Fractions

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

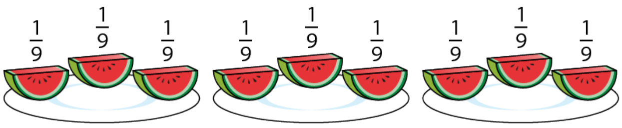
Question 10

How confident are you that you understand and can support children to understand the multiplication of whole numbers and fractions as both repeated addition and scaling?

1 ☑ 2 ☐ 3 ☐ 4 ☐

How would you respond …?

a. Can you write three addition and multiplication expressions to match the image?



b. Can you explain, using a representation, why this is not correct?

$$\frac{3}{7} \times 2 = \frac{6}{14}$$

c. Can you complete this equation in different ways?

$$\frac{24}{25} = \square \times \frac{\square}{25}$$

c.

Responses

Note your responses to the questions here before you engage with the rest of this section:

Did you notice that…?

a. The plates are used to group the watermelon slices so the expressions should not have included unit fractions:

$$\frac{3}{9}+\frac{3}{9}+\frac{3}{9}$$

In this expression, there is recognition that on each plate there are $\frac{3}{9}$ of a watermelon and this is repeated. This has been shown as a repeated addition:

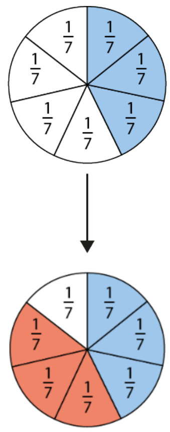
$$3\times\frac{3}{9}$$

In this expression, we can see quantities of fractions: there are 3 plates each containing $\frac{3}{9}$:

$$\frac{3}{9}\times 3$$

As children understand the law of commutativity, this image can also be expressed in this way. As they develop their understanding of scaling, they will begin to understand that fractions of quantities can also be expressed and this relates to their developing understanding that multiplying by a proper fraction will result in a smaller quantity.

b. The child has applied their understanding of multiplication to multiply both the numerator and denominator by two. They have not applied their understanding of the fractional amount $\frac{2}{7}$ being multiplied by two. Where children are taught a process, they do not necessarily have the understanding to recognise when to apply the process correctly. This can be a particular issue with some of the 'tricks' applied to the teaching of fractions. Starting with a representation and building understanding, using models, will mitigate against these errors occurring. After repeated practice with models, children can then make generalisations and move to equations with the prerequisite understanding in place; this avoids 'forgetting' the process.



The most likely error that children make is multiplying both the numerator and the denominator by the whole number.

In this example, $\frac{3}{7}$ x 2 = ☐

Ask children to imagine what it would look like if $\frac{3}{7}$ was multiplied by two.

Emphasise that the denominator doesn't change because the unit has not changed; several lots of the same units are combined.

c. This example relies on the ability to unitise, recognising the unit is twenty-fifths. The focus is therefore on the numerator. The numerator is 24 so the question being asked is *'How do I make 24?'*

Children can then work systematically:

$$1\times\frac{24}{25}\quad 2\times\frac{12}{25}\quad 3\times\frac{8}{25}\quad 4\times\frac{6}{25}\quad 6\times\frac{4}{25}\quad 8\times\frac{3}{25}\quad 12\times\frac{2}{25}\quad 24\times\frac{1}{25}$$

Multiplication of whole numbers and fractions

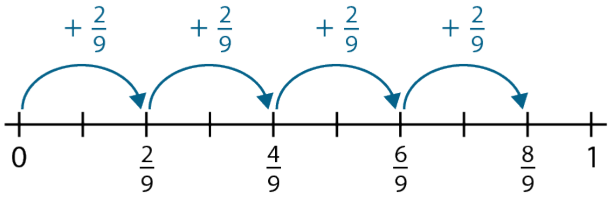
Children will first be introduced to multiplication with whole numbers and fractions in Year 4. This concept builds on children's previous learning that repeated addition of whole numbers can be rewritten as multiplication. They will take this previous learning with whole numbers and apply it to fractional quantities.

The focus is on two main structures through which to consider multiplication of a whole number by a fraction. The first is through interpreting multiplication by a whole number as repeated addition, which will draw heavily on their understanding of unitising. The second is to think about multiplication by a proper fraction as 'scaling down'.

It is important for children to use concrete manipulatives to support them as they learn these new concepts; rods are particularly helpful in building comprehension of the concept of scaling.

Structure 1: Repeated addition

The calculation $4 \times \frac{2}{9}$ (or $\frac{2}{9} \times 4$) uses the knowledge that four groups of $\frac{2}{9}$ is $\frac{8}{9}$.



In this approach, the fraction ($\frac{2}{9}$) is the quantity being worked with and the whole number (4) is the operator: *'There are four <u>of</u> that quantity.'*

Structure 2: Scaling

The scaling structure interprets multiplication as 'of': *finding a fraction 'of' a quantity.*

For example:

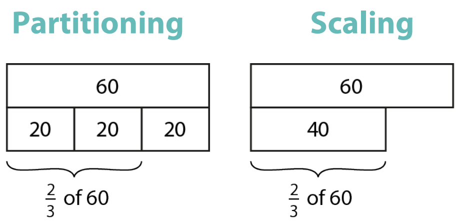
$\frac{2}{3} \times 60$ (or $60 \times \frac{2}{3}$)

$\frac{1}{3}$ of 60 is 20

so

$\frac{2}{3}$ must be 40

In time, children should see that 40 is the result of a 'scaling down by $\frac{2}{3}$ of 60'. This is effectively 'shrinking' the 60 to two-thirds of its original size, rather than seeing it as partitioning 60 into three parts and taking two of those parts. When multiplying by a number greater than one, scaling UP occurs. When multiplying by a number less than one, scaling DOWN occurs.



In this approach, the whole number (60) is the quantity being worked with and the fraction ($\frac{2}{3}$) is the operator: '*I have two-thirds <u>of</u> that quantity.*'

It might be helpful to think of these approaches as 'quantities of fractions' (structure 1) versus 'fractions of quantities' (structure 2). With time, children will become confident interchanging between these, for example seeing that $4 \times \frac{2}{9}$ can be thought of as scaling $\frac{2}{9}$ of 4 and as repeated addition: 4 lots of $\frac{2}{9}$.

Our focus in the primary phase is helping children develop the flexibility to move between the two models and decide on the most helpful way of thinking about a specific calculation. For example, asking a child to find $\frac{2}{9}$ of 4 may be beyond the remit of primary but finding 4 lots of $\frac{2}{9}$ is not.

When children first learn about multiplying whole numbers by fractions, they may not yet know about equivalent fractions, so even when answers to calculations can be simplified (e.g. $4 \times \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$), leave them in their un-simplified form ($4 \times \frac{1}{8} = \frac{4}{8}$). This will also help maintain the focus on the structure of, in this example, four one-eighths.

Common errors in this area may include:

- children multiplying the numerator and the denominator by the whole number
- children thinking multiplication always makes the answer bigger.

What to look for

Can a child:

- use their knowledge of unitising to identify that they are multiplying by the numerator?

Links to supporting materials:

NCETM Primary Professional Development materials, Spine 3: Fractions:

- Topic 3.6: Multiplying whole numbers and fractions

Notes:

Key learning from support material and self-study:

What I will focus on developing in my classroom practice: