NCETM
NATIONAL CENTRE FOR EXCELLENCE IN THE TEACHING OF MATHEMATICS

Multiplicative Reasoning

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

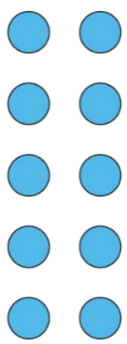
Question 4

How confident are you that you understand and can support children to recognise and use associative and commutative laws in multiplication?

1 ☑ **2** ☐ **3** ☐ **4** ☐

How would you respond …?

a. How can an array be used to develop the idea of commutativity?



b. Decide whether each pair of expressions has the same product. Draw a picture, or use counters, to explain your answer, linking to the idea of commutativity.

	Products equal	**Products *not* equal**
2×3 and 3×2		
5×4 and 4×5		
5×5 and 6×5		
12×3 and 13×2		
3×22 and 23×2		

c. How can the commutative or associative laws make this question easier to efficiently calculate?

'A school sets out chairs in the school hall to show a film. There are 12 rows and 8 chairs in each row. If all the chairs are full and each person pays £5, how much money is collected?'

Responses

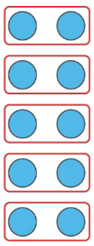
Note your responses to the questions here before you engage with the rest of this section:

Did you notice that…?

a. Using the same array, $2 \times 5 = 10$, the structure of how a multiplication equation can have two different grouping interpretations is exposed.

b. An array can be used to demonstrate when the products are equal, circling in groups, as above. The first two have equal products; the bottom three do not.

The first representation could be read as:

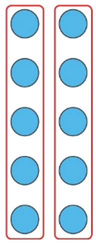


'There are five groups of two.'

'Five groups of two is equal to ten.'

'Five times two is equal to ten.'

The second representation could be read as:



'There are two groups of five.'

'Two groups of five is equal to ten.'

'Two times five is equal to ten.'

c. There are several ways to solve this problem. Here is one example:

$$12 \times 8 \times 5 = 12 \times (8 \times 5) = 12 \times 40 = 480$$

Commutative and associative laws

In this section, commutative and associative laws will be explored with a focus on introducing the idea of commutativity.

Children may have some understanding of commutative and associative laws when working with addition and these principles are related to commutativity in multiplication. The commutative law can be applied in contexts where there is only one pair of factors and one operation, whereas the associative law is only applicable where there are more than two numbers involved.

Children may initially explore commutativity in multiplication when exploring how the factors in a multiplication equation can be written in either order, with the product remaining the same.

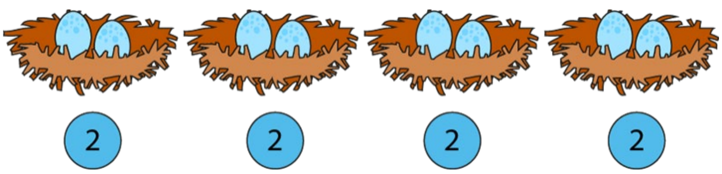
Initially the concept of one equation, two interpretations, could be introduced.

- number of groups × group size = product
- group size × number of groups = product

For example, in 4×2 the first factor represents the number of groups (4) and the second factor represents the group size (2). This could be expressed as *'four times two' or 'four twos'*.

If you write the factor 2 (the group size) first, 2×4, this would then be expressed as '*two, four times*'.

- *'There are four groups of two eggs. There are eight eggs altogether. 4 × 2 = 8'*
- *'There are two eggs, four times. There are eight eggs altogether. 2 × 4 = 8'*



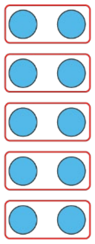
Notice that the language of *'multiplied by'* is not used, as this implies that the multiplicand (the group size) is written first followed by the multiplier (the number of groups). Children instead use the language '*groups of*', '*times*' (not '*times by*') and '___ *twos*' (for example, *'four twos'*).

When children first express multiplication facts, they will write the factors in the order '*number of groups × group size*'. However, once commutativity is introduced, they can form the generalisation '*factor × factor = product*', as the factors can be written in either order.

Building on from this, understanding of commutativity can be further developed through the use of arrays as a representation.

Using the same array, 2 × 5 = 10, the structure of how a multiplication equation can have two different grouping interpretations is exposed.

The first representation could be read as:

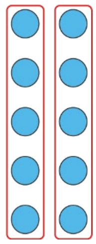


'There are five groups of two.'

'Five groups of two is equal to ten.'

'Five times two is equal to ten.'

The second representation could be read as:

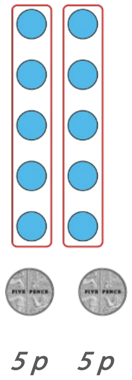


'There are two groups of five.'

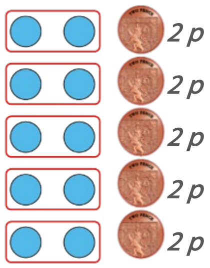
'Two groups of five is equal to ten.'

'Two times five is equal to ten.'

This can then be linked to unitising, where the unit is equal to the group size.



'Two times five can represent two groups of five.'



'Two times five can also represent five groups of two (or two, five times).'

Children will be able to apply commutativity when solving problems that do not initially appear to be using a familiar multiplication table. For example, *'There are seven players on a netball team. When two teams are playing, how many players are there altogether?'*

Initially, this problem may appear to be using the seven times table. However, by applying their understanding, children will be able to see they can use the two times table to find the product.

This can lead to the generalisation: *'The product of ___ and ___ is equal to the product of ___ and ___.'*

This can then be simplified to: *'___ times ___ is equal to ___ times ___.'*

It is important that children understand that division is not commutative.

I have 15 flowers. If I put 5 flowers in each vase, how many vases do I need?

□ × 5 = 15   5 × □ = 15

Which calculation represents this problem?

15 ÷ 5 = □ ✓

5 ÷ 15 = □ ✗

Common errors in this area may include:

- children only operate the calculation form left to right
- children are not able to link their description of the calculation to the appropriate image

What to look for

Can a child:

- use the idea of commutativity to calculate efficiently?

Links to supporting materials:
NCETM Primary Professional Development materials, Spine 2: Multiplication and Division: • Topic 2.3: Times tables: groups of 2 and commutativity (part 1) • Topic 2.5: Commutativity (part 2), doubling and halving • Topic 2.10: Connecting multiplication and division, and the distributive law

Notes:
Key learning from support material and self-study:
What I will focus on developing in my classroom practice: