NCETM
NATIONAL CENTRE FOR EXCELLENCE IN THE TEACHING OF MATHEMATICS

Core concept 5.3: Probability

This document is part of a set that forms the subject knowledge content audit for Key Stage 3 maths. The audit is based on the NCETM Secondary Professional Development materials and there is one document for each of the 17 core concepts. Each document contains audit questions with check boxes you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications and explanations, and further support links. At the end of each document there is space to type reflections, targets and notes. The document can then be saved for your records.

5.3.1 Explore, describe and analyse the frequency of outcomes in a range of situations

How confident are you that you understand and can explain the concepts of randomness, outcomes that are equally likely and those that are not?

1 ☐ 2 ☐ 3 ☐ 4 ☐

How confident are you that you understand and can explain how to order events on a scale in order of likelihood and how to determine likelihood by designing and carrying out an experiment?

1 ☐ 2 ☐ 3 ☐ 4 ☐

Often, students mistakenly believe that an event with only two possible outcomes has an 'even' chance of happening, or that the probability of one event occurring when there are *n* possible outcomes is 'one in *n*'. Students should be exposed to examples of when this is true and when this is not true and discuss what's the same and what's different about the situations.

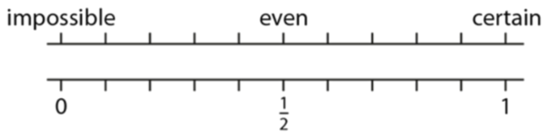
Before they quantify probabilities, students need to appreciate that, where an event has different possible outcomes, some of these outcomes may be more or less likely than others for different possible reasons.

One factor that underpins uncertainty is that of randomness. A key awareness is to understand that although an individual event might be random, reasoning about uncertain events can be fruitful when they are repeated many times. Given enough time, trends in apparently random behaviour can become predictable by analysing the frequency of outcomes.

Predictions of likelihood do not predict individual events. Rather, experimental data will tend towards this theoretical value (for example, knowing that flipping a head or a tail on a coin has an even chance of occurring does not mean these outcomes will occur an equal number of times).

Specific and precise language is key to working with probability. For example, the distinctions between an event (for example, flipping a coin) and an outcome (for example, a coin landing on heads) or between probability and possibility (for example, it is possible that it will snow in summer, but not probable).

A probability scale supports students in quantifying between everyday terms such as 'likely', 'impossible' and 'certain'.



Where events are mutually exclusive and exhaustive the total of their probabilities is one.

For example:

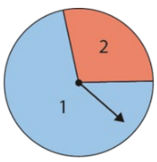
a) Which of these events do you think will select a number at random?

i) *Rolling a six-sided dice numbered 1 to 6.*

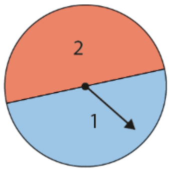
ii) *Rolling a nine-sided dice numbered 1 to 9.*

iii) *Rolling a nine-sided dice numbered with four 1s and five 2s.*

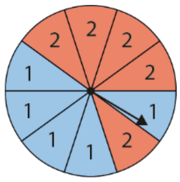
iv) Spinning this spinner.



v) *Spinning* this spinner.



vi) Spinning this spinner.



vii) Asking the person sitting next to you to pick a whole number between 1 and 10.

b) Do you think that any of the above are 'more random' than the others? Explain why.

Which of the above do you think is a 'fair' way to select a random number? Explain why the other ones are not fair.

Further support links

- NCETM Secondary Professional Development materials: 5.3 Probability, pages 8–13
- NRICH: Taking chances extended (article): https://nrich.maths.org/4728
- NRICH: The random world (article): https://nrich.maths.org/5975

5.3.2 Systematically record outcomes to find theoretical probabilities

How confident are you that you understand and can explain how to use probability tree diagrams and sample space diagrams to systematically record all possible outcomes for more than two events?

1 ☐ 2 ☐ 3 ☐ 4 ☐

How confident are you that you understand and can explain how to calculate theoretical probabilities from sets of outcomes organised in a systematic way from a range of appropriate representations?

1 ☐ 2 ☐ 3 ☐ 4 ☐

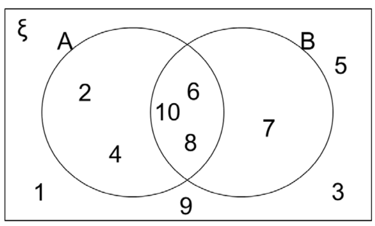
Identifying the range of possible outcomes (the sample space) for an event is key to being able to reason about the likelihood of one of those outcomes occurring. For situations with equally likely outcomes, the greater the possible number of different outcomes, the less likely each individual outcome becomes. Different ways to record and represent outcomes include lists, tables, grids and Venn diagrams.

Example 1:

- ξ integers between 1 and 10 inclusive.
- A – even numbers
- B – numbers > 5

What is:

a) P(getting an even number)
b) P(getting a number > 5)
c) P(picking a number > 5 OR even number)
d) P(picking a number > 5 AND even number)



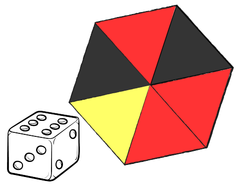
As the number of equally likely events is increased, consideration of the sample space becomes more crucial. For example, when flipping two coins, many students may say that an outcome of two heads, two tails or a head and a tail are all equally likely. The use of a probability space diagram, where outcomes are assigned probabilities, can help make sense of this misconception.

Example 2:

Finley drew this sample space diagram for all the outcomes when you spin the coloured spinner and roll the six-sided dice.

Number on dice

Colour	1	2	3	4	5	6
R	R,1	R,2	R,3	R,4	R,5	R,6
B	B,1	B,2	B,3	B,4	B,5	B,6
Y	Y,1	Y,2	Y,3	Y,4	Y,5	Y,6



It is incorrect. Why?

How can you change the sample space to reflect this?

Further support links

- NRICH: An introduction to tree diagrams (article): https://nrich.maths.org/tree-diagram-intro

5.3.3 Calculate and use probabilities of single and combined events

How confident are you that you understand and can explain how to calculate probabilities for single events and combined events using a variety of representations?

1 ☐ 2 ☐ 3 ☐ 4 ☐

How confident are you that you understand and can explain that the probabilities of all possible outcomes sum to one?

1 ☐ 2 ☐ 3 ☐ 4 ☐

Probability is quantified using proportion, and this proportion is usually represented as a fraction, although a decimal or percentage can also be used. Probability is also frequently quantified using a ratio, which implies a slightly different perspective on probability.

Consider a situation in which two blue counters and three red counters are in a bag, and a counter is repeatedly taken out of the bag and then replaced. The probability that a blue counter is drawn can be quantified as $\frac{2}{5}$; that is, for every five counters selected, two of them can be expected to be blue. When represented as a ratio, this becomes $2:3$, with the implicit interpretation that, for every two blue counters drawn out, three red counters remain.

The total chance of all the outcomes of an event sums to one and this can be illustrated on a number line, and links to this relationship:

[the chance of an outcome not happening] = 100% – [the chance of it happening]

For example:

Am I lucky today?

For each scenario, roll a dice 30 times and record the resulting number of points.

a) You score a point every time you roll an even number.
b) You score a point every time you roll a 6.
c) You score a point every time you roll a number less than 6.
d) You score a point every time you roll a number greater than 2.
e) You score a point every time you roll a square number.
f) You score a point every time you roll a number less than 10.

In each case, decide whether you were luckier or not than expected. Explain your reasoning by comparing your results with the expected number of occurrences

Further support links

- NCETM Secondary Professional Development materials: 5.3 Probability, pages 14–19

Notes