digit whole numbers by 100. Remind children that 1000 is equal to ten 100s, or 100 tens (*Spine 1: Number, Addition and Subtraction*, segment *1.22*).

Use place-value counters and unitising language, and then a place-value chart, in the same way as in steps *1:6* and *1:7*, but now for multiplying by 100.

Repeat for several different two-digit starting values, each time emphasising the change in value of the digits after multiplying by 100, and the 'new' tens and ones digits of zero.

Multiplicand = 15; multiplier = 100; place-value counters:

'I have fifteen. This is one ten and five ones. How much is one hundred times this amount?'

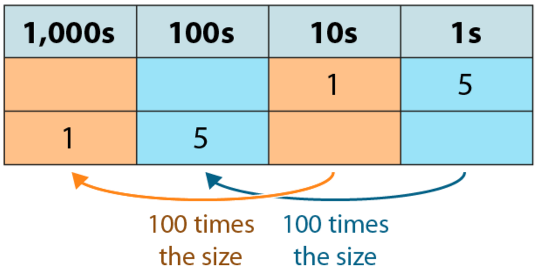


- *'Fifteen multiplied by one hundred is equal to one thousand five hundred.'*

  15 × 100 = 1500
- *'One thousand five hundred is one hundred times the size of fifteen.'*

Multiplicand = 15; multiplier = 100; place-value chart:

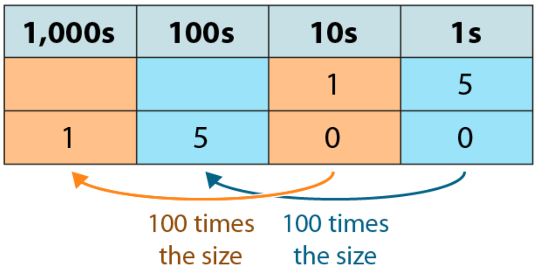
Step 1 – move each of the digits <u>two</u> places to the left



1,000s	100s	10s	1s
		1	5
1	5		

↓ × 100 *Think of '15' and make it 100 times the size.*

Step 2 – introduce zeros in the tens and ones places



1,000s	100s	10s	1s
		1	5
1	5	0	0

↓ × 100 *Think of '15' and make it 100 times the size.*

Step 3 – summarise and compare the value of the digits

15	×	100	=	**1,500**
- *'What is the value of the "5" in fifteen?'* - *'five'* - 5				- *'What is the value of the "5" in one thousand five hundred?'* - *'five hundred'* - 500
- *'What is the value of the "1" in fifteen?'* - *'ten'* - 10				- *'What is the value of the "1" in one thousand five hundred?'* - *'One thousand'* - 1000

'We had fifteen <u>ones</u>. We now have fifteen <u>hundreds</u>.'

3:7 Work systematically through the numbers 10–20, extending the ratio chart from step *3:5*.

× 100 ↓	…	9	10	11	12	13	14	15	16	17	18	19	20
	…	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800	1,900	2,000

3:8 Provide children with practice multiplying by 100/making quantities 100 times the size.

Example word problems:

- *'Bethany has fifteen marbles; Nasir has one hundred times as many. How many marbles does Nasir have?'*
- *'Ian has twenty pence. Tom has one hundred times as much. How much money does Tom have?'*

Include multiplicands up to and including 25.

Also support children to realise that they can use the *strategy* they have learnt for multiplying by 100 to solve grouping problems as well as scaling problems; for example:

- *'If one bike costs £100. How much do fourteen bikes cost?'*
  (groups of 100)
- *'There are one hundred football teams in a particular league. Each team has seventeen players, including substitutes. How many players are there altogether?'*
  (100 equal groups)

Missing-number problems:

'Fill in the missing numbers.'

$\boxed{14} \xrightarrow{\times 100} \boxed{}$

$\boxed{} \xrightarrow{\times 100} \boxed{1{,}700}$

$\boxed{20} \xrightarrow{\text{100 times the size}} \boxed{}$

$\boxed{} \xrightarrow{\text{100 times the size}} \boxed{2{,}500}$

$5 \times 100 = \boxed{}$

$\boxed{} = 19 \times 100$

$\boxed{} \times 100 = 600$

$1{,}500 = \boxed{} \times 100$

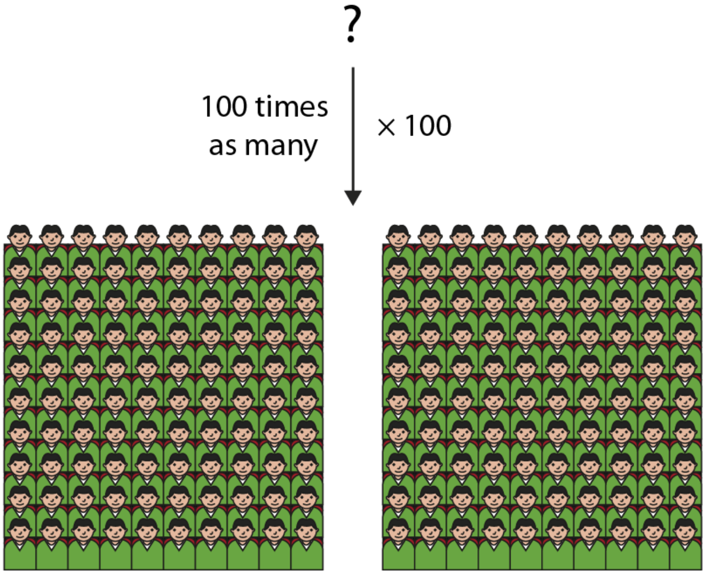
$7 \times \boxed{} = 700$

$2{,}100 = \boxed{} \times 100$

Teaching point 4:

To divide a multiple of 100 by 100, remove the final two zero digits (in the tens and ones places) from that number.

Steps in learning

	Guidance	**Representations**
4:1	Now, building on the learning about *multiplying* by 100, work to develop children's understanding of *dividing* by 100. As in *Teaching point 2*, the understanding of division as scaling will be based on the inverse of '100 times as many'/'100 times the size'. Again, it is important to avoid language such as *'This afternoon there were one hundred times fewer people in the cinema'* or *'one is one hundred times smaller than one hundred'*, since 'one hundred times' implies multiplication, and it is not possible to multiply by a whole number and get a smaller product. Begin by working through a cardinal problem similar to that in step *3:1*, but with the multiplicand (of '1') unknown; for example: '*This evening there are one hundred people in the cinema, one hundred times as many people as there were this afternoon. How many people were in the cinema this afternoon?'* Connect the missing-multiplicand equation (*? × 100 = 100*) with the corresponding division equation (*100 ÷ 100 = ?*), using the language developed in step 2*:1*, now adapted for division by 100: - ***'___ multiplied by one hundred is equal to ___.'*** - ***'___ is one hundred times the size of ___.'***	Example – multiplicand = ?; multiplier = 100; product = 200: *'This evening there are two hundred people in the cinema, one hundred times as many people as there were this afternoon. How many people were in the cinema this afternoon?'*  $\square \times 100 = 200$ $200 \div 100 = \square$ - *'Two multiplied by one hundred is equal to two hundred.'* **2** × 100 = 200 - *'Two hundred divided by one hundred is equal to two.'* 200 ÷ 100 = **2** - *'Two hundred is one hundred times the size of two.'* - *'There were two people in the cinema.'*

(Emphasise the quantity that is made 100 times the size, as indicated in the example on the previous page.)

- ***'___ divided by one hundred is equal to ___.'***

Then connect back to the cinema context using the following stem sentence: ***'There were ___ in the cinema.'***

Repeat the problem, increasing the unknown multiplicand by one each time. For more detailed guidance, see step *2:1*.

Work towards the following generalisation: '***To find the inverse of one hundred times as many, divide by one hundred.'***

As before, teachers should note that this language does not apply to negative numbers. This will be developed further in segment *2.17 Structures: multiplication and division as scaling*, where children will use the language of fractions, for example, *'To find one hundredth as many, divide by one hundred.'*

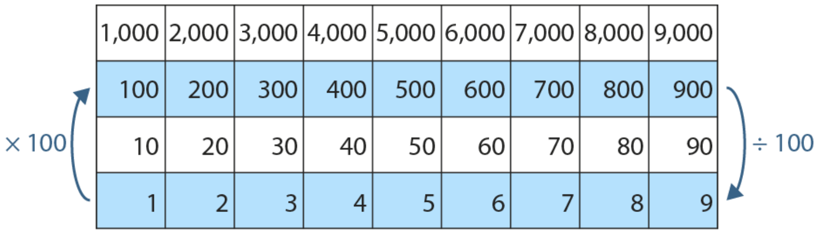
4:2 Now use the Gattegno chart to review, systematically, dividing each of the multiples of 100 from 100–900 by 100. Use the following stem sentences to link division by 100 to the inverse of multiplication by 100:

- ***'___ multiplied by one hundred is equal to ___.'***
- ***'___ is one hundred times the size of ___.'***
- ***'___ divided by one hundred is equal to ___.'***

For each calculation, write out the accompanying multiplication and division equations.

Draw attention to the fact that all of the numbers in the 'hundreds' row are multiples of 100, and that all of the dividends in the division equations are multiples of 100.

Gattegno chart:



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

4:3

Similarly to step *2:3*, now use a place-value chart to explore the strategy of removing two zeros to divide a multiple of 100 by 100.

Take a multiple of 100, such as 900, and divide it by 100, writing an equation and describing it as shown below (see 'using known facts'; alternatively use place-value counters and unitising language as in step *2:3*, i.e. *'We had nine hundreds. We now have nine ones.'*). Then show how we can record the same calculation on the place-value chart. When moving the digits to divide by 100, ask children what direction we would move the digits if we were *multiplying* by 100 (to the left), and then ask them to reason what direction we need to move the digits in to *divide* by 100 (to the right). Following division, emphasise that the '9' is now in the ones column and we no longer need the two zeros, by asking children to compare the value of the digit before and after, as shown below.

Repeat for several different multiple-of-100 starting values, each time emphasising that when we've divided by 100, the 100s digit is now in the ones place, and two zeros have been removed.

Generalise: ***'To divide a multiple of one hundred by one hundred, remove two zeros (from the tens and ones places).'***

Dividing by 100 – using known facts:

900 ÷ 100 = ?

- *'I know that nine times one hundred is equal to nine hundred.'*

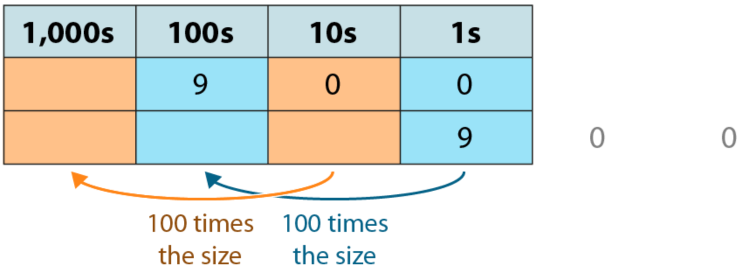
9 × 100 = 900

- *'So, nine hundred divided by one hundred is equal to nine.'*

900 ÷ 100 = 9

Dividing by 100 – place-value chart:

Divide '900' by 100. ↓ ÷ 100



1,000s	100s	10s	1s		
	9	0	0		
			9	0	0

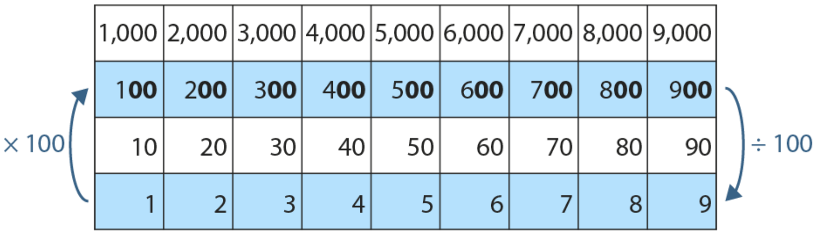
Summarise and compare the value of the digits:

900	÷	100	=	**9**
'What is the value of the "9" in nine hundred?' • *'nine hundred'* • 900				*'What is the value of the "9" in nine?'* • *'nine'* • 9
'We had nine hundreds. We now have nine ones.'				

4:4

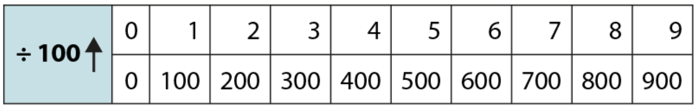
Briefly review how we can visualise the generalisation from step *4:3* on the Gattegno chart and on a ratio chart.

Gattegno chart:



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	**200**	**300**	**400**	**500**	**600**	**700**	**800**	**900**
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Ratio chart:



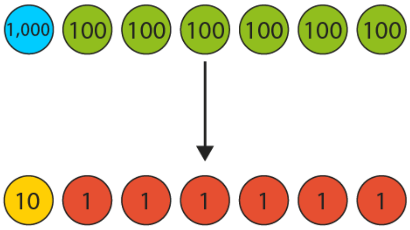
÷ 100 ↑	0	1	2	3	4	5	6	7	8	9
	0	100	200	300	400	500	600	700	800	900

4:5

Finally, explore how the generalisation can be applied to dividing four-digit multiples of 100 by 100. Work through several different dividends, each time emphasising the change in value of the digits after dividing by 100, and removal of two zeros (removing the zeros from the tens and ones places).

Dividing by 100 – place-value counters:

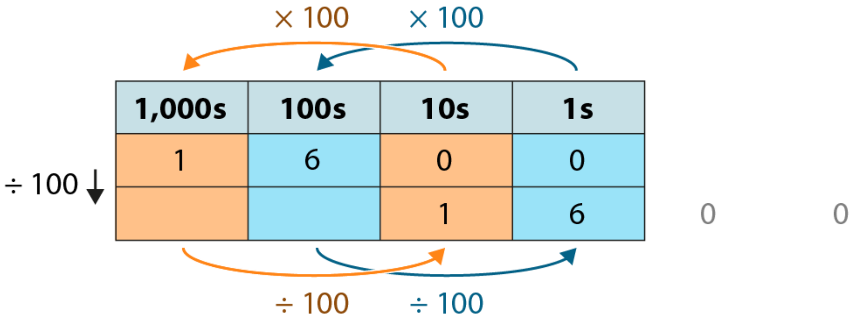
'I have one thousand six hundred. This is one thousand, and six hundreds. How much will I have if I divide by one hundred?'



1,600 ÷ 100 = 16

Dividing by 100 – place-value chart:

Divide '1,600' by 100.



1,000s	100s	10s	1s
1	6	0	0
		1	6

Summarise and compare the value of the digits:

1,600 ÷ 100 = **16**

- *'What is the value of the "6" in one thousand six hundred?'*
  - *'six hundred'*
  - 600
- *What is the value of the "1" in one thousand six hundred?'*
  - *'one thousand'*
  - 1,000

- *'What is the value of the "6" in sixteen?'*
  - *'six'*
  - 6
- *'What is the value of the "1" in sixteen?'*
  - *'ten'*
  - 10

'We had sixteen hundreds. We now have sixteen ones.'

4:6 Now work systematically to extend the ratio chart from step *4:4*, dividing each of the multiples of 100 from 1,000–2,000 by 100.

÷ 100 ↑	…	9	10	11	12	13	14	15	16	17	18	19	20
	…	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800	1,900	2,000

4:7 Provide children with practice dividing by 100, including:

- missing-number problems (including those that support the link between multiplying and dividing by 100)
- true/false style problems
- word problems, for example:
  - *'Freya is doing a jigsaw puzzle with three thousand pieces. Freya's puzzle has one hundred times as many pieces as Eliza's puzzle; how many pieces does Eliza's puzzle have?'*
  - *'Nathan bought some sweets for a party. He bought some flying-saucers and some cola-bottles. He bought three hundred cola-bottles, one hundred times as many as the number of flying-saucers. How many flying-saucers did Nathan buy?'*

Include dividends up to and including 2,500.

Missing-number problems:

'Fill in the missing numbers.'

$$\boxed{11} \underset{\div 100}{\overset{\times 100}{\rightleftarrows}} \boxed{} \qquad \boxed{} \underset{\div 100}{\overset{\times 100}{\rightleftarrows}} \boxed{3{,}500}$$

$$\boxed{600} \xrightarrow{\div 100} \boxed{} \qquad \boxed{} \xrightarrow{\div 100} \boxed{13}$$

Also support children to realise that they can use the *strategy* they have learnt for dividing by 100 to solve quotitive and partitive division problems; for example:

- *'Ina has collected 1,600 p. There are one hundred pennies in a pound, so how many pound coins could she swap her pennies for?'*
  (quotitive)
- *'Danilo cooks eight hundred cocktail sausages for his party. One hundred guests come to the party. How many sausages will each guest get if they share them equally?'*
  (partitive)

100 times the size
←
1,900 ☐

100 times the size
←
1,800 ☐

14 × 100 = ☐

27 × 100 = ☐

1,400 ÷ 100 = ☐

☐ ÷ 100 = 27

400 ÷ 100 = ☐

☐ = 17 ÷ 100

☐ ÷ 100 = 8

1,200 = ☐ ÷ 100

☐ ÷ 100 = 7

2,000 = ☐ ÷ 100

True/false problem:

'Decide whether each calculation will result in a multiple of 100 or not.'

	Answer is a multiple of 100: true (✓) or false (✗)?
25 × 100	
2,500 ÷ 100	
500 ÷ 100	
12 × 100	
9 × 100	
2,000 ÷ 100	
6 × 100	

Teaching point 5:

Multiplying a number by 100 is equivalent to multiplying by 10, and then multiplying the product by 10. Dividing a multiple of 100 by 100 is equivalent to dividing by 10, and then dividing the quotient by 10.

Steps in learning

5:1

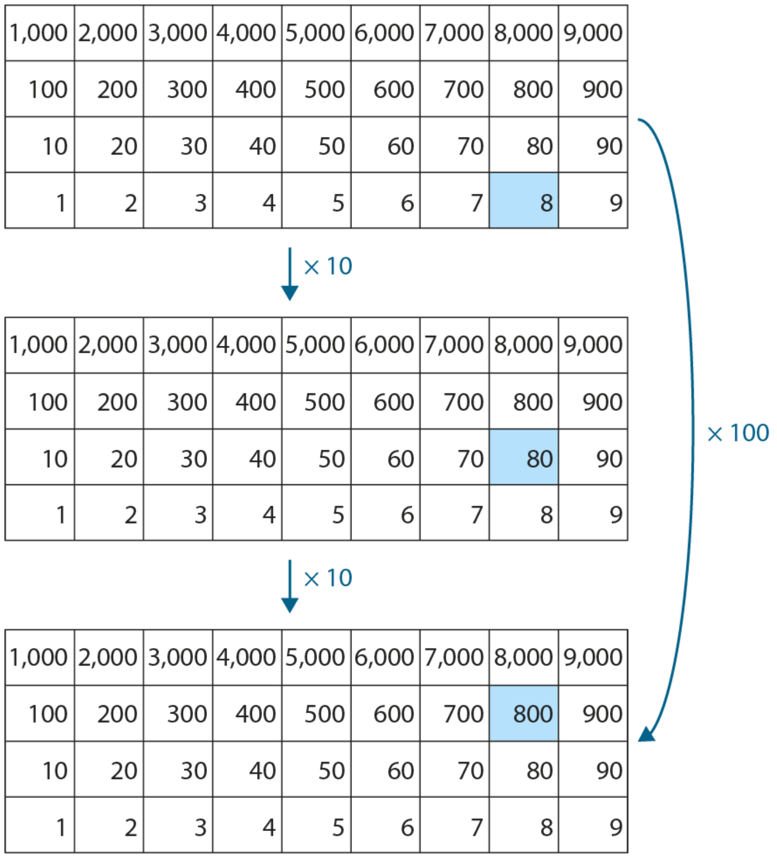
This teaching point connects the learning from *Teaching points 1–4*. Begin by using the Gattegno chart to explore the link between multiplying by 100 and multiplying by 10 and 10 again. For example, instruct children:

- *'Put your finger on "8".'*
- *'Move your finger to multiply by ten.'*
- *'Move your finger to multiply by ten again.'*
- *'What number are you on?'*
  (800)

Then:

- *'Put your finger on "8".'*
- *'Move your finger to multiply by one hundred.'*
- *'What number are you on?'*
  (800)
- *'What do you notice?'*

Repeat with other single-digit starting values, working towards the generalisation: ***'Multiplying by one hundred is equivalent to multiplying by ten, and then multiplying by ten again.'***



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

5:2 Write equations and use a place-value chart, as below, to show how the generalisations children have already learnt for multiplying by 10 and by 100 (by placing one or two zeros as the final digits, respectively) support this understanding.

Ensure that children do not write equations such as:

$8 \times 10 = 80 \times 10 = 800$ ✗

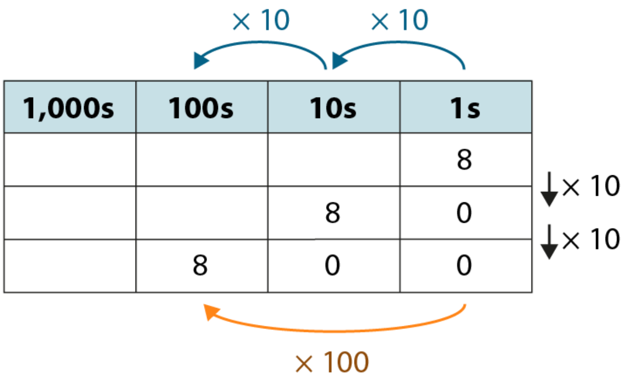
If any children do this, remind them of the meaning of the '=' sign, and ask them to write each operation as a separate equation.

Equations:

$8 \times 10 = 80$	8 multiplied by 10: place a '0' in the ones place.
$80 \times 10 = 800$	80 multiplied by 10: place a '0' in the ones place.
$8 \times 100 = 800$	8 multiplied by 100: place two '0's.

$8 \times 10 \times 10 = 8 \times 100$

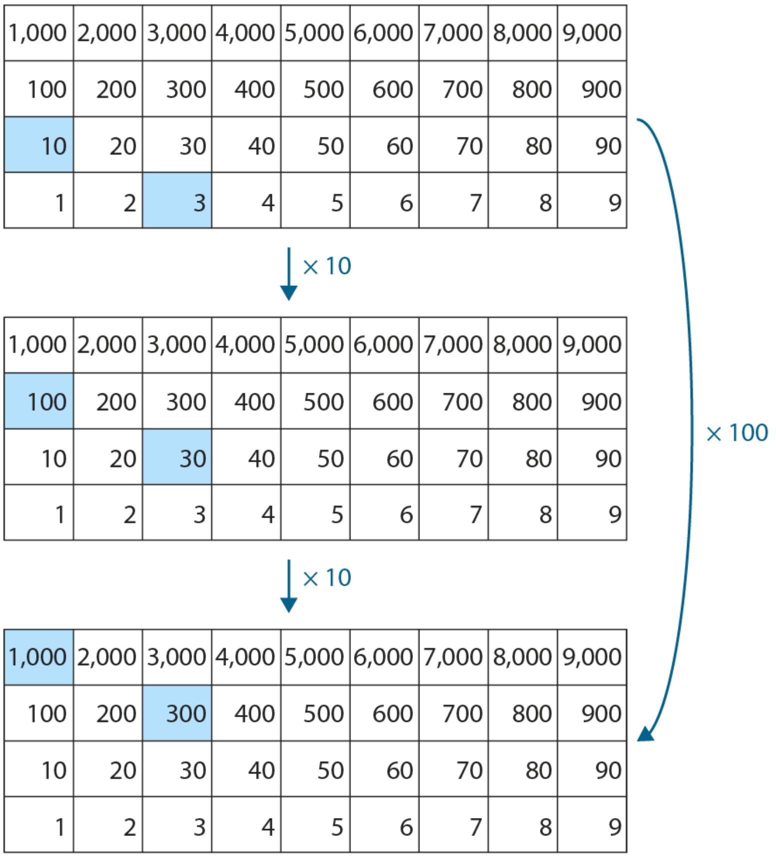
Place-value chart:



1,000s	100s	10s	1s
			8
		8	0
	8	0	0

5:3 Now progress beyond a multiplicand of '9', using the Gattegno chart as shown below for selected multiplicands. Then, as a class, build a ratio chart. If necessary, write equations or use a place-value chart for support as in the previous step.

Gattegno chart and equations:



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

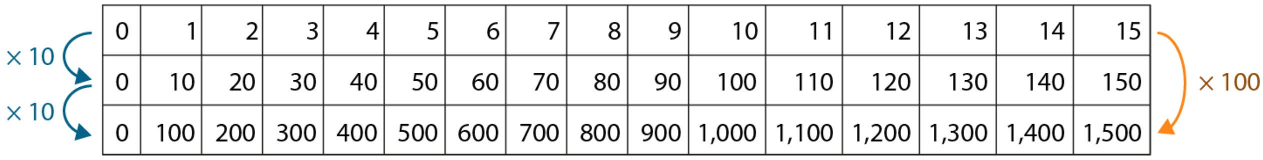
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

13 × 10 = 130
130 × 10 = 1300

13 × 100 = 1300 13 × 10 × 10 = 13 × 100

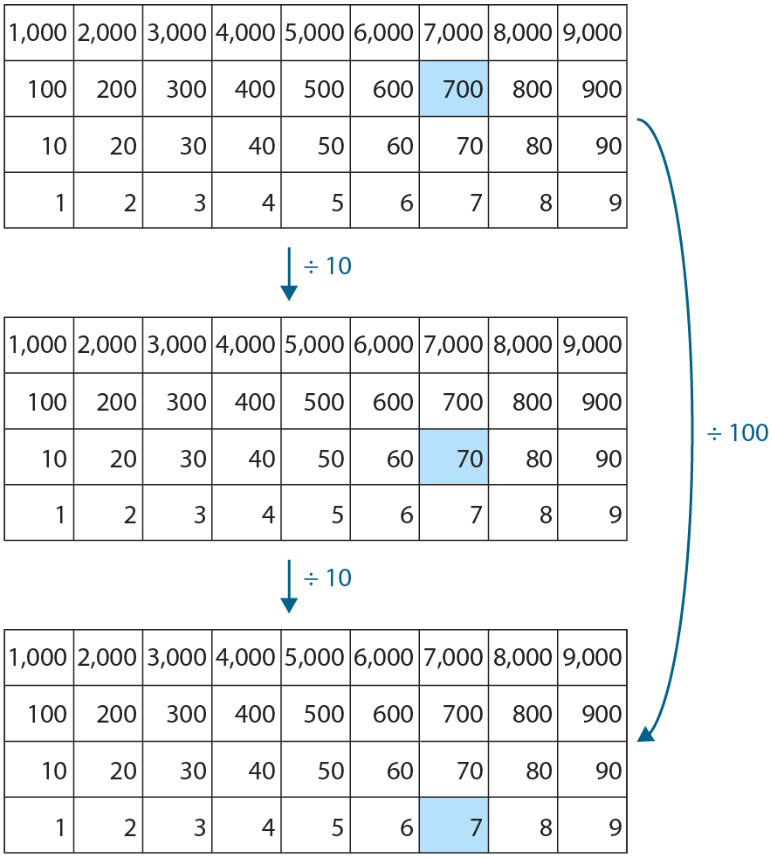
Ratio chart:



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
0	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500

5:4 Now use the Gattegno chart to explore the link between dividing by 100 and dividing by 10 and 10 again. Follow a similar procedure to that described in step *5:1*, now starting with a multiple of 100 (e.g. 700) and asking children to move down as they divide by 10 (to 70), and by 10 again (to 7); then ask them to divide the multiple of 100 (700) by 100, moving down two rows at once (to 7).

Repeat with other multiple-of-100 starting values, working towards the generalisation: '***Dividing by 100 is equivalent to dividing by ten, and then dividing by ten again.***'

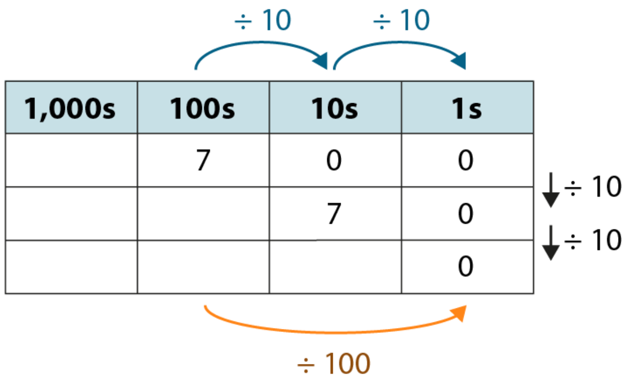


1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

5:5 Again, write equations and use a place-value chart to show how the generalisations children have already learnt for dividing by 10 and by 100 (by removing one or two zeros, respectively) support this understanding. Note that we do not write, e.g. *700 ÷ 10 ÷ 10*, since unlike the three-factor multiplication equation (e.g. *8 × 10 × 10*), without brackets this equation could cause confusion (some children may interpret this as *700 ÷ 10 ÷ 10 = 700 ÷ (10 ÷ 10) = 700 ÷ 1 = 700*).

<table>
<tr>
<td></td>
<td>
<p>Equations:</p>
<p>700 ÷ 10 = 70   700 divided by 10: remove ‘0’ from the ones place.
70 ÷ 10 = 7   70 divided by 10: remove ‘0’ from the ones place.</p>
<p>700 ÷ 100 = 7   700 divided by 100: remove two ‘0’s.</p>
<p>Place-value chart:</p>


<table>
<tr><th>1,000s</th><th>100s</th><th>10s</th><th>1s</th></tr>
<tr><td></td><td>7</td><td>0</td><td>0</td></tr>
<tr><td></td><td></td><td>7</td><td>0</td></tr>
<tr><td></td><td></td><td></td><td>0</td></tr>
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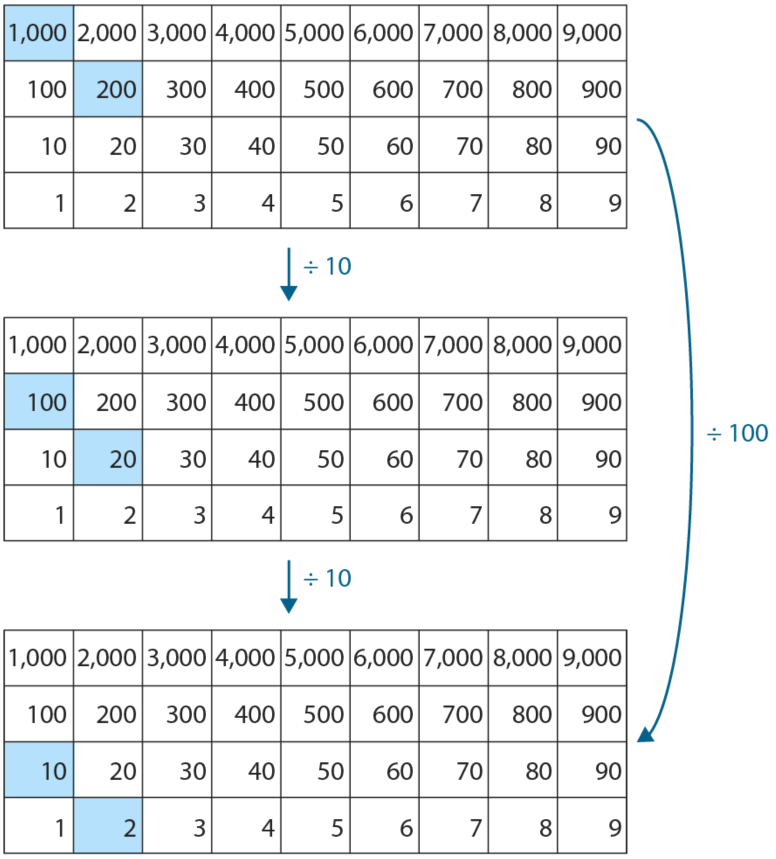
<tr>
<td>5:6</td>
<td>Progress beyond a dividend of 900, using the Gattegno chart for selected dividends. Then build a ratio chart together, as a class. If necessary, write equations or use a place-value chart for support as in the previous step.</td>
</tr>
</table>

Gattegno chart and equations:



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

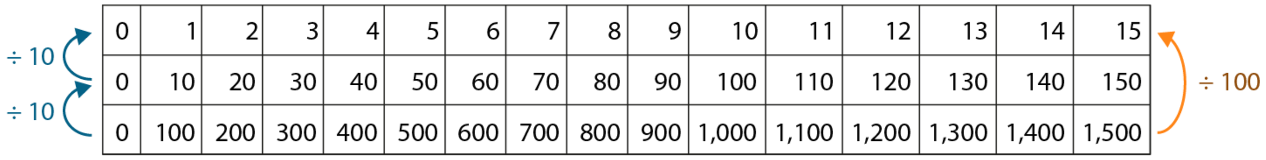
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1200 ÷10 = 120

120 ÷10 = 12

1200 ÷ 100 = 12

Ratio chart:



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
0	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500

5:7 Provide children with practice to reinforce the links between multiplying and dividing by 10 and multiplying and dividing by 100.

Example contextual problems:

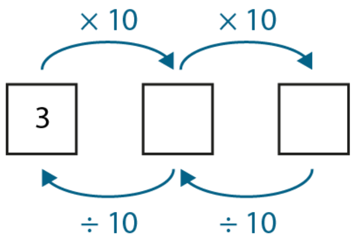
- *'Yulia started an art club; on Monday, she was the only member. On Tuesday there were ten times as many people in the art club as there were on Monday. On Wednesday there were ten times as many people in the art club as there were on Tuesday. How many people were in the art club on Wednesday?'*

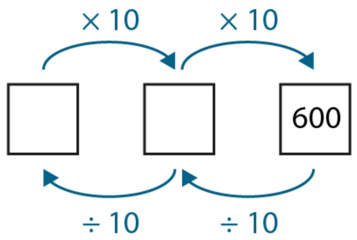
- *'Eugene has six hundred stamps in his collection now. This is ten times as many as he had last year, which is ten times as many as he had two years ago when he started his collection. How many stamps did Eugene have in his collection at the start?'*
- Dòng nǎo jīn:
  - *'Emily read ten times as many books as Cathrynne.'*
  - *'Cathrynne read ten times as many books as Jamie.'*

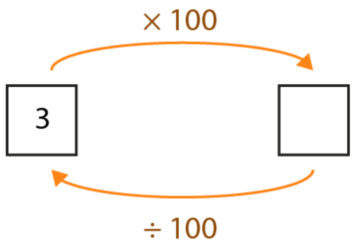
  'If they read more than 300, but less than 500 books altogether, how many books could each person have read?'

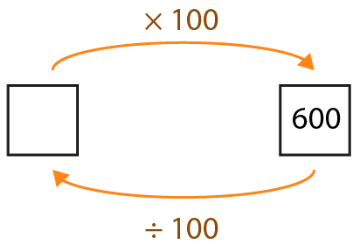
Missing number problems:

'Fill in the missing numbers.'









$3 \times 10 \times 10 = \square \times 100$

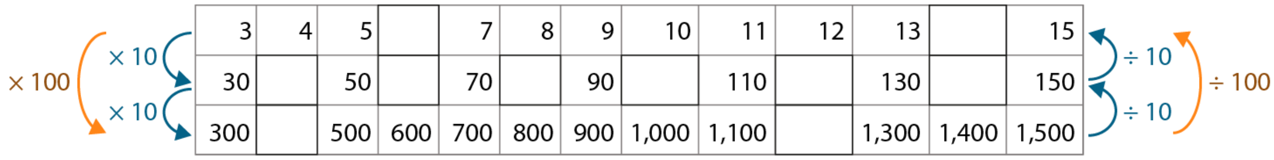
$\square \times 10 \times 10 = 4 \times 100$

$5 \times 10 \times 10 = \square \times 100$

$7 \times \square \times 10 = 7 \times 100$

$11 \times 10 \times 10 = 11 \times \square$

$23 \times \square \times 10 = 23 \times 100$



3	4	5		7	8	9	10	11	12	13		15
30		50		70		90		110		130		150
300		500	600	700	800	900	1,000	1,100		1,300	1,400	1,500

Teaching point 6:

If one factor is made 10 times the size, the product will be 10 times the size. If the dividend is made 10 times the size, the quotient will be 10 times the size.

Steps in learning

6:1

Guidance

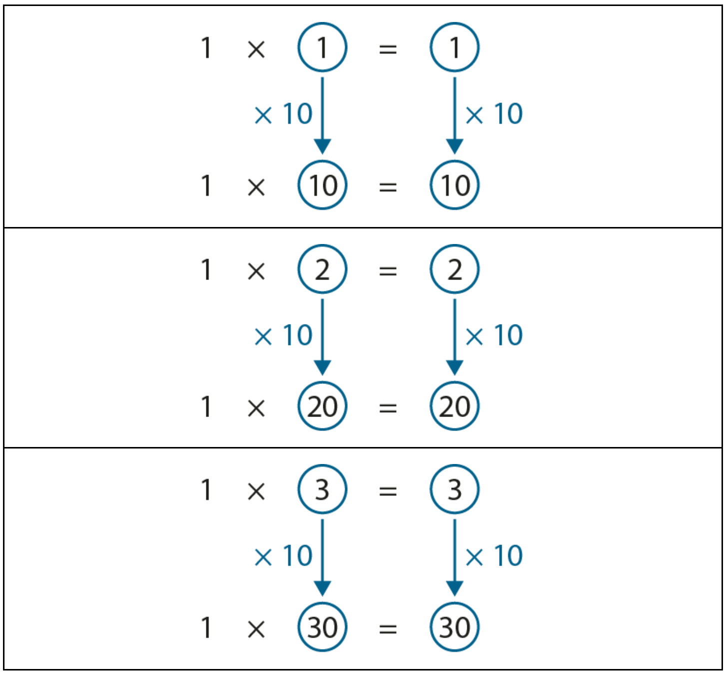
In steps *6:1–6:4*, children will apply their knowledge of multiplying by ten to understand how the product changes if one factor is made ten times the size (e.g. *3 × 4 = 12* vs *3 × 40 = 120*) and how this can be used as a strategy to solve calculations where one of the factors is a multiple of ten. They then follow a similar learning sequence, in steps *6:5–6:7*, applying their knowledge of dividing by ten to understand how the quotient changes if the dividend is made ten times the size (e.g. *12 ÷ 3 = 4* vs *120 ÷ 3 = 40*) and how this can be used as a strategy to solve calculations where the dividend is a multiple of ten.

Begin by comparing pairs of linked equations in which the second factor is made ten times the size. Initially use equations where the first factor is '1' and the second factor is varied, as shown in *set 1* opposite. Then compare equations where the first factor is '2' and second factor is varied, as shown in *set 2* opposite. Work towards the generalisation: ***'If one factor is made ten times the size, the product will be ten times the size.'***

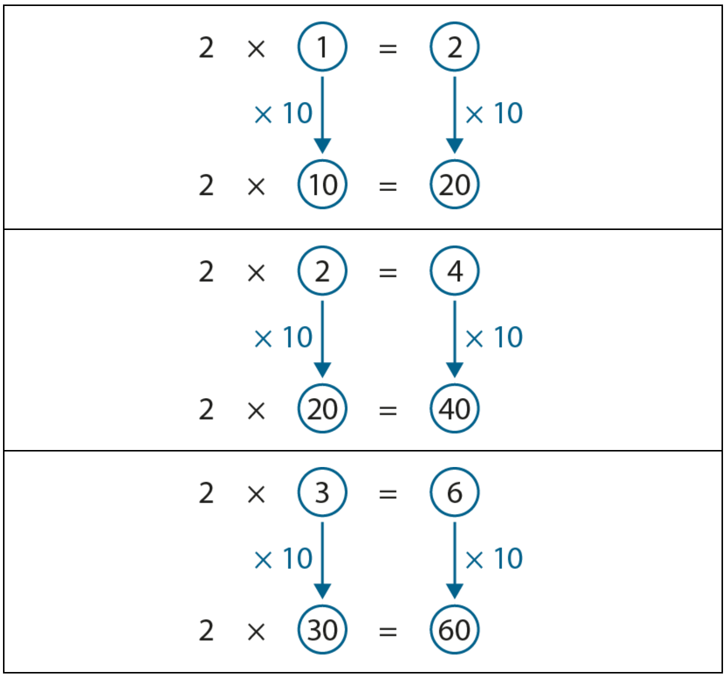
Then choose a different example to model how we can use known multiplication facts (e.g. *3 × 4 = 12*) to solve related calculations that involve a multiple of ten (e.g. *3 × 40 = 120*).

Representations

Making the second factor ten times the size – set 1:



Making the second factor ten times the size – set 2:



		Using known facts: $$\begin{tikzcd} 3 & \times & 4 \arrow[d, "\times 10"'] & = & 12 \arrow[d, "\times 10"] \\ 3 & \times & 40 & = & \boxed{120} \end{tikzcd}$$ - *'Three times four is equal to twelve, so three times four tens is equal to twelve tens.'*
6:2	Now apply the generalisation from step *6:1* to examples where the *first* factor is made ten times the size.	Making the first factor 10 times the size – example 1: $$\begin{tikzcd} 4 \arrow[d, "\times 10"'] & \times & 2 & = & 8 \arrow[d, "\times 10"] \\ 40 & \times & 2 & = & \boxed{80} \end{tikzcd}$$ - *'Four times two is equal to eight, so four tens times two is equal to eight tens.'* Making the first factor 10 times the size – example 2: $$\begin{tikzcd} 3 \arrow[d, "\times 10"'] & \times & 5 & = & 15 \arrow[d, "\times 10"] \\ 30 & \times & 5 & = & \boxed{150} \end{tikzcd}$$ - *'Three times five is equal to fifteen, so three tens times five is equal to fifteen tens.'*

6:3

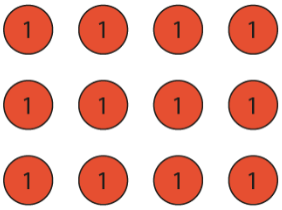
Now, apply this learning to a contextual problem, such as: *'There are four classes. There are thirty children in each class. How many children are there altogether?'*

Use equations, and arrays of place-value counters, to support children's understanding, and model the reasoning using the language exemplified opposite.

'There are four classes. There are thirty children in each class. How many children are there altogether?'

- *'If there were <u>four</u> classes of <u>three</u> children, there would be twelve children altogether.'*

  $4 \times 3 = 12$

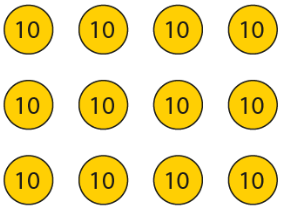


- *'But there are <u>four</u> classes of <u>thirty</u> children; there are ten times as many children in each class. So there are ten times as many children altogether.'*

  $4 \times 3 = 12$ and $12 \times 10 = 120$

so

$4 \times 30 = 120$



6:4

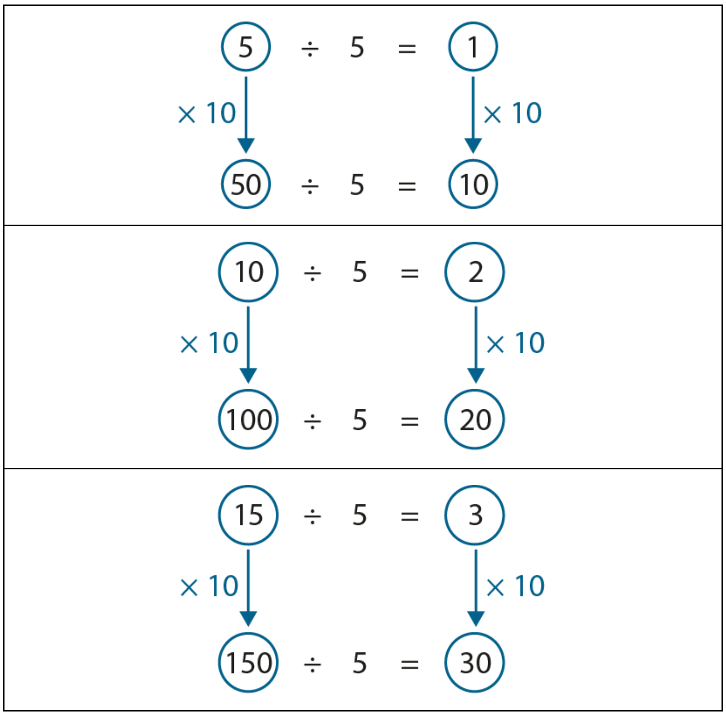
At this point, provide children with some practice, including:

- matching related pairs of calculations
- missing-number problems (initially scaffold these by providing links to known facts, and then provide problems without scaffolding so that children must identify the required known fact themselves)
- contextual problems, for example:
  - *'Party bags can hold forty sweets. If there are five bags, how many sweets are there in total?'*
  - *'Each child in a class needs four exercise books. There are thirty children. How many books are needed?'*
  - *'Three classes each raise £60 for charity. How much have they raised altogether?'*

Matching related pairs of calculations:

'Draw lines to match up the pairs of calculations.'

I can use this calculation…	**…to help me solve this calculation.**
$6 \times 3 =$	$8 \times 20 =$
$5 \times 4 =$	$60 \times 3 =$
$8 \times 2 =$	$9 \times 30 =$
$9 \times 3 =$	$50 \times 4 =$

		Missing-number problems: *'Fill in the missing numbers.'* $2 \times 12 = \square$ $\quad$ $5 \times 7 = \square$ so $\quad$ so $20 \times 12 = \square$ $\quad$ $5 \times 70 = \square$ $30 \times 3 = \square$ $\quad$ $70 \times 9 = \square$ $9 \times 40 = \square$ $\quad$ $8 \times 70 = \square$ Dòng nǎo jīn: *'Fill in the missing numbers.'* $6 \times 70 = 7 \times \square$ $80 \times 4 = \square \times 40$
6:5	Now follow a similar progression for division, considering related pairs of equations where the dividend is made ten times the size. As in step *6:1*, begin by comparing pairs of linked equations. Work towards the generalisation: ***'If the dividend is made ten times the size, the quotient will be ten times the size.'*** Then choose a different example to model how we can use one division problem (e.g. *12 ÷ 3 = 4*) to solve the related calculation where the dividend is a multiple of ten (e.g. *120 ÷ 3 = 40*).	Making the dividend ten times the size: 

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Using one division calculation to solve another:

$$\begin{tikzcd} 12 \arrow[d, "\times 10"'] & \div & 3 & = & 4 \arrow[d, "\times 10"] \\ 120 & \div & 3 & = & \boxed{40} \end{tikzcd}$$

'Twelve divided by three is equal to four, so twelve tens divided by three is equal to four tens.'

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<tr>
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6:6

</td>
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Now apply this learning to a contextual partitive (sharing) division problem, such as: *'Eighty children are going on a trip. There are four coaches. If the children are divided equally between the four coaches, how many children will there be on each coach?'*

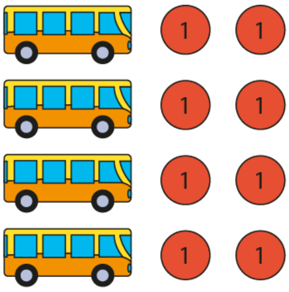
Use equations, and arrays of place-value counters, to support children's understanding, and model the reasoning using the language exemplified opposite. As described in segment *2.6 Structures: quotitive and partitive division*, *Teaching point 3*, when dividing eight children (each represented by a one-value place-value counter) between the four coaches, simultaneously distribute four children at a time (one to each coach); then, when dividing eighty children (each represented by a ten-value place-value counter) between the four coaches, simultaneously distribute forty children at a time (ten to each coach).

</td>
<td>

'Eighty children are going on a trip. There are four coaches. If the children are divided equally between the four coaches, how many children will there be on each coach?'

- *'If there were <u>eight</u> children divided between <u>four</u> coaches, there would be two children on each coach.'*

  $8 \div 4 = 2$

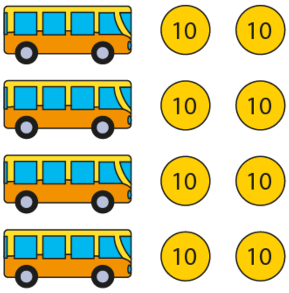


- *'But there are <u>eighty</u> children divided between <u>four</u> coaches; there are ten times as many children altogether. So there are ten times as many children on each coach.'*

  $8 \div 4 = 2$ and $2 \times 10 = 20$

so

$80 \div 4 = 20$



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<td>6:7</td>
<td>Provide children with some practice similar to that described in step <i>6:4</i>.

Example word problems:
- <i>‘Libby needs to buy three hundred squash balls. Squash balls come in packs of six. How many packs does Libby need to buy?’</i>
(quotitive division)
- <i>‘A school council raises £480 for eight classes to spend on books. If the money is shared equally between the eight classes, how much do they each get?’</i>
(partitive division)

Also include some problems that allow children to make links between multiplication and division.</td>
<td>Matching related pairs of calculations:
<i>‘Draw lines to match up the pairs of calculations.’</i>

<table>
<tr><th>I can use this calculation…</th><th>…to help me solve this calculation.</th></tr>
<tr><td>24 ÷ 3 =</td><td>120 ÷ 2 =</td></tr>
<tr><td>49 ÷ 7 =</td><td>240 ÷ 3 =</td></tr>
<tr><td>12 ÷ 2 =</td><td>480 ÷ 6 =</td></tr>
<tr><td>48 ÷ 6 =</td><td>490 ÷ 7 =</td></tr>
</table>

Missing-number problems:
<i>‘Fill in the missing numbers.’</i>

<table>
<tr><td>14 ÷ 2 = ☐
so
140 ÷ 2 = ☐</td><td>54 ÷ 9 = ☐
so
540 ÷ 9 = ☐</td></tr>
<tr><td>200 ÷ 2 = ☐</td><td>500 ÷ 5 = ☐</td></tr>
<tr><td>160 ÷ 4 = ☐</td><td>400 ÷ 8 = ☐</td></tr>
<tr><td>350 ÷ 5 = ☐</td><td>320 ÷ 4 = ☐</td></tr>
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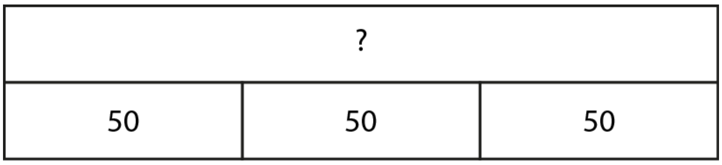
Linking multiplication and division:

- *'Complete the equations to describe the bar model.'*



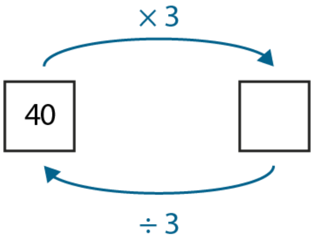
50 + ☐ + 50 = ☐

3 × ☐ = ☐

50 × ☐ = ☐

150 ÷ ☐ = 50

- *'Fill in the missing numbers.'*



20 × 4 = ☐

☐ ÷ 4 = 20

70 × ☐ = 560

560 ÷ ☐ = 70

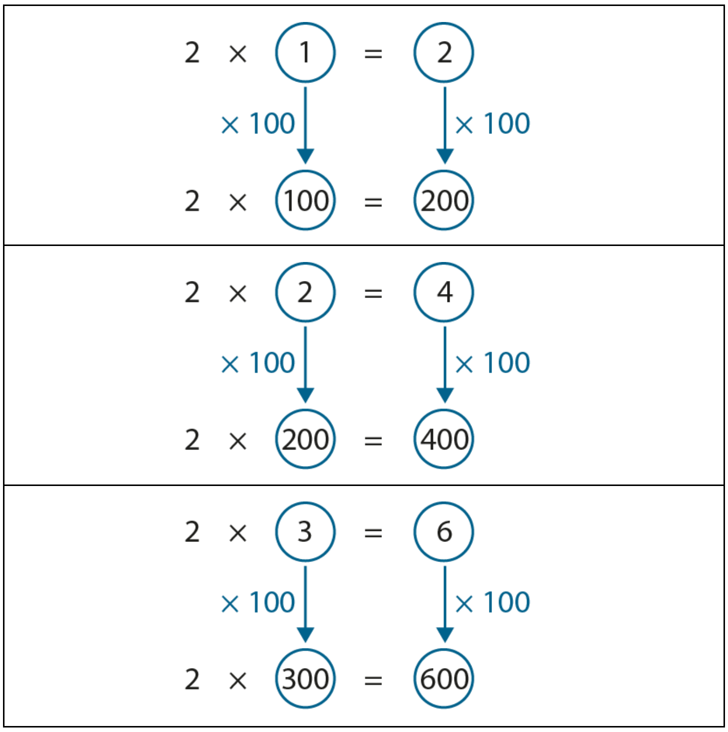
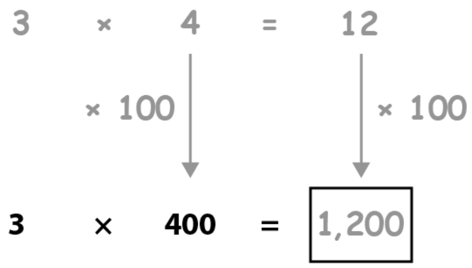
☐ × 3 = 240

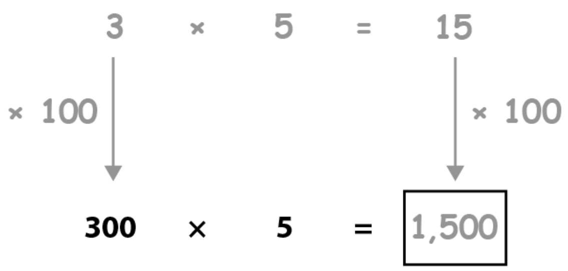
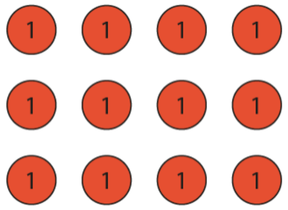
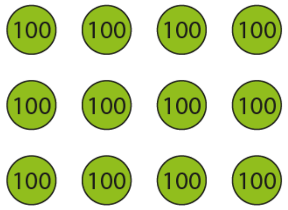
240 ÷ 3 = ☐

Teaching point 7:

If one factor is made 100 times the size, the product will be 100 times the size. If the dividend is made 100 times the size, the quotient will be 100 times the size.

Steps in learning

	Guidance	Representations
7:1	This teaching point follows a similar progression to that for *Teaching point 6*, but now considers the effect of making a factor or the dividend 100 times the size. Guidance here is kept brief, and key representations are shown; for more detail see *Teaching point 6* and adapt for scaling the factor/dividend by 100 instead of ten. Begin, as in step *6:1*, by: • comparing pairs of linked equations in which the second factor is made 100 times the size • making the following generalisation: ***'If one factor is made one hundred times the size, the product will be one hundred times the size.'*** • modelling how we can use known multiplication facts to solve related calculations that involve a multiple of 100.	Making the second factor 100 times the size:  Using known facts:  • *'Three times four is equal to twelve, so three times four hundreds is equal to twelve hundreds.'*

7:2	Now apply the generalisation from step *7:1* to examples where the *first* factor is made 100 times the size.	Making the first factor 100 times the size:  - *'Three times five is equal to fifteen, so three hundreds times five is equal to fifteen hundreds.'*
7:3	Now, apply this learning to a contextual problem, using equations and arrays of place-value counters, to support children's understanding, and model the reasoning using the language exemplified opposite.	*'There are four jars of marbles. Each jar contains three hundred marbles. How many marbles are there altogether?'* - *'If there were four jars of three marbles, there would be twelve marbles altogether.'* 4 × 3 = 12  - *'But there are four jars of three hundred marbles; there are one hundred times as many marbles in each jar. So there are one hundred times as many marbles altogether.'* 4 × 3 = 12 and 12 × 100 = 1,200 so 4 × 300 = 1,200 

7:4

At this point, provide children with some practice similar to that described in step *6:4*.

Example word problems:

- *'There are eight classes in a school. Each class has two hundred books in their book corner. How many books are there in the school?'*
- *'A shop sells squash balls in packs of six. If there are three hundred packs in the store room, how many squash balls are there altogether?'*
- *'Six classes each raise £400 for charity. How much have they raised altogether?'*

Matching related pairs of calculations:

'Draw lines to match up the pairs of calculations.'

I can use this calculation…	**…to help me solve this calculation.**
7 × 4 =	9 × 300 =
6 × 5 =	700 × 4 =
9 × 3 =	5 × 800 =
5 × 8 =	600 × 5 =

Missing-number problems:

'Fill in the missing numbers.'

2 × 12 = ☐

so

200 × 12 = ☐

5 × 7 = ☐

so

5 × 700 = ☐

100 × 7 = ☐

8 × 700 = ☐

9 × 400 = ☐

400 × 4 = ☐

Dòng nǎo jīn:

'Fill in the missing numbers.'

7 × 800 = 8 × ☐

900 × 5 = ☐ × 500

7:5

Now follow a similar progression for division, considering related pairs of problems where the dividend is made 100 times the size.

As in step *6:5*, begin by comparing pairs of linked equations. Work towards the generalisation: ***'If the dividend is made one hundred times the size, the quotient will be one hundred times the size.'***

Then choose a different example to model how we can use one division problem (e.g. *12 ÷ 3 = 4*) to solve the related calculation where the dividend is a multiple of 100 (e.g. *1,200 ÷ 3 = 400*).

Making the dividend 100 times the size:

$$\begin{tikzcd} 3 \arrow[d, "\times 100"'] & \div & 3 & = & 1 \arrow[d, "\times 100"] \\ 300 & \div & 3 & = & 100 \end{tikzcd}$$

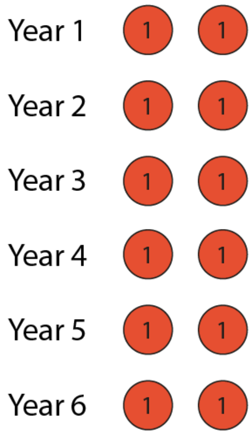
$$\begin{tikzcd} 6 \arrow[d, "\times 100"'] & \div & 3 & = & 2 \arrow[d, "\times 100"] \\ 600 & \div & 3 & = & 200 \end{tikzcd}$$

$$\begin{tikzcd} 9 \arrow[d, "\times 100"'] & \div & 3 & = & 3 \arrow[d, "\times 100"] \\ 900 & \div & 3 & = & 300 \end{tikzcd}$$

Using one division calculation to solve another:

$$\begin{tikzcd} 12 \arrow[d, "\times 100"'] & \div & 3 & = & 4 \arrow[d, "\times 100"] \\ \mathbf{1{,}200} & \div & \mathbf{3} & = & \boxed{400} \end{tikzcd}$$

- *'Twelve divided by three is equal to four, so twelve hundreds divided by three is equal to four hundreds.'*

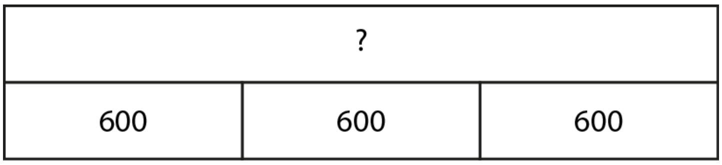
7:6	Now, apply this learning to a contextual partitive (sharing) division problem, as in step *6:6*. For the example shown, when dividing 12 books (each represented by a one-value place-value counter) between the six year groups, simultaneously distribute six books at a time (one to each year group); then, when dividing 1,200 books (each represented by a 100-value place-value counter) between the six year groups, simultaneously distribute six hundred books at a time (100 to each year group).	*'A school buys 1,200 exercise books. These are shared equally between six year groups. How many books does each year group get?'* - *'If there were twelve books divided between six year groups, there would be two books for each year group.'* $12 \div 6 = 2$  - *'But there are twelve hundred books divided between six year groups; there are one hundred times as many books altogether. So each year group gets one hundred times as many books.'* $12 \div 6 = 2$ and $2 \times 100 = 200$ so $1{,}200 \div 6 = 200$ 

7:7	Provide children with some practice similar to that described in step *6:7*. Example word problems: • *'A factory makes five hundred and forty squash balls every hour. They are put into packs of six. How many packs are made every hour?'* (quotitive division) • *'A school council raises £4800 for eight classes to spend on equipment. If the money is shared equally between the eight classes, how much do they each get?'* (partitive division)	Matching related pairs of calculations: *'Draw lines to match up the pairs of calculations.'* **I can use this calculation…** 36 ÷ 3 = 56 ÷ 7 = 24 ÷ 2 = 30 ÷ 6 = **…to help me solve this calculation.** 2,400 ÷ 2 = 3,600 ÷ 3 = 3,000 ÷ 6 = 5,600 ÷ 7 = Missing-number problems: *'Fill in the missing numbers.'* 55 ÷ 5 = ☐ so 5,500 ÷ 5 = ☐ 63 ÷ 7 = ☐ so 6,300 ÷ 7 = ☐ 800 ÷ 2 = ☐ 4,800 ÷ 8 = ☐ 2,700 ÷ 9 = ☐ 1,800 ÷ 2 = ☐ 3,500 ÷ 7 = ☐ 2,800 ÷ 4 = ☐

7:8

Linking multiplication and division:

- *'Complete the equations to describe the bar model.'*



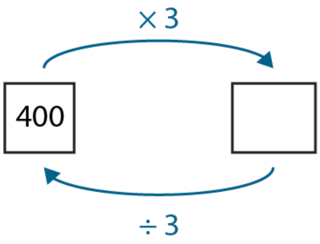
600 + ☐ + 600 = ☐

3 × ☐ = ☐

600 × ☐ = ☐

1,800 ÷ ☐ = 600

- *'Fill in the missing numbers.'*



300 × 5 = ☐

☐ ÷ 5 = 300

☐ × 4 = 2,800

2,800 ÷ 4 = ☐