## \#mathscpdchat 8 November 2022

## How do you help students gain a deep understanding of non-linear sequences? <br> Hosted by Kathryn Darwin

This is a summary of the discussion - to see all the tweets, follow the hashtag \#mathscpdchat in Twitter


The links shared during this discussion were:

Quadratic nth term which is a collection of tasks created by Don Steward. It was shared by Nathan Day

Fibonacci patterns which are more tasks created by Don Steward. Students are challenged to prove algebraically results about Fibonacci patterns and sequences that they have established in other ways. It was shared by Nathan Day

Directed number grid which is a ask presented by Don Steward, originally created by Martin Wilson. 'It is good for practice of directed number addition and subtraction because sometimes you are developing the grid forwards and sometimes backwards, Fibonacci-like.' It was shared by Nathan Day

1 Step, 2 Step Poster which is a poster from NRICH. It presents the NRICH problem 1 Step 2 Step. Students are invited to explore the different ways of going down 12 steps if they can go down one step or two steps at a time in any combination of those two possibilities. It was shared by Nathan Day

Step Up which is a presentation from Play with your math of the step problem described above. In this case students are explicitly challenged to investigate it for different numbers of steps. It was shared by Nathan Day

Fibonacci Numbers which is an attractively-illustrated interactive Mathigon article in which users are challenged to respond to questions about pairs of breeding rabbits, pine cones and sunflowers. They can also read illustrated notes about the history of the person Fibonacci. It was shared by Dave

## Taylor

The Slightly Spooky Recaman Sequence - Numberphile which is a YouTube video about an unusual sequence of numbers which is described on this page of the On-Line Encyclopedia of Integer
Sequences. It was shared by Dave Taylor

Quadratic Sequences which is an illustrated blog by Miss Konstantine in which she presents clearly and attractively some of her own original tasks. It was shared by Miss Konstantine

Ideas from Points of Departure which are activities from the ATM Points of Departure books which have been collated by Mike Ollerton. They include an interactive spreadsheet file by means of which an infinite number of sequences can be generated, and their general terms explored. It was shared by

## Mary Pardoe

Geometric Series: Sum to Infinity which is an article in the archived NCETM Secondary Magazine 137. It was shared by Mary Pardoe

Visual Patterns which is a very extensive and varied collection of patterns, created by Fawn Nguyen, who was a middle school teacher for 30 years, prior to her present role on Special Assignment in California. Many of the patterns generate sequences that can be explored in various ways. It was shared by David Butler

How Equal Temperament Ruined Harmony (and Why You Should Care) which is a book by Ross Duffin about some (mathematical) aspects of music, including harmonic 'disadvantages' of the equal division of the octave into twelve notes that has become our present standard tuning method. It was shared by Mark Dawes

Linear Sequences - differentiating and making connections which is a blog post by Rob Southern. It was shared by Rob Southern

King Arthur's Problem which is a PDF document which poses a classic problem presented as an illustrated story. It was shared by Mr Hawes

Summing Up Fibonacci which is a blog by Andrew Stacey. It was shared by Andrew Stacey

Overhanging bricks which is a YouTube video of a presentation by David Bedford at a Maths Jam Conference in 2014. It was shared by David Bedford

An illustrated summary of the discussions in this \#mathsCPDchat follows.

The host followed her introductory message ...


Kathryn MCCT @Arithmaticks • 15h
Gooood evening everyone! $\&$ Thank you for joining in with \#MathsCPDChat tonight.
As usual, one rule and one rule only... use the hashtag to help us find and follow the conversation!

... by tweeting this poll ...


Kathryn MCCT @Arithmaticks • 15h
Replying to @Arithmaticks
So let's kick off... I blooming love sequences. I am going to imagine you do too... But which ones are your favourite? Why? \#MathsCPDChat

| Linear | $12.5 \%$ |
| :--- | ---: |
| Quadratic © | $50 \%$ |
| Fibonacci | $34.4 \%$ |
| Geometric | $3.1 \%$ |

32 votes . Final results
... which generated two discussions. This was a conversation about teaching quadratic sequences ...

MrHawesMaths @HawesMaths • 15h

## Replying to @Arithmaticks

Love a quadratic sequence linking in with linear. \#mathscpdchat
Kathryn MCCT @Arithmaticks • 15h
Replying to @HawesMaths
Can you elaborate? \#MathsCPDChat

... and the next tweet prompted a long conversation which included some interesting tweets about maths applied to music:


RobotMaths @robotmaths • 15h
Replying to @Arithmaticks
Harmonic

Kathryn MCCT @Arithmaticks • 15h
Replying to @robotmaths
I was only going for the ones on the GCSE spec as I only had 4 options! But YES
Please tell us more about it/how you use it in the classroom?
\#MathsCPDChat


RobotMaths @robotmaths • 15h
...
Replying to @Arithmaticks
So a harmonic sequence is formed by taking the reciprocals of every term in an arithmetic sequence. The obvious one is $1 / 1,1 / 2,1 / 3$ etc which is illustrated by how far you can lean books on top of each other without them overbalancing


David Bedford @DavidB52s • 15h
Replying to @Arithmaticks and @robotmaths
Just get yourself some bricks :)

youtube.com
David Bedford 'Overhanging bricks', Maths Jam Conference 2014

Kathryn MCCT @Arithmaticks • 15h
Replying to @robotmaths
Gorgeous! How might you use this in lessons? \#MathsCPDChat

RobotMaths @robotmaths.15h
Replying to @Arithmaticks
To show that a decreasing sequence doesn't necessarily have a sum that reaches a limit
sigma $1 / r$ has no limit
sigma $1 / r^{\wedge} 2$ has a limit of $p^{\wedge} 2 / 6$


Kathryn MCCT @Arithmaticks • 15h
Replying to @robotmaths
Lovely stuff - lots of A Level potential. Would you show it to a GCSE class? \#MathsCPDChat


RobotMaths @robotmaths. 15h
Replying to @Arithmaticks
We do series with Year 10 in our A Level Further Maths taster day, mainly covering sigma $r r^{\wedge} 2$ and $r^{\wedge} 3$, but introducing reciprocals as well


RobotMaths @robotmaths • 15h
Replying to @robotmaths and @Arithmaticks
confusingly the "harmonic series" in music is actually geometric the frequency of the pitches of different notes an octave apart go ..., 110, $220,440,880,1760, \ldots$
and the wavelengths are not a harmonic series because they are the reciprocals of a geometric not arithmetic seq.

## Kathryn MCCT @Arithmaticks • 15h

Replying to @robotmaths
This has really upset me haha... I don't know what lots of those musical terms really mean, but it feels like someone's made a big error here $\widehat{6}$ \#MathsCPDChat


Kathryn MCCT @Arithmaticks • 15h
Replying to @robotmaths
This has really upset me haha, I am not 'theoretically' musical (I just sing by ear in choir) so I didn't know this. Feels like a HUGE oversight from the universe... 冬 \#MathsCPDChat


Mark Dawes @mdawesmdawes • 15h
Replying to @Arithmaticks and @robotmaths
But, but, but ...
This is true if you only like octaves!
The unit fractions of the wavelength of the fundamental of the string/pipe give you lots of other notes. A bugle can only play those notes (known as 'harmonics'!).
\#MatheMusicalPedantry

Mark Dawes @mdawesmdawes • 15h
Replying to @mdawesmdawes @Arithmaticks and @robotmaths
On a related issue, to go up from one note by a perfect 5th (eg from C up to G) you multiply the frequency by 1.5

If you keep going up by a 5 th a dozen times you get back to the note you started on ( ${ }^{*}$ ) - only 7 octaves higher.
The freq has been multiplied by $1.5^{\wedge} 12=129.75$
1/


Mark Dawes @mdawesmdawes • 15h
Replying to @mdawesmdawes @Arithmaticks and @robotmaths
Going up by an octave involves multiplying the frequency by 2 . Going up 7 octaves means $2^{\wedge} 7=128$
The discrepancy between 128 and 129.75 is noticeable and is known as (drumroll ...) a "Pythagorean Comma"!
2/2

Kathryn's first main question ...


Kathryn MCCT @Arithmaticks • 17h
How do you introduce NON-LINEAR sequences to your students?
\#MathsCPDChat
... prompted many replies. This was one conversation:

Miss Konstantine @giftedHKO.18h
Replying to @Arithmaticks
Our yesterday 11 are looking at quadratic sequences at the moment. We look at images that represent linear sequences first then quadratic.
\#mathsCPDchat


Kathryn MCCT @Arithmaticks • 18h
Replying to @giftedHKO
This is stunning! Did you have to scaffold how to 'see' them?
\#mathscpdchat
Miss Konstantine @giftedHKO.18h
Replying to @Arithmaticks
As a class we discussed a few. They worked in pairs.
Kathryn MCCT @Arithmaticks • 18h
Replying to @giftedHKO
I really like it - particularly the ones with 3 terms (Can you please send it to me? : ${ }^{\circ}$ ) \#MathsCPDChat

Miss Konstantine @giftedHKO • 18h
Replying to @Arithmaticks
Yes. I'll link in a min. Gotta pop out. So when I'm back \#mathscpdchat

Miss Konstantine @giftedHKO • 17h
Replying to @giftedHKO and @Arithmaticks
mathshko.com/2018/04/24/qua...
\#mathscpdchat


Brooke Hunter @BrookeEHunter•18h
Replying to @giftedHKO and @Arithmaticks
This is beautiful!!

In this conversation sequences identifiable within Pascal's triangle, and in patterns derived from it, were mentioned:


Mary Pardoe @PardoeMary • 17h
Replying to @Arithmaticks
There is this ... from @ATMMathematics
\#mathscpdchat
atm.org.uk/Ideas-from-Poi...

## Ideas from Points of Departure



Below is a list of activities from the ATM Points of Departure books that has been collated by Mike Ollerton

Activities from Points of Departure Books 1 - 4
The Skewed Pascal is from Points of Departure 3
Interactive spreadsheet file containing an infinite amount of sequences can be generated and their functions can be explores.


Kathryn MCCT @Arithmaticks • 21h
This has reminded me of the power of Pascal's triangle... What is your favourite sequence hidden within it?
\#MathsCPDChat


Mary Pardoe @PardoeMary • 21h
Replying to @Arithmaticks
I have always really loved triangle numbers $\qquad$ since I was about four! \#mathscpdchat
Kathryn MCCT @Arithmaticks • 21h
Replying to @PardoeMary
I have a thing for them too. I love that they seem so logical and bonkers all at once! Their nth term is so complex comapared to how easy they are to generate! \#MathsCPDChat

MrHawesMaths @HawesMaths • 21h
Replying to @Arithmaticks and @PardoeMary
I like the investigation (king Arthur's problem) where you have to use your knowledge of powers of 2 and linear sequences to problem.
people.math.sc.edu/pme/year/2010-...


Nathan Day @nathanday314•21h
Replying to @Arithmaticks
Partitions! (well technically, compositions)

## \#MathsCPDChat

Nathan Day @nathanday314. Oct 18
I've particularly enjoyed thinking about this task since @StudyMaths's \#mathconf30 workshop.

This is my favourite visualisation of the problem yet.
Look closely and you may be able to see how systematically these partitions are being generated and ordered. twitter.com/nathanday314/s...


[^0]Mary Pardoe @PardoeMary.16h
Replying to @Arithmaticks
Visual images can be a way in ...


The number of squares in the $n$th image $=n \times 2 n-n(n-1)$

$$
=2 n^{2}-n^{2}+n
$$

$$
=n^{2}+n
$$



Rebecca Atherfold @becatherfold • 17h
Replying to @PardoeMary and @Arithmaticks
These are all great! I'm bookmarking away! \#mathscpdchat


Kathryn MCCT @Arithmaticks • 17h
Replying to @PardoeMary
These are very nice! I love a visualisation of a quadratic sequence, but normally go for counters! \#MathsCPDChat


Mary Pardoe @PardoeMary . Nov 8
Replying to @Arithmaticks
Yes ... lots of possibilities ... also focus on people's different ways of seeing (1/n)
\#mathscpdchat
Quadratic expressions
An expression for the number of small triangles in the $n^{\text {th }}$ image is ... ?


1+3 $1+3+5 \quad 1+3$

An expression for the number of small triangles in the $n^{\text {th }}$ image is

$$
\begin{gathered}
1+3+5+7+\ldots(2 n-5)+(2 n-3)+(2 n-1)+(2 n+1) \\
1+(2 n+1)+3+(2 n-1)+5+(2 n-3)+7+\ldots \ldots \\
(2 n+2)+(2 n+2)+(2 n+2)+7+\ldots \ldots \\
1 / 2(n+1) \times(2 n+2) \\
1 / 2(n+1) \times 2(n+1) \\
(n+1) \times(n+1) \\
(n+1)^{2} \\
\hline
\end{gathered}
$$


$\frac{1 \times 2}{2}$

Nathan Day @nathanday314•17h
Replying to @Arithmaticks and @PardoeMary
I love these from Don.
donsteward.blogspot.com/2017/04/quadra...

The patterns in the answers are delightful, too.

## \#MathsCPDChat




Nathan Day @nathanday314•17h
Replying to @nathanday314 @Arithmaticks and @PardoeMary His Fibonacci questions (donsteward.blogspot.com/2012/04/fibona...) clinch them as my favourite type of sequence though!

Especially when combined with @vihartvihart's legendary Fibonacci spirals videos, which my Y10's loved last week -youtube.com/watch?v=ahXIMU...

1) add up any three consecutive Fibonacci numbers
what happens?
2) show that the sum of four consecutive Fibonacci numbers is the sum of two Fibonacci numbers
3) for any four consecutive Fibonacci numbers add the $1^{\text {st }}$ and last ( $\left.4^{\text {th }}\right)$ and divide by 2
what happens?
4) for any five consecutive Fibonacci numbers sum the 1 st and last ( $5^{\text {th }}$ ) and divide by 3
what happens?
5) for any seven consecutive Fibonacci numbers add the 3rd and 7th and divide by 3 what happens?
6) for any nine consecutive Fibonacci numbers sum the 1st and 9th and divide by 7 what happens?
7) total any six consecutive Fibonacci numbers and divide by 4 what happens?
8) add up any ten consecutive Fibonacci numbers and divide by 11 what happens?
9) add up the first two, first three, first four, etc. Fibonacci numbers what do the results have to do with a Fibonacci number?
10) for any nine consecutive Fibonacci numbers, subtract the 6th from the 9th and divide by 2
what happens? $10 \quad 55$
$\begin{array}{llll}\text { 144) for any eight consecutive Fibonacci } & 11 & 89\end{array}$
numbers, subtract the 5th from the 8th $12 \quad 144$
and divide by 2
what happens? 13233
$14 \quad 377$
11) for any eleven consecutive Fibonacci numbers add the 3rd and the 11th and 15610 divide by 7
what happens?


Nathan Day @nathanday314•17h
Replying to @nathanday314 @Arithmaticks and 2 others
Ooh, especially especially when followed up with my favourite @playwyourmath / @nrichmaths poster problem. playwithyourmath.com/2017/07/27/7-s... /nrich.maths.org/7199

## 1 Step, 2 Step



## 7 Step Up

In how many ways can you climb* 3 steps?

5 steps?
6 steps?
15 steps?
$n$ steps?



Kathryn MCCT @Arithmaticks • 17h
Replying to @nathanday314 and @PardoeMary OBSESSED with these. This whole post is gorgeous. \#MathsCPDChat


David Butler @DavidKButlerUoA • 16h
Replying to @PardoeMary and @Arithmaticks
You may be interested in @fawnpnguyen's Visual Patterns visualpatterns.org

Kathryn MCCT @Arithmaticks • 17h
Replying to @nathanday314 @PardoeMary and @vihartvihart I love this one too! \#KingDon \#MathsCPDChat


Kathryn MCCT @Arithmaticks • 17h
Replying to @nathanday314 @PardoeMary and @vihartvihart \#MathsCPDChat


Nathan Day @nathanday314.17h
Replying to @Arithmaticks and @PardoeMary


This was a different 'way in' to non-linear sequences, less dependent on visual images ...


Dr Anna @Dr_anna_maths • 19h
Replying to @Arithmaticks
My favourite starter activity is just to write $2,4, \ldots$ up on the board several times and then ask students to continue in different ways. After the obvious they go nonlinear


Kathryn MCCT @Arithmaticks • 19h
Replying to @Dr_anna_maths
How do you then 'channel' that conversation? Does it lead to formal definitions? \#MathsCPDChat

Dr Anna @Dr_anna_maths • 19h ...
Replying to @Arithmaticks
I broke the first rule of \#mathscpdchat 给 Yes, something to keep returning to as you work through geometric, Fibonacci and quadratic sequences. Sometimes depending on the students, I 'seed' an answer or two around the class to help channel that flow
... and, in response to the host's first main question, there was a comment about a resource and also two more teaching-approach suggestions:


MathsWithMsB MathsWithMsB • 16h
Replying to @Arithmaticks
There was a really good @NCETM resource about this - growing sequences.


Mike Thain @ThainMike • 18h

## Replying to @Arithmaticks

Starting with quadratic sequences I show how they are made by adding a linear sequence to the square numbers, then deconstructing them back into their constituent parts.


MrHawesMaths @HawesMaths • 18h
Replying to @Arithmaticks
Patterns. Use of multi link works well for this. As we could look at them in a 2d for and 3d too. \#mathscpdchat

Kathryn's second main question ...


Kathryn MCCT @Arithmaticks • 19h
What is your go-to resource for teaching students about the different types of sequence? \#MathsCPDChat
... prompted this reply:

Rebecca Atherfold @becatherfold • 19h
Replying to @Arithmaticks
@goteachmaths resources are great - been using the sequences ones recently \#mathscpdchat alongside multi link cubes

To the host's third main question ...
Kathryn MCCT @Arithmaticks • 19h
Quadratic is winning so far!
Why do you love it so much?
How do you teach it?
We've seen some images shared already - do you have any other tricks up your sleeve?
\#MathsCPDChat

| So let's kick off... I blooming love sequences. I am going to imagine you do <br> too... But which ones are your favourite? Why? \#MathsCPDChat |  |
| :--- | ---: |
| Linear | $13.3 \%$ |
| Quadratic | $46.7 \%$ |
| Fibonacci | $33.3 \%$ |
| Geometric | $6.7 \%$ |
| 15 votes $\cdot 42$ minutes left |  |

... there were more replies. This discussion was generated by Sam's suggestions:

Sam Blatherwick @blatherwick_sam•19h
Replying to @Arithmaticks
On quadratic sequences that "occur"... if you follow a path from an ulam spiral outwards it forms a quadratic sequence... so $3,13,31,57,91$ is quadratic.

There's neat stuff you can spot from answers here
\#mathscpdchat

| 196 | 195 | 194 | 193 | 192 | 191 | 190 | 189 | 188 | 187 | 186 | 185 | 184 | 1 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 145 | 144 | 143 | 142 | 141 | 140 | 139 | 138 | 137 | 136 | 135 | 134 | 133 | 1 |
| 3 | 146 | 101 | 100 | 99 | 98 | 97 | 96 | 95 | 94 | 93 | 92 | 91 | 132 | 1 |
| 5 | 147 | 102 | 65 | 64 | 63 | 62 | 61 | 60 | 59 | 58 | 57 | 90 | 131 | 1 |
| 1 | 148 | 103 | 66 | 37 | 36 | 35 | 34 | 33 | 32 | 31 | 56 | 89 | 130 | 1 |
| 2 | 149 | 104 | 67 | 38 | 17 | 16 | 15 | 14 | 13 | 30 | 55 | 88 | 129 | 1 |
| 3 | 150 | 105 | 68 | 39 | 18 | 5 | 4 | 3 | 12 | 29 | 54 | 87 | 128 | 1 |
| 4 | 151 | 106 | 69 | 40 | 19 | 6 | 1 | 2 | 11 | 28 | 53 | 86 | 127 | 1 |
| 5 | 152 | 107 | 70 | 41 | 20 | 7 | 8 | 9 | 10 | 27 | 52 | 85 | 126 | 1 |
| 5 | 153 | 108 | 71 | 42 | 21 | 22 | 23 | 24 | 25 | 26 | 51 | 84 | 125 | 1 |
| 7 | 154 | 109 | 72 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 83 | 124 | 1 |
| 8 | 155 | 110 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 123 | 1 |
| 8 | 156 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 | 121 | 122 | 1 |
| 3 | 157 | 158 | 159 | 160 | 161 | 162 | 163 | 164 | 165 | 166 | 167 | 168 | 169 | 1 |
| 212 | 213 | 214 | 215 | 216 | 217 | 218 | 219 | 220 | 221 | 222 | 223 | 224 | 2 |  |

Kathryn MCCT @Arithmaticks • 19h
Replying to @blatherwick_sam
I haven't seen this before! Thank yiou for sharing... any more insights for us? \#MathsCPDChat

Sam Blatherwick @blatherwick_sam•19h
Replying to @blatherwick_sam and @Arithmaticks
Also, simply looking at diagonals on a times table grid, then asking students to factorise their nth term leads to interesting results
\#mathscpdchat


Kathryn MCCT @Arithmaticks • 19h
Replying to @blatherwick_sam
Well. I'm going to do this as soon as the chat is over... \#MathsCPDChat


## Sam Blatherwick @blatherwick_sam•19h

Replying to @Arithmaticks
There was an AQA L2 FM question many years ago that gave two linear sequences and multiplied the terms together and you were asked to find the nth term of that sequence. It was a really cool question and has neat depth to explore. \#mathscpdchat


Kathryn MCCT @Arithmaticks • 19h
Replying to @blatherwick_sam
I think I remember this one actually! Gorgeous question \#MathsCPDChat
Tayyub Majeed @tm_maths • 19h
Replying to @blatherwick_sam and @Arithmaticks
May be wrong but sure @mrsouthernmaths had something similar he posted about.


Rob Southern @mrsouthernmaths • 19h
Replying to @tm_maths @blatherwick_sam and @Arithmaticks
It was finding the sum and the difference of two linear sequences. I blogged about it here:
mrsouthernmaths.wordpress.com/blog/
mrsouthernmaths.wordpress.com
Blog
b) 5

Follow My Blog Linear Sequences - differentiating and making connections - 16/10/2021 Firstly, a bit...

Also, there was this comment about exploring quadratic sequences ...

## MrHawesMaths @HawesMaths • 19h

Replying to @Arithmaticks
Again I use the multi link to create the squares in one colour and then use another colour to generate the linear part. \#mathscpdchat
Kathryn MCCT @Arithmaticks • 19h
Replying to @HawesMaths
I was just about to ask this in reply to your last tweet! A little like @giftedHKO 's sheet, but concrete? \#MathsCPDChat
... and this:

## Miss Konstantine @giftedHKO • 19h

\#mathscpdchat now linking this with quadratic sequences.

Miss Konstantine @giftedHKO • 20h
Not posted a MCQ sheet for a while. Made one for y 11 as they are looking at quadratic sequences and we will want to be revising topics for the mock too. mathshko.com/multiple-choic...

| 1.) Generate the sequence with nth term $n^{2}+3 n-10$ <br> Which sequence below shows the first 4 terms? |  |  |  |  |  |  |  | 2.) Factorise the expression below: $x^{2}+3 x-10$ <br> Use it solve the equation $x^{2}+3 x-10=0$ |  |  |  | 3.) Substitute $x=-3$ <br> Into the expression $y=x^{2}+3 x-10$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $-6,-3,2,9 \ldots$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| B |  | $-6,0,8,18, \ldots$ |  |  |  |  |  | A | $\begin{gathered} x=5 \\ x=-3 \end{gathered}$ | B | $x=-5$ $x=-2$ | A | -10 | B | -28 |
| C | $-6,-1,5,13, \ldots$ |  |  |  |  |  |  | C |  |  | $x=-5$ |  |  |  |  |
| D | $-5,0,2,10, \ldots$ |  |  |  |  |  |  |  | $x=-2$ | D | $x=2$ | C | 8 | D | -13 |
| 4.) Complete the table for the equation$y=x^{2}+3 x-10$ |  |  |  |  |  |  |  | 5.) Which graph is the graph of $y=x^{2}+3 x-10$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | Graph A |  | Graph B |  | Graph C |  | Graph D |  |
| $\boldsymbol{x}$ | $-3-2-1$ | 0 | 1 | 2 | 3 | 4 | 5 |  |  |  |  |  |  |  |  |
| $y$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Kathryn's fourth main question, as she kept an eye on results emerging from her poll, ...
Kathryn MCCT @Arithmaticks • 20h
Fibonacci is in close second, but is definitley my favourite!
How do you use Fibonacci sequences to inspire your students?
(Aside from the fact they confuse the number of $B / N / C$ 's in the name, and you can say "BUT THEY FOLLOW THE SEQUENCE.. 1B, 1N \& 2N'S!") \#MathsCPDChat

Kathryn MCCT @Arithmaticks • 25m
So let's kick off... I blooming love sequences. I am going to imagine you do too... But which ones are your favourite? Why? \#MathsCPDChat

| Linear | $18.2 \%$ |
| :--- | :--- |
| Quadratic | $45.5 \%$ |
| Fibonacci | $31.8 \%$ |
| Geometric | $4.5 \%$ |

[^1]... resulted in two suggestions. This one ...

## Nathan Day @nathanday314•20h

Replying to @Arithmaticks
I love a Directed Number Grid (both when teaching Fibonacci and directed number).
They're like doing Fibonacci in two-dimensions at once!
donsteward.blogspot.com/2020/03/direct...

(2)

|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
|  | 3 | 3 |  |  |
|  | 3 | 1 |  |  |
|  |  |  |  |  |

(3)

|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :---: |
|  |  |  |  |  |
|  |  |  | 5 | 8 |
|  |  |  | 6 | 13 |
|  |  |  |  |  |

(4)

|  |  |  | 3 | 11 |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  | 4 | 2 |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

(5)

|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  | 19 |  |
|  |  |  |  |  |
|  | 16 |  | 20 |  |
|  |  |  |  | 35 |

(6)

(7)

|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | 13 |  |  | 1 |
|  |  |  |  |  |
|  |  |  |  |  |
|  | 13 |  |  | 49 |

(8)

| -19 |  |  | 5 |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
| -3 |  |  | 13 |  |
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|  |  |  |  |  |
| 57 |  |  | 23 |  |

Kathryn MCCT @Arithmaticks • 20h
Replying to @nathanday314
I haven't seen these before... Thank you for sharing! \#MathsCPDChat

Dave Taylor @taylorda01.20h
Replying to @Arithmaticks
I like this to model the problem from Liber Abacci.
mathigon.org/course/sequenc.)\ was\ an\ Italian\ mathematicia n.

mathigon.org
Fibonacci Numbers - Sequences and Patterns - Mathigon
Learn about some of the most fascinating patterns in mathematics, from triangle numbers to the Fibonacci sequence and Pascal's triangle.

Brooke Hunter @BrookeEHunter•20h
Replying to @taylorda01 and @Arithmaticks
This is lovely!


Kathryn MCCT @Arithmaticks • 20h
Replying to @taylorda01
I definitley don't use Mathigon enough! \#MathsCPDChat

Dave Taylor @taylorda01.21h
I've been putting a child to bed, and then I'm off for a run, but are we just choosing our favourite sequences? \#mathscpdchat

youtube.com
The Slightly Spooky Recamán Sequence - Number... Check out Brilliant (and get 20\% off their premium service): https://brilliant.org/numberphile ...


Kathryn MCCT @Arithmaticks • 21h
Replying to @taylorda01
Not JUST... but I appreciate this! \#MathsCPDChat

The host's (Kathryn Darwin's) fifth main question, which included an image showing the poll results as they were at that instant, ...

Kathryn MCCT @Arithmaticks • 21h
Oooh poor Geometric... why do we dislike it so much?
How can we spice it up? \#MathsCPDChat

| Kathryn MCCT @Arithmaticks $\cdot 39 \mathrm{~m}$ <br> So let's kick off... I blooming love sequences. I am going to imagine you do <br> too... But which ones are your favourite? Why? \#MathsCPDChat |  |
| :--- | ---: |
| Linear | $14.8 \%$ |
| Quadratic | $48.1 \%$ |
| Fibonacci | $33.3 \%$ |
| Geometric | $3.7 \%$ |
| 27 votes 19 minutes left |  |

... generated three conversations and two interesting comments, which the (linked-to-Twitter) screenshots below show. In those replies and conversations only you can click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

The longest of the three conversations was initiated by an observation from Tom Bowler, and included contributions from Kathryn Darwin, Andrew Stacey, David Bedford, Susan Whitehouse and Jonathan Hall:

Tom Bowler @Ridermeister • 21h
Replying to @Arithmaticks
Mustn't forget that Fibonacci is effectively geometric in the long term!
Kathryn MCCT @Arithmaticks • 22h
Replying to @Ridermeister
Would you explore this element with students? Bring in the golden ratio? \#MathsCPDChat


Tom Bowler @Ridermeister • 20h
Replying to @Arithmaticks
Definitely, especially as both are explicitly in GCSE now so it's lovely to have that link. Showing them the nth term is great as well because it brings in surds too. I have derived the nth term with a class but they were an extremely strong GCSE group.


Andrew Stacey (@loopspace@mathstodon.xyz) @mathforge.21h
Replying to @Ridermeister and @Arithmaticks
Depending on the students, you could try some of the ideas in:

|  |  |
| :--- | :--- |
| 므 | loopspace.mathforge.org <br> Summing Up Fibonacci |

Tom Bowler @Ridermeister • 21h
Replying to @Ridermeister and @Arithmaticks
And it's lovely to look at the sum to infinity $0.9+0.09+0.009+\ldots$ as another way to justify that 0.999... is equivalent to $1.000 \ldots$

## David Bedford @DavidB52s • 22h

Replying to @Ridermeister and @Arithmaticks
That's really the only way - everything else is smoke and mirrors :)


## Susan Whitehouse @Whitehughes • 21h

Replying to @Ridermeister and @Arithmaticks
I love finding the sum to $n$ terms by changing into a different base, e.g. $1+2+4+8$ in base 2 is 1111 which is $2^{\wedge} 4-1$

Kathryn MCCT @Arithmaticks • 21h
Replying to @Whitehughes and @Ridermeister
This is very fancy. @mrshawthorne7 and @StudyMaths would love that haha \#MathsCPDChat


Jonathan Hall @StudyMaths • 20h
Replying to @Arithmaticks @Whitehughes and 2 others
It reminds me of when I do my multi-base monsters task and pupils are amazed how fast I know 11111111 in binary is 255 .


Kathryn MCCT @Arithmaticks • 19h
Replying to @StudyMaths @Whitehughes and 2 others I mean. I need to know more about this immediately.


Jonathan Hall @StudyMaths • 13h
Replying to @Arithmaticks @Whitehughes and 2 others

| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | $\mathbf{1 5 2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | $\mathbf{1 5 2}$ |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | $\mathbf{2 5 5}$ |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | $\mathbf{2 5}$ |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | $\mathbf{6 1}$ |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | $\mathbf{3 6}$ |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | $\mathbf{3 6}$ |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 | 102 |

A shorter conversation was a dialogue between Mary Pardoe and Kathryn Darwin


Mary Pardoe @PardoeMary.21h
Replying to @Arithmaticks
I don't dislike it!!!! There is this, that I loved writing ...
ncetm.org.uk/media/pasliscp..
\#mathscpdchat

... and a question ...
What froction of each large circle is block?


Kathryn MCCT @Arithmaticks • 21h
Replying to @PardoeMary
Go on then... sell it to us! $-\underset{\text { \#MathsCPDChat }}{ }$


Mary Pardoe@PardoeMary.21h
Replying to @Arithmaticks
\#mathscpdchat
Lots of lovely visual possibilities to explore ...


A question about this frieze that can be answered if one has investigated, and thereby developed understanding of, geometric sequences, is:

If $T$ is the area of the largest triangle,
what (in terms of $T$ ) will be the total-area-to-infinity of the parts of the frieze that are green?

$\ldots$ and there was this short discussion between Peter Williams, Kathryn Darwin and Charlotte Hawthorne:

Peter Williams @MathsImpact • Nov 8
Replying to @Arithmaticks
Geometric progression exam questions have the distinct advantage of being asked in a context that makes sense and doesn't feel contrived.

I think it's easy to forget that we use them far more than all the other sequences via growth and decay.

## \#mathscpdchat



## Kathryn MCCT @Arithmaticks • Nov 8

Replying to @MathsImpact
(Well timed on the 'edit'!)
I LOVE them for this exact reason... was literally talking to @mrshawthorne7 about this on my way home tonight! \#MathsCPDChat
Charlotte Hawthorne @mrshawthorne7 • Nov 8
Replying to @Arithmaticks and @Mathslmpact
We were!
MatheMusician shared some of her interesting knowledge about maths and music:


MatheMusician @Mathe_Musician. Nov 8
Replying to @Arithmaticks
How can you *not* like Geometric progressions. Great fun when pupil comes for a music theory lesson and gets a lesson on sum of GPs instead. (Adding repeated dots to notes.)
MatheMusician @Mathe_Musician.Nov 8
Replying to @Mathe_Musician and @Arithmaticks
Oh, and there's a Dvorak symphony that starts with quadruply dotted notes...

There was only one reply to the host's last main question ...


Kathryn MCCT @Arithmaticks • Nov 8
OK, 10 mins to go... Best NON-LINEAR sequences resource/idea you have? \#MathsCPDChat
... by a contributor who did NOT read the question carefully:
Mary Pardoe @PardoeMary • Nov 8
Replying to @Arithmaticks
Not sure about 'best', but this is a strategy l've used and students have enjoyed it ... and learned from it!
\#mathscpdchat
give examples of tasks that provide opportunities for pupils to see visual patterns in various different ways, and thereby arrive at equivalent quadratic expressions. At the same time pupils can use facts about first and second differences and the coefficients of general quadratic expressions to derive expressions for nth terms. They can then use those results to check the expressions that they reached by examining visual patterns. Because the images are on square grids they could all be made into 3-D objects using multilink cubes, in which case it would be the number of cubes (rather than squares) that are counted.

A tweet from Mr Hawes (which was not a reply to any one of Kathryn's main questions) ...

## MrHawesMaths @HawesMaths • Nov 8

Another of my favourites is What is the next number in this sequence: 1,11 , 21, 1211, 111221, 312211, 13112221? \#mathsepdchat
... received the following replies. The contents of the last two of them (which were answers to Mr Hawes' question) have been covered in the screen shots in order that those tweets are not 'spoilers' for readers of this summary.


Jonathan Payne @DrPMaths • Nov 8
Replying to @HawesMaths
The thing I find crazy about this sequence is the ratio between terms converges, and the limit is the solution of a polynomial with strangely high degree

marc schofield @hezooss • Nov 8
Replying to @HawesMaths


# Janette Ruth @JanetteRuth $15 \cdot 19 \mathrm{~h}$ <br> Replying to @HawesMaths 

(To go to the actual tweet click here.)

(To go to the actual tweet click here.)

This was the host's final tweet of the chat:

## Kathryn MCCT @Arithmaticks • Nov 8

Thank you SO much for tonight - I have enjoyed really digging deep into sequences. I think I was right to assume you'd love them as much as me! Enjoy the rest of the half term! \#MathsCPDChat



[^0]:    This next conversation featured various different sequences of images:

[^1]:    22 votes - 34 minutes left

