NCETM
NATIONAL CENTREfor EXCELLENCE in the TEACHING of MATHEMATICS

## Multiplicative Reasoning

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are ( $1=$ not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

## Question 1

How confident are you that you understand and can support children to recognise the role of unitising in multiplicative reasoning?
1 V
2
3
$\square$
$4 \square$

## How would you respond ...?

a. How can money be used to develop children's understanding of unitising as a precursor to multiplication?
Which purse would you rather have?


A


B
b. How is the structure of this problem linked to unitising?

Dinesh is making a necklace of beads. Each bead is 0.6 cm long. How long could the necklace be?

Tick the possible lengths of the necklace:


48 cm

## Responses

Note your responses to the questions here before you engage with the rest of this section:

## Did you notice that...?

a. When developing their understanding of unitising in multiplication, children initially begin with $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s , as they are familiar with skip counting in these multiples. The denominations of coins link to this, enabling unitising to be explored through the values of the coins.

For example, different coins have different values; they do not all have the value of 1 p. A 5 p coin is 1 unit in its own right. If we compare the two purses, the blue purse has six coins. Each coin has a value of 5 p so there is 30 p. The yellow purse has 10 coins but each coin has a value of $2 p$ so there is 20 p. This initial understanding builds the foundations for the structure of multiplication as repeated addition of equal groups.
b. In this question, known multiplication facts can be used to derive whole number $\times$ decimal fraction facts. Each bead is 0.6 cm so children can use their understanding of multiplication. For example, 6 times 6 ones equals 36 ones, so 6 times 6 tenths would be 36 tenths; therefore, 3.6 cm would be a possible length for the necklace. Other possible answers for this question would be 6.0 cm or 48 cm .

## Unitising and multiplication

This section looks at the role of unitising in multiplication, paying attention to multiplying whole numbers by decimals and within the context of converting units of measure.
Children require an understanding that a 'unit' does not need to have a value of 1 . This means considering one group of objects as a unit in its own right.
For example, if we look at some shoes, we might see ten shoes (with a unit equal to one shoe) or five pairs of shoes (with a unit being equal to two shoes).


Children are able to connect five groups of two with ten, counting two ways. This is sometimes referred to as dual counting.

- 'One group of two, two groups of two, three groups of two...'
- 'Two, four, six...'

By counting in different units, children will learn to recognise the number of units (five units in the example of five pairs of shoes); the unit size (two in the example of five pairs of shoes) and the total value (ten shoes in the example of five pairs of shoes). Although this could be abbreviated to 'one two, two twos, three twos...,' it is important that children are exposed to the language of groups in preparation for future work on multiplication as repeated addition of equal groups, which will be explored in more detail in Question 2.

The ability to see the same collection of objects in two different ways, is essential in many areas of maths, including multiplication, place value and fractions.

$4 \times 7=28$
$4 \times 7$ ones $=28$ ones

0.70 .00 .70 .7
$4 \times 0.7=2.8$
$4 \times 7$ tenths $=28$ tenths

Unitising supports children to draw upon their known number facts when multiplying whole numbers by tenths or hundredths.

Using arrays to represent the calculation will help children to compare how the same layout can be used to represent two related facts by changing the place value counters.
Stem sentences will support this. For example: 'Four times seven ones is equal to 28 ones, so 4 times seven tenths is equal to 28 tenths.'

The same principles apply when multiplying by hundredths. Children can explore triplets of related facts, deepening their understanding of unitising.

Have a go at writing the multiplication facts and stem sentences for the related facts in the image below.

0.40 .40 .4
0.040 .040 .04

As children develop their multiplicative reasoning and place value, they will multiply and divide numbers by 10 , 100 and 1000, linking this to real-life contexts and conversions between different units of measure.
Use children's understanding that dividing by $10 / 100$ is equivalent to multiplying by $0.1 / 0.01$ to write the equivalent multiplication equations, extending this understanding to multiplying by 0.001 and dividing by 1,000.

Dividing by 1,000 / Multiplying by 0.001 - Place Value Chart

$\therefore 1,000 \downarrow$| $\mathbf{1 , 0 0 0}$ | $\mathbf{1 0 0}$ | $\mathbf{1 0 s}$ | $\mathbf{1 s}$ | $\mathbf{0 . 1 s}$ | $\mathbf{0 . 0 1 s}$ | $\mathbf{0 . 0 0 1 s}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 0 | 0 | 0 |  |  |  |
|  |  |  | 6 |  |  |  |$\downarrow \times 0.001$

Unitising language is used to explain this structure. For example:

| 6,000 | $\div$ | 1,000 | $=$ |  |
| :--- | :--- | :--- | :--- | :--- |
| 6,000 | $\times$ | 0.001 | $=$ |  |
| 'What is the value of the '6' in six thousand?' <br> 'six thousand' <br> 6,000 |  |  |  |  |
| 'What is the value of the " 6 " in six?' <br> 'six ones' <br> 6 |  |  |  |  |

Problem-solving questions provide opportunities for children to apply their understanding in different contexts. For example, 'Nia measures the height of her desk. It is 64 cm tall. How tall is Nia's desk in metres?'

## Common errors in this area may include:

- children linking the value of a coin to its size
- believing that multiplication always increases the value.


## What to look for

## Can a child:

- understand that units can be made up of more than one object?
- draw on multiplication facts and unitising to multiply whole numbers by decimals?
- recognise that multiplying a positive number can result in a smaller value the one started with, for example when multiplying by decimals or fractions?


## Links to supporting materials:

NCETM Primary Professional Development materials, Spine 2: Multiplication and Division:

- Topic 2.1: Counting, unitising and coins
- Topic 2.19: Calculation: $\times / \div$ decimal fractions by whole numbers


## Notes:

Key learning from support material and self-study:

What I will focus on developing in my classroom practice:

