Primary
& Early Years
Magazine 58

National Centre
for Excellence in the
Teaching of Mathematics

Welcome to Issue 58 of the Primary Magazine. In this issue, The Art of Mathematics features the artist Henri Rousseau. A Little Bit of History continues its series on inventions: in this issue we look at paper clips. Focus On... features an article on place value and Roman Numerals by Barbara Carr, and Maths to Share looks at addition.

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Editor's extras

In *Editor's Extras* we have a reminder of the NCETM PD Lead Support events and the growing NCETM suite of videos to support the implementation of the new primary curriculum. We also have a video clip of an amazing card trick.

The Art of Mathematics

In this issue, we explore the life and works of the French artist Henri Rousseau, whose child-like paintings were ridiculed during his lifetime but are now considered works of genius. If you have an artist that you would like us to feature, please let us know.

Focus on…

In this issue we have the first of three articles by Barbara Carr on whether we really make the most of our wonderful number system. If you have anything that you would like to share, please let us know.

A little bit of history

This is the ninth in our series about inventions. In this issue we look at another important piece of classroom equipment – the paper clip! If you have any history topics that you would like us to make mathematical links to, please let us know.

Maths to share – CPD for your school

In *Maths to Share* we look at the development of the column method for addition to ensure conceptual understanding and the importance of encouraging the children to look at calculations and make decisions on the most efficient methods to use to solve them. If you have any other areas of mathematics that you would like to see featured please let us know.

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Editor's extras

The National Curriculum

Last term, we published a new 'Essentials' page for implementing the National Curriculum, the final version of which has now been published. Implementing the new curriculum is a 'one-stop shop' with links to resources on the NCETM website that will be helpful to subject leaders who are beginning to consider how to support teachers in readiness for the new programme of study.

As part of this support we have produced a suite of 16 videos focusing on calculation and the associated skills and understanding (for example, the concepts of place value and exchange). The videos seek to demonstrate how fluency and conceptual understanding can be developed in tandem. One of the aims of the new National Curriculum, that children should 'reason mathematically', is demonstrated throughout. Each set of videos has an accompanying presentation to stimulate thought and discussion about teaching and learning. We hope you enjoy the videos and find them helpful in supporting teacher professional development. We'd be delighted to receive your feedback, and to learn how you use them (either by commenting below or emailing us at info@ncetm.org.uk). In the near future this suite will include videos focusing on fractions, algebra and division. So keep a look out for these!

The NCETM Professional Development Lead Support Programme (PDLSP)

We're pleased to confirm more new dates for our programme of national free face-to-face events for Primary CPD leads, the NCETM Professional Development Lead Support Programme (PDLSP).

Those who complete the programme are accredited by the NCETM to provide professional development in the priority areas of arithmetic proficiency in primary schools; to date over 140 participants in the programme have been accredited, with more to come.

The dates and locations for the new Primary cohorts are:

Places	Date	Location	Region
20	13 Dec 2013 7 Feb 2014	Rochester, Kent	SE
20	6 & 7 Jan 2014 17 & 18 Mar 2014	Bristol	SW
20	17 Jan 2014 21 Mar 2014	Cheshire	NW
20	5 & 6 Mar 2014 28 & 29 Apr 2014	London, venue tbc	London
20	14 Mar 2014 9 May 2014	Nottingham, venue tbc	EM

Note: Rochester and Cheshire are being run as two one-day events, times to be confirmed; the London Primary Cohort (5/6 March and 28/29 April) is particularly aimed at Primary SLEs.

The PDLSP microsite has full details of the programme - including support materials, and information about how to book your free place.

Colleagues who have completed the first cohorts have said about the programme:

> *'I really valued the input from experienced colleagues and the diversity of viewpoints was very refreshing.'*
>
> *'One of the main criteria for successful PD is that it stimulates new thinking – it certainly did that for me.'*
>
> *'The course is definitely impacting on my daily work.'*

New online material for subject leaders to support high attainers in mathematics in primary schools

Have you seen the section of our website which aims to support schools in evaluating and supporting their provision for high attaining pupils in mathematics in primary school? High Attaining Pupils in Primary Schools will help subject leaders, senior leaders and teachers to identify and support pupils who are attaining higher than expected standards in mathematics, not just in Year 6 but from the time they begin school.

And finally…

It's a month late but still worth sharing… 9 November 2013 (9.11.13) was a mathematical phenomenon so rare that it only occurs five times every 100 years. This is the last time for 92 years that we will have a day that features three consecutive odd numbers. It is also the last one of the 21st century. You could ask the children to work out when the next one will be and to work out how old they might be when it occurs! They could also work out when the previous one was and when there will next be three consecutive even numbers.

If you like card tricks, take a look at The best Card Trick that you will see this year - it's amazing!

The Art of Mathematics
Henri Rousseau

Henri Rousseau was born in Laval, France on 21 May 1844. His family were plumbers and, as a small child, he was made to help in the family business. He went to Laval High School. At first he was a day pupil but he became a boarder when his father ran into debt and their home was repossessed. At school he was apparently mediocre at most subjects but won prizes for music and drawing. When Henri left school he worked for a lawyer and studied law. That didn't go too well for him and he left to join the army. He was in the army for four years. In 1868 he moved to Paris to support his mother after his father died. He got a job as a government worker. In the same year he married his landlord's 15 year old daughter, Clémence Boitard. They had six children, but sadly, only one survived. His wife died in 1888 and in 1898 he married again, this time to Josephine Noury.

In 1871, Henri was appointed as a tax collector of Paris, collecting taxes on goods entering Paris. This resulted in the nickname he was given, which is also how he is sometimes known – *Le Douanier* (the customs officer).

He started painting seriously in his early forties. When he was 49 he retired from his government job to work on his art full-time. He claimed that he had 'no teacher other than nature' although he did ask for advice from two established painters, Félix Auguste Clément and Jean-Léon Gérôme. He was considered to be rather a naïve painter because his paintings were very childlike and this caused him to be ridiculed by others. After his death he came to be known as a 'self-taught genius whose works were of high artistic quality'.

Apparently, one of Henri's paintings was being sold on the street in Paris as a canvas to be painted over when Pablo Picasso saw it, recognise his genius and wanted to meet him. In 1908 he held a banquet in his studio in Henri's honour.

The Snake-charmer

Henri's best known paintings are of jungle scenes. Never having left France or seen a jungle he got his inspiration for his paintings from picture books, magazines, the botanical gardens in Paris and tableaux of taxidermies of wild animals. Apparently when he was in the army he met soldiers who had been on a French expedition to Mexico, and he listened to their stories which described the subtropical conditions they lived in. He remembered these stories and they also influenced his jungle paintings.

Along with his jungle scenes he also painted pictures of people, Paris and its suburbs.

In 1893, Henri officially retired and moved to a studio in Montparnasse where he lived and painted until his death on 2 September 1910. He supplemented his small pension with part-time jobs and work such as playing a violin in the streets. In 1897, he produced one of his most famous paintings, The Sleeping Beauty (*La Bohémienne Endormie*). A few months before his death he exhibited his final painting, The Dream, at the 1910 Salon des Indépendants.

Apparently one of his paintings was used as an inspiration for the animated film *Madagascar*, and another painting was the basis of the Joni Mitchell song, *The Jungle Line*.

Information sourced from:

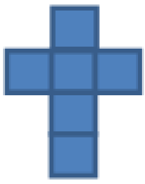
- Henri Rousseau - Paintings, biography, and quotes.

Now for some mathematics!

Show The Sleeping Gypsy

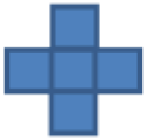
Ask the children to identify the items that they can see in it. Can they tell you what the musical instrument is? It appears to be a lute. You could ask the children to use the internet to find all the stringed instruments that they can and to sort these in a way they choose. Can they count how many strings the lute in the painting has?

The children could make a simple stringed instrument, for example they could make an open cube or cuboid as the main body of the instrument. Of course you would need to explore nets for these first. A good activity for this is to use plasticine or a similar malleable material. Ask them to make a sphere, discuss its properties including the curved surface and that it has no edges because there are no flat faces, it has no vertices because there are no edges to form any. Discuss what it can do and where the children would see one in real life. Then ask them to turn their sphere into a cube. Ask them to describe what they are doing – flattening the curved surface to create faces. Discuss the properties of the cube, including number of faces, edges, vertices and the shape of the faces. You could explore the properties of the squares made by the faces, including angles and lines of symmetry. Next ask them to imagine opening their cube up by pulling down the square faces and to visualise what they would see and then draw it on paper. They may well draw this:



Discuss what an open cube would look like.

They might describe it as a box without a lid. Ask them to draw a net for an open cube on paper:



Give them pieces of card and ask them to draw their net for an open cube onto the card, measuring accurately to ensure the faces are squares. They then cut out, fold and finally stick their net to make the cube. Give them three or four different sizes of elastic band to put around their cube. They should be able to pluck these to make sounds. What do they notice about the different sounds made by the different bands? Hopefully they should notice that the smaller bands make higher pitched sounds. They could measure their elastic bands to prove this.

You could explore other elements of the painting, for example, the lion. They could find a picture of the face of a lion and explore symmetry, drawing their own symmetrical lion face. What else in the picture also shows symmetry?

You could ask them to make a drawing of the gown the gypsy is wearing with vertical coloured stripes of specific lengths.

Show Woman Walking in an Exotic Forest

This is a good painting for counting and exploring symmetry. Ask the children to count the blue flowers and the orange fruit. Can they estimate how many petals are on the blue flowers? What would be an efficient way to count to check their estimate? How many leaves can they see on each of the flowers? What is an efficient way to count these? This could lead to some practice of doubling.

The blue flower with the leaves on the left of the painting shows symmetry. The children could practice making a pattern using coloured cubes with a vertical line of symmetry. They could then draw or paint their own flower with the same line of symmetry as the one in the painting.

What shape do they think the orange fruits are? This would be an opportunity for some work on 3-D shape. What 2-D shape do the fruit appear to be? This could lead into work on circles. You could encourage them to draw accurate circles as described in Issue 50. You could give them different sizes of radii to practise first. They could then draw trees in a similar style to Henri's with accurate circles to represent the fruit.

Show Artillerymen

Ask the children to tell you what they could count in the painting. Together count their suggestions inefficient ways, for example, if counting the artillerymen, can they subitise the four men on the right and then count on from four in twos?

As in the previous painting you could explore circles again using the suggestions in Issue 50. This time they could make two circles out of card for the wheels of the gun and then explore other 3-D shapes which would be suitable for the cannon part of the gun. They could make nets for their shapes and then create them to make the whole artillery gun. This would of course involve accurate measuring!

Show Carnival Evening

Ask the children to describe what they can see in the picture. What season do they think is depicted and why?

You could spend a few minutes discussing the seasons we have and the months in each. You could focus on the heights of the trees.

If you can print out copies of this painting, the children could measure different trees and compare their heights in centimetres and millimetres. They could measure the heights of the two people to the nearest centimetre and work out what to multiply them by to scale them up to the height of an average person. As children are often measured in feet and inches they could do some conversions from these to metres and centimetres. Once they have done this they could multiply the heights of the trees by the same amount to find out how tall they would actually be in real life. They could then stick strips of paper together to represent these heights!

They could use the heights they measured to create a drawing or painting similar to *Carnival Evening*.

Show Surprised! (or *Tiger in a Tropical Storm*), one of Henri's jungle paintings

Ask the children to consider any mathematics that they can explore using this painting. Work through some activities that relate to their suggestions. Examples could include estimating and counting leaves, exploring shapes and the colour of the different shades of green.

The last idea would be a great opportunity to do some paint mixing and explore ratio. Can they mix similar colours, using blues and yellows to get results like Henri has achieved? What ratios of blues and yellows have they made? They could paint a jungle scene using their mixed paints.

The ideas here are just to give you a taster of the mathematical activities that could be involved when looking at artists such as Henri Rousseau. We know you can think of plenty of others! If you try out any of these ideas or those of your own, please share them with us!

Explore further!
If you've enjoyed this article, don't forget you can find all the other *Art of Mathematics* features in the archive, sorted into categories: *Artists*, *Artistic styles*, and *Artistic techniques*.

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Focus on…
Base 10 and Roman numerals

In this three-part series, Barbara Carr, an independent consultant who is also an accredited PD lead for the NCETM, outlines how to use Dienes (Base 10) equipment and the Roman Numeral system to help children develop better conceptual understanding of how place value in our number system works.

Do we really make the most of our wonderful number system?

There has been surprise, possibly annoyance, even outrage at the introduction of Roman Numerals in the 2014 mathematics curriculum.

Year 4 draft programme of study:

- Read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.

The guidance states:

> *Roman numerals should be put in their historical context so pupils understand that there have been different ways to write whole numbers and that the important concepts of zero and place value were introduced over a period of time.*

Year 5 draft programme of study:

- Read Roman numerals to 1 000 (M) and recognise years written in Roman numerals
- Count forwards and backwards in steps of powers of 10 for any given number up to 1 000 000.

The guidance states:

> *Pupils identify the place value in large whole numbers.*
>
> *They continue to use number in context, including measurement. Pupils extend and apply their understanding of the number system to the decimal numbers and fractions that they have met so far.*

As a 'non-mathematician' primary mathematics specialist who was taught arithmetic by rote in the sixties, most of my mathematical understanding has emerged through passion in my work. I too, could not really see much point teaching Roman Numerals until I unearthed 'Primary Mathematics Today' by EM Williams and Hilary Shuard (1980). They say we move in cycles in education and I can honestly say that, I have gained from taking time to explore this in more detail.

This article is my attempt to explain why it's helpful to include Roman Numerals in our new curriculum.

In part 1, I will provide a brief history of ancient number systems, including Roman Numerals. This will help us to see how number systems have come and gone over time and gradually developed over centuries.

In part 2, in a future issue of the Primary Magazine, I will explore:

- how we can use Dienes and base boards to help children to 'see' and 'feel' our place value system
- how specifically teaching children about 'powers of ten' will help them to make sense of the value of each digit in a number
- the notion of teaching children Latin prefixes to help them name headings on place value boards for units of measure and converting units of measurement
- the place value of our units of measure.

For future parts of my article, you will need some Base 10 Dienes equipment on hand and some A3 paper (laminated if possible). It is essential that you try out my ideas practically. It is only by handling the equipment that all will (hopefully) become clear.

In part 3, I will be considering the place value of our units of measure.

Early Number Systems

Bablylonian number systems – Cuneiform.

Wedges were stamped into clay. There may not have been a counting system as such but animals were grouped and these groups were given names. The system consisted of the names of groups and separate objects. (Similar to our Tens and Units)

Because finger counting is so useful, grouping in tens was common in many countries. Before this there is evidence of grouping in 5s, 12s, 20s and 60s in other ancient civilisations.

Egyptian Numerals

Two horseshoes in the example above represent 20.

The swirls represent 100 so there are 5 of these making 500.

At first, there was no fixed order of where to write the symbols. Hundreds could appear after the Tens. It is more convenient for addition to have the symbols appear in a consistent order so over time this became the convention.

The use of an abacus fixes the position of the groupings; structured apparatus would also need to be positioned in the correct order.

Egyptian Multiplication

The Egyptian number system relied on addition. Multiplication was carried out by repeated addition. For some multiplications this was cumbersome so they came up with a method using repeated doubling:

17 x 13

17 = 1 x 17
34 = 2 x 17
68 = 4 x 17
136 = 8 x 17

Now 13 is made up of 8 + 4 + 1
So...
136 (8x17) + 68 (4 x 17) + 17 = 221

They also used doubling and adding for long division.

The Egyptians use of fractions was interesting. It was based on halving. There were no decimals in the Egyptian number system.

The Romans counted in powers of 10:

X = 10

C = 10 x 10

M = 10 x 10 x 10

They recorded groupings by repeating symbols.

231 (Hindu-Arabic) was represented CCXXXI (Roman)

A special feature of Roman notation was the introduction of a symbol for the five of each grouping (a half-way point)

V = 5

L = 50

D = 500

The Egyptians would have to write 6 shapes for 600 and 6 tens for 60. The Romans could write DC (500 + 100) and LX (50 + 10) which was a more efficient recording system.

Roman Numerals

Familiar letters are used in place of numerals.

Some of these letters are repeated.

Numbers are formed by adding the symbols together.

There is no zero in the system.

Symbols are placed to the right in order of value (largest value first).

In order to avoid having too many strokes IIII, subtractive notation is used.

Symbol	Value
I	1
V	5
X	10
L	50
C	100
D	500
M	1 000

Rules:

- The numeral I (1) is placed before V (5) and X (10) to make 4 and 9. Another way of seeing this is 'one before five' and 'one before ten'.
- X (10) can be placed before L (50) and C(100) to make 40 and 90
- C (100) can be placed before D (500) and M (1 000) to make 400 and 900.

The numerals 1-10 would look like this:

I, II, III, IV, V, VI, VII, VIII, IX, and X.

To write multiples of 10:

X, XX, XXX, XL, LX, LXX, LXXX, XC

To write multiples of 100:

C, CC, CCC, CD, D, DC, DCC, DCCC, CM and M.

The use of Roman numerals declined in the 14th Century onwards but we still use them sometimes today.

Wikipedia lists current uses of Roman numerals:

- names of monarchs and Popes, e.g. Elizabeth II of the United Kingdom, Pope Benedict XVI. These are referred to as monarchical ordinals; e.g. "II" is pronounced "the second". This tradition began in Europe sporadically in the Middle Ages, gaining widespread use in England only during the reign of Henry VIII. Previously, the monarch was not known by numeral but by an epithet such as Edward the Confessor
- male generational titles
- the year of production of films, television shows and other works of art within the work itself, which according to BBC News was originally done "in an attempt to disguise the age of films or television programmes." Outside reference to the work will use regular Hindu–Arabic numerals
- hour marks on timepieces. In this context 4 is usually written IIII.
- the year of construction on building faces and cornerstones
- page numbering of prefaces and introductions of books
- book volume and chapter numbers

- sequels of films, video games, and other works
- outlines
- a recurring grand event, such as the Olympic Games
- Super Bowl, NFL's yearly championship games

In astronomy, the natural satellites or "moons" of the planets are traditionally designated by capital Roman numerals.

The Arabic System

This system was invented 2 000 years ago although it has changed significantly over time. The Hindus used numerals 1-9 to make numbers.

2 763 would be written 2 thousands 7 hundreds 6 tens and 3 (in Arabic).

An unknown scholar decided to introduce a symbol to indicate when none of a particular grouping occurred (e.g. 4 032 – where there are no hundreds). This new symbol is now called zero and an empty circle (like an empty bowl) was used (0). If we used an abacus to show 4 032 we would place 4 beads on the Th spike, no beads on the H spike, 3 beads on the T spike and 2 beads on the U spike.

Number is infinite so all we need to do is create new names for each new column created and this simple idea continues to hold true.

A big thank you to Barbara for sharing the first part of her article with us. In part 2, which will appear a future issue, Barbara will be discussing practical ways of teaching place value.

You might be interested in exploring in more depth the ancient number systems which were written about in early issues of the Primary Magazine. They can all be found in the NCETM Essentials.

Explore further!
If you've enjoyed this article, don't forget you can find all previous *Focus on...* features in our archive.

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A little bit of history – paper clips

As we all know a paper clip is something that we use to hold sheets of paper together. As with all the other items featured in this series, it is yet another useful piece of equipment used in many walks of life including… the classroom! A paper clip is usually made of steel wire that is bent in a looped shape. Most paper clips come from the Gem type which was introduced back in the 1890s or earlier… more about that later.

Have you ever wondered how paper clips work?

They are very simple – but clever! Paper clips usually have almost two full loops of wire which hold the paper. They are able to do this in the way that they do because of the twisting action and the elasticity of the metal and also the friction between the wire and the paper. When enough sheets of paper are inserted between the 'tongues' of the paper clip, these tongues are forced apart. This causes a twist in the bendy part of the wire which grips the sheets together. If you put too many sheets into a paper clip, the metal's ability to twist and grip will be lost and it will be permanently damaged. Have you ever tried to hold too much paper with a paper clip and ruined it? Now you know why!

It is unclear who the actual inventor of the paper clip was. Several people have been attributed with its design. According to the Early Office Museum, the first patent was given to Samuel B. Fay, from the US, in 1867. It was originally intended for attaching tickets to fabric. However he wasn't the only person to receive a patent for this device. He was one of 50 other people! Edman J Wright, also from the US, was one of them. He advertised his as a device that could fasten newspapers.

'Clip 317'

The most common type of paper clip which we still use today is the Gem. The Gem Manufacturing Company in the UK began its production in the early 1870s. A few years after production began, the paper clips were praised in an article for being 'better than ordinary pins', 'binding together papers on the same subject, a bundle of letters, or pages of a manuscript'. The first picture of them was seen in an advert in 1894 and showed that they were similar to the ones we use today. In 1899 William Middlebrook from Connecticut built a machine for making them and production increased rapidly.

Over the years, countless variations of paper clip have been produced. Some have pointed ends, some have the end of one loop bent slightly to make it easier to insert sheets of paper and some have wires with undulations or barbs to get a better grip. Different shapes, such as triangular or round ones have been produced as well as many colours. The original Gem paper clip has proved to be the most practical, and consequently by far the most popular for over 100 years.

Did you know…

- the Swedish word for paper clip is gem; the French equivalent is trombone
- paper clips are used for many tasks that require a thin wire, for example, ejecting CDs from a CD ROM drive, ejecting the SIM card from a mobile device, replacing fuses, picking locks, and unfastening handcuffs. The last three are not recommended!
- a paper clip image is the standard image for an attachment in an email.

Information sourced from:

- Inventors
- The Office Museum.

Now for a few mathematical ideas for paper clips...


'Paper Clips'

You could make a collection of different paper clips and ask the children to sort them according to their own criteria, e.g. shape of the ends, colour, size. They could sort them into a Carroll or Venn diagram with one or two criteria.

You could ask the children to explore the timeline of paper clips from the Office Museum website. You could print out copies, cut the pictures and dates out, jumble them up and give to groups of children to order. They could then stick these on a number line beginning at 1850.

You could give a pile of paper clips to each child and time them to see how many they can link together in a minute. You could ask them to use this information to estimate how many they could link together in 2 minutes, 1 hour, 7 hours... They could then compare their lengths and find the differences between them using a counting on strategy. They could also find totals of different lengths using a mental calculation strategy.

You could ask the children to measure one of their paper clips and then multiply by the number in the length they made. They could then measure their length. Are the two the same? It is unlikely that they will be, can the children figure out why this is? They could also work out the difference between their calculation and the actual length. They could rehearse converting the length of one clip or the whole length from centimetres to inches.

Using your collection of different types of paper clip, you could explore fractions, percentages, ratio and proportion. For example, using two different sizes, ask the children to link five together. They can then tell you what fraction is larger and what fraction is smaller, for example 3/5 of the chain is made from large paper clips and 2/5 from smaller ones. They could tell you what ratio and proportion the larger ones are to the smaller ones. You could then repeat this for 10 paper clips, exploring equivalent fractions, ratios and proportions. They could tell you what percentage of the paper clips are large/small.

You could give the children 2cm strips of A4 paper to use as 'show me strips' or 'bar models'. Tell them that the strip represents an amount, for example 10p. Then call out different amounts and ask them to show you where these would go on the strip. You could repeat this for many other numbers, for example, 1kg, 1 whole, 1cm, 100, 2 litres, 30.

You could ask them to make a bracelet using 8 paper clips. They could then work out the cost if one paper clip cost 2p, 3p, 4p etc. This would be a good opportunity to practice counting in steps of different sizes or rehearsing multiplication facts.

If you mix the paper clips and cost them accordingly – largest being the most expensive, the children could work out the prices of their bracelets and compare them. They could order them from least to most expensive and work out the difference in prices. They could also find out how much it would cost to buy different bracelets of different amounts.

You could ask them to explore how they could buy paper clips by visiting stationery shops online, or using Google. How many come in a box? They could find out and compare different prices from different shops. They could work out the cost of one paper clip to the nearest whole or half penny.

You could ask the children to make a pattern with their paper clips, for example, using the same size clip, place one vertically then one horizontally and repeat this. They could then work out in what position the 12th one would be and then the 97th. Once they have, ask them to tell you how they know.

You could ask them to make strings of paper clips and fold them to show different angles, for example acute, obtuse, straight line and right angles, estimate 45º, 135º.

They could also make symmetrical patterns with their paper clips in two and then four quadrants.

You could use the paper clips to practice rotation. The children could place the paper clip on a piece of paper, draw round it and then put their pencil in one of the loops and rotate it a quarter turn, draw round it again. Repeat this for four turns. You could ask them to do this for turns of 45º.

You could give the children squared paper and ask them to make different translations.

They could use strings of paper clips to make shapes… how many do they need to make the smallest equilateral triangle / square / regular pentagon etc.? If they linked 10 paper clips together, what regular and irregular shapes can they make?

We hope that this article has inspired you to make a more mathematical use of your classroom paper clips! If there is any area of history that you would like us to make mathematical links to, please let us know.

Explore further!
If you've enjoyed this article, don't forget you can find all previous *A little bit of history* features in our archive, sorted into categories: *Ancient Number Systems*, *History of our measurements*, *Famous mathematicians*, and *Topical history*.

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Primary
& Early Years
Magazine 58

National Centre
for Excellence in the
Teaching of Mathematics

Maths to share – CPD for your school

The National Curriculum requires teachers to teach addition and subtraction of numbers with up to 3 digits, using formal written methods of columnar addition and subtraction from Year 3. It also requires teachers to teach multiplication of two-digit and three-digit numbers by a one-digit number using formal written layout from Year 4 and division of numbers up to 4 digits by a one-digit number using the formal written method of short division from Year 5.

Over the next four issues of the Primary Magazine we will exploring ways in which you could do this so that you can help your children to develop their conceptual understanding of the four operations.

We begin with addition. For more about this operation, see Issue 23 which explores the basics of this concept. In this issue we will focus on the development of the written columnar method.

For the meeting you will need equipment such as straws and base ten equipment.

If you have them, place value counters would be helpful or simply sets of three different coloured counters to represent hundreds, tens and ones.

Begin your staff meeting by writing these calculations on the board:

- 125 + 127
- 231 + 142
- 163 + 99
- 3 245 +1 678

Give colleagues a few minutes to discuss ways to solve each calculation. Take feedback, discussing the different strategies they have used.

You could ask colleagues questions such as:

- When asked to calculate what is your initial response? Do you begin to calculate mentally, reach for pencil and paper or a calculator?
- Were you taught mental methods at school or were you just taught standard written methods (algorithms)?
- As a result of your learning of mathematics at school, do you think this has equipped you for adult life?

There are several ways to answer the calculations above. It might be worth highlighting the more obvious methods, such as:

125 + 127: doubling 125 and adding 2

231 + 142: sequencing (231 + 100 + 40 + 2)

163 + 99: add 100 and adjust

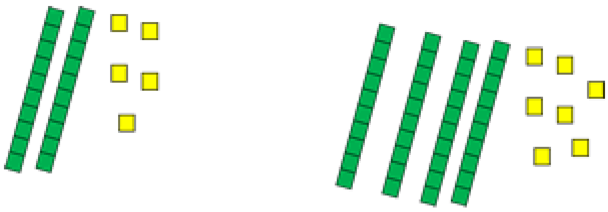
3 245 + 1 678: this may best be answered using the column method.

Ideally, we would all want our children to develop the ability to look at a calculation and decide which method is the appropriate one to use for adding the numbers. Sometimes it might be that a mental calculation strategy is the most efficient, sometimes it might be the column method. This means that teaching mental calculation strategies remains important. In the notes and guidance section for each year group there is an expectation that children use mental calculation strategies, for example, in Year 6 it states that 'they undertake mental calculations with increasingly large numbers and more complex calculations'.

You might like to follow some of the suggestions for oral and mental starter activities from Issue 47 to recap and rehearse some of the strategies the children need to learn.

When teaching the column method it is probably wise to begin with a simple calculation and model the approach in a similar way to this…

Ask colleagues to make 25 and 47 using the equipment that you have available. If possible they should explore this with straws, base ten and counters. If you are using different coloured counters assign a different place value to each and tell colleagues what each colour represents.



Now ask them to add the two numbers together. Observe how they do this. Do they add the tens first or the ones? Having worked with the most significant digits first for most of their school life, the children may well add the tens first and then the ones. They are likely to have learned to add by partitioning in KS1 and continued this method into Year 3:

```
 25
+47
 60
 12
 72
```

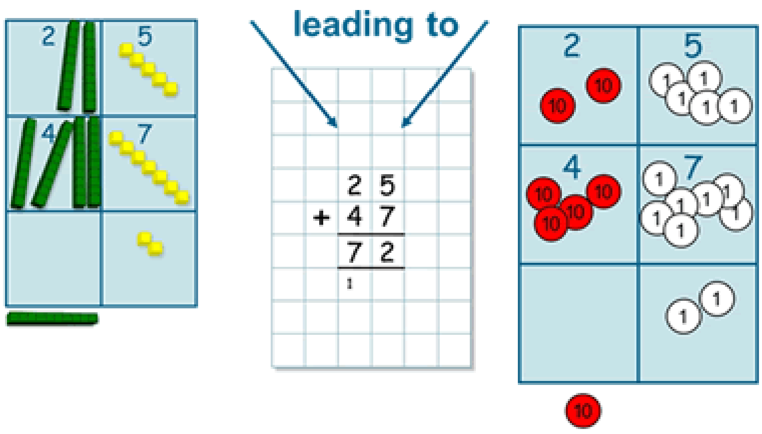
Agree that whichever way they add, they will have 60 and 12. Ask colleagues how they can make 60 12 into a number that we say. Agree that they need to take ten of the ones and exchange them for a ten giving 72. Stress the fact that when we change one unit into another we use the word exchange.

You could lead a discussion on a way to move the children from adding the most significant digit first to the least significant digit in order to miss out the middle part of the partitioning strategy and move directly to the answer. You might like to consider this: when you add 20 and 40 you get 60, so 6 goes in the tens position, when you add 5 and 7 you get 12. What is wrong with this…?

  25
+47
612

It is clear that the answer isn't 612, to make this correct means that you have to do some rubbing or crossing out in order to add the ten to the sixty. It is therefore more efficient to add using the least significant digits.

Next model the column method showing how this links to the process colleagues have carried out using the manipulatives:



Ask colleagues to tell you what is the same and what is different about these models. These are great questions to ask the children, it helps develop their reasoning skills.

You could play this game to show colleagues how to introduce the idea of exchange to their children:

Try this with place value counters or Dienes:

1. Throw the dice
2. Count that number of ones
3. Keep going exchanging every time you make a 10
4. First person to make 50.

Lead a discussion on which manipulatives to use in which year group. Is there a progression? It might be that straws are best used in KS1 so that children can see and feel the actual numbers they are working with. Dienes might come next as it is a representation that can be linked easily to hundreds, tens and ones by the divisions on the pieces of equipment. Place value counters could be used when the children have an understanding of the size of hundreds, tens and ones and can cope with a more abstract representation.

You might like to show the video clip Using resources to develop fluency and understanding, which has been produced by the NCETM, showing how Year 2 children use straws and Dienes to add 2-digit numbers.

Finish the meeting by asking colleagues to use the different manipulatives to solve these problems:

- Jak has 158 football cards. Tammy has 167. How many do they have altogether?
- Ben scored 234 points in the game. Natasha scored 145. How many points did they score in total?

As they solve the problems, ask colleagues to consider:

- How well can the manipulatives help children to solve the problems
- How well do the manipulatives help to move pupils towards written methods?

Encourage colleagues to try using manipulatives to practice exchanging, using an activity such as the game above, or to move from partitioning to the column method. For older children who currently use the column method for addition, encourage colleagues to give them manipulatives in order to explain how their written method works. This is a good assessment opportunity to find out if they really understand the procedure that they use.

Ask colleagues to be prepared to share what happened at a future meeting.

If you decide to use it for staff professional development, please let us know (either by posting a comment below or emailing us at info@ncetm.org.uk) - we'd love to hear what you did.

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