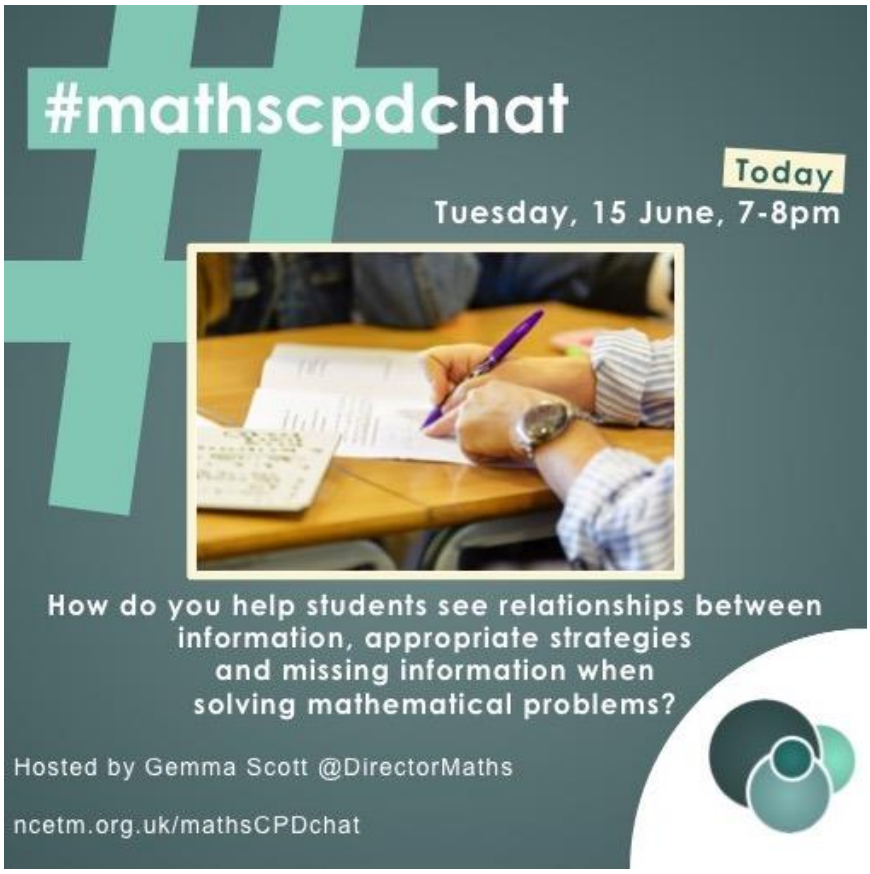
#mathscpdchat 15 June 2021

How do you help students see relationships between information, appropriate strategies and missing information when solving mathematical problems?
Hosted by Gemma Scott

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Among the links shared during the discussion were:

Lesson Study: Problem Solving Approaches in Mathematics Education as a Japanese Experience which is a paper by Masami Isoda that was presented at the International Conference on Mathematics Education Research 2010. The author explains the 'Problem Solving Approach' as a teaching approach in which children learn mathematics by/for themselves in Japan. It was shared by J Dempsey

Teachers Using NRICH 3 which is an NRICH article by Sharon Walter (now Sharon Malley) and Jennifer Piggott. Sharon talks about her experiences when she was one of four NRICH Teacher Fellows who worked on embedding NRICH materials into their teaching during 2008/09. It was shared by Sharon Malley

Goal Free Problems which is a website set up by Pete Mattock in order to allow teachers to access and share goal free problems, which are intended to support pupils in improving their mathematical knowledge and understanding. It was shared by Pete Mattock

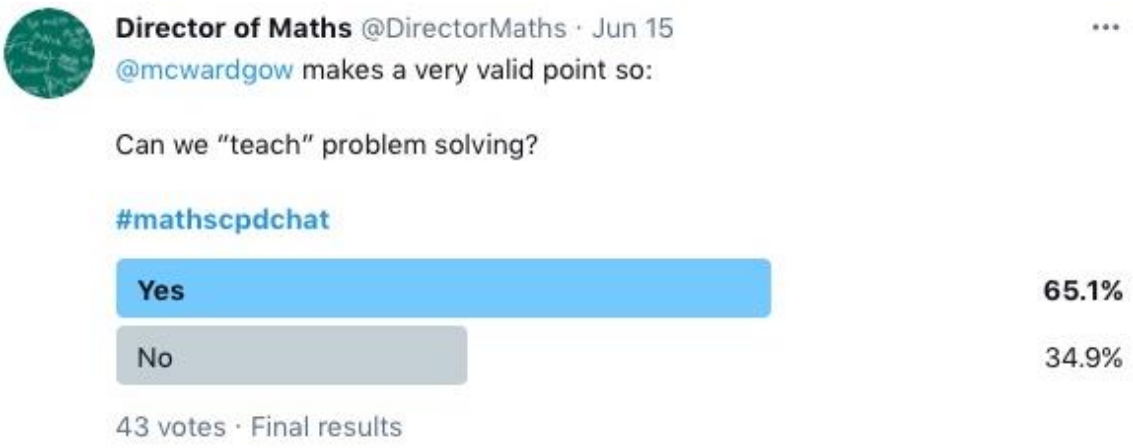
Same Surface, Different Depth (SSDD) Problems which is a website set up by Craig Barton where teachers will find special sets of problems that may look as if they are of the same kind at first glance, but which require different mathematical ideas to solve them. It was shared by Gemma Scott

Ofsted research review: mathematics was published on 25 May 2021. It was shared by Jenny Hill-Parker

Andrew Jeffrey Explains Why Not to Buy a Lottery Ticket which was a TV interview. It was shared by Andrew Jeffrey,

The screenshots below, of chains of tweets posted during the chat, show parts of two conversations: about whether it is possible to teach explicitly the skills pupils need in order to solve mathematical problems, and about factors that may be (removable?) obstacles in the way of pupils acquiring such skills. **Click on any of these screenshots-of-a-tweet to go to that actual tweet on Twitter.**

The conversations were generated by this tweet from Gemma Scott:



and included these from mrsouthernmaths, Alice Ward-Gow, J Dempsey and Peter Williams:

@mrsouthernmaths @mrsouthernmaths · Jun 15

Replying to @DirectorMaths and @mcwardgow

I think it is possible to equip students with a set of strategies eg: draw a diagram, add a line, solve a simpler problem. However, I think our students would be much better problem solvers if we used problem solving as a means of teaching rather than the goal. #mathscpdchat

Miss Ward-Gow @mcwardgow · Jun 15

What do you mean by "as a means of teaching" ? #mathscpdchat

J Dempsey @Dempseymaths · Jun 15

Replying to @mcwardgow @mrsouthernmaths and @DirectorMaths

@mcwardgow Have a look at this sciencedirect.com/science/articl...

Lesson Study: Problem Solving Approaches in Mathem...

Lesson study is a scientific activity for Japanese teachers on their meaning. In Japan, research topics f...

sciencedirect.com

Peter Williams @MathsImpact · Jun 15

Replying to @DirectorMaths and @mcwardgow

Can you teach someone to be an artist?

I'm not sure, but you can teach different brush techniques, and you can study other people's paintings.

Then just maybe when you hand someone a blank canvas they can create something...

these from mrsouthernmaths, Helen Scott, Gemma Scott and Mr HawesMaths:

@mrsouthernmaths @mrsouthernmaths · Jun 16

For me, problem solving is using skills and information you have to find something out with minimal guidance. So as often as possible, I try to give minimal guidance and instead ask the students to suggest strategies.

@mrsouthernmaths @mrsouthernmaths · Jun 16

This can be done by subtly varying the conditions of a problem, so that they have to use skills they already have in a slightly different way. This can be as simple as asking them to find a length given an area, once they are happy with how to find an area.

Helen Scott @HelenScott88 · Jun 15

Replying to @DirectorMaths and @mcwardgow

We can model approaches, thought processes etc. Not sure we can explicitly teach how to solve every problem. Perhaps we can teach "strategies" and develop thinking that help to solve problems... Maybe I should have said yes after all!
#mathscpdchat

Director of Maths @DirectorMaths · Jun 15

Hahaha it's a tough one to be definitive either way! I always say to my students that it's impossible for me to predict every scenario they might be asked about. I give them the tools but ultimately they have to use them #mathscpdchat

MrHawesMaths @HawesMaths · Jun 15

Replying to @DirectorMaths and @mcwardgow

It is. I like to think of it like beating a defender in sport. You need to develop the skill set that will enable you to choose many options to assist your decision making. Some will work@some won't. As you continue to develop, you will naturally work out what is required.

Director of Maths @DirectorMaths · Jun 15

A bit like being an early career teacher! How big a role do you think resilience plays in this? #mathscpdchat

MrHawesMaths @HawesMaths · Jun 15

Huge amounts. This culture of having to get it right first time and having to have information instantly needs to be addressed. Have sometimes gone the old fashioned way of getting students to look at a book to research strategies and apply. It's okay to fail. #mathscpdchat

these from Gemma Scott, Andrew Jeffrey, Catherine Edwards and Simon Ball:

Director of Maths @DirectorMaths · Jun 15

Wow the time is flying tonight! On to our final question!

Q6 Are there any other barriers to mathematical problem solving that aren't necessarily maths based? How can we overcome these?

Andrew Jeffrey @AJMagicMessage · Jun 15

Replying to @DirectorMaths

1/ Language language language
2/ Misunderstanding of what maths IS.

#MathsCPDchat

Catherine Edwards @Edwards08C · Jun 15

Literacy is a huge one for my classes. Also what I've heard referred to as the hinterland of knowledge. Not a problem solving example, but the number of times I've had a scales lesson derailed because I've forgotten to explain what a speedometer is! #mathscpdchat

Simon Ball @ballyzero · Jun 15

Replying to @Edwards08C and @DirectorMaths

That's what I was going to say: language and knowledge can be huge barriers. A former colleague told me she stopped a GCSE exam to explain to the pupils what an electricity meter was, because they all bought electricity meter cards locally. **#mathscpdchat**

and these from Mr J-W, Andrew Jeffrey, Gemma Scott, and Julia Smith:

Mr J-W @Trudgeteacher · Jun 15

Replying to @DirectorMaths

Low self esteem? I am no good at this? #mathscpdchat

Andrew Jeffrey @AJMagicMessage · Jun 15

Replying to @Trudgeteacher and @DirectorMaths

Yup, definitely that. I remember being told once, 'If you see dead wood, ask yourself what killed it?'

How on earth did we let kids come to see themselves as failures? It's a disgrace really. None of us sets out with that intention. #mathsCPDchat

Mr J-W @Trudgeteacher · Jun 15

Moving through the curriculum before pupils have succeeded in understanding. Its like shifting sand. #mathscpdchat

Director of Maths @DirectorMaths · Jun 15
And I guess when it comes to problem solving this could be a vicious cycle! How might we combat? #mathscpdchat

Mr J-W @Trudgeteacher · Jun 15
Keep problem solving within context of maths students have confidence in. Its a myth that problem solving always has to be around 'harder' topics. #mathscpdchat

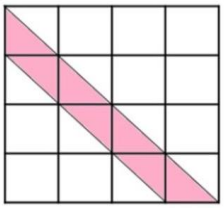
Tessmaths @tessmaths · Jun 15
Replying to @DirectorMaths
A6 words words words...making sense of them and also the context @PixiMaths has a poster 'if its tricky, draw a piccy' - helps students get into the problem to find a starting point #mathscpdchat

Director of Maths @DirectorMaths · Jun 15
A really important skill to have, translating into a diagram! #mathscpdchat

(to read the discussion sequence generated by any tweet look at the 'replies' to that tweet)

Other areas where discussion focused were:

the host asked what teachers believe to be characteristics of students who are good at solving mathematical problems, and posted some images, including the following two, asking what a student would need to be able to do in order to respond 'successfully' to each question:



What fraction of the shape is shaded?

- 1/4
- 7/16
- 7/32
- 25/32

What is the value of A?

- 2/5
- 2/9
- 1/4
- 3/10

- teachers commented that a student would need to be able 'to link fractions in the written and diagram form', and **find equivalent fractions easily in order to compare those with**

different denominators … they would understand the essential role of equality in parts of a fraction image;

- many people mentioned that **students who are good at solving maths problems are resilient**, knowing that 'temporary failure is a normal part of learning' … they enjoy sharing and discussing their difficulties, and do not adopt a competitive approach to learning maths in general or to problem solving in particular;
- several teachers commented that working on **'Goal free problems'** (link provided above) helps to build the confidence that good problem solvers need;
- teachers agreed that successful problem solvers 'are **willing to start routes of inquiry without being able to see the full set of steps to get them to the end goal**';
- many teachers commented that, in order to develop students' problem-solving abilities 'during the early days', it is important to **ensure that the problems that they encounter require only mathematical knowledge of which they have a sound grasp** … that a student's attention is more easily directed to the reasoning involved when facts that need to be recalled fluently are relatively 'trivial' (simple) … this image was tweeted as an example:

- there was a short discussion about developing **'a culture in the classroom whereby students are willing to try things out, discuss and learn from each other'** … that it is helpful for students to use individual whiteboards '(if it's wrong, just clear it), and encourage discussion of mistakes. Build on them and go off on tangents';
- other useful tasks that were mentioned included those in which students are presented with mathematical information about a situation, and challenged to extract and **represent as much further information as possible by drawing on their existing mathematical knowledge** … some examples were tweeted, including this one:

C

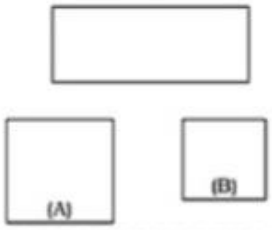
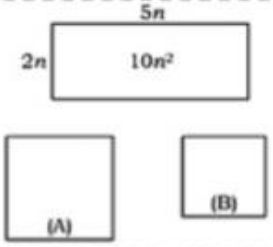
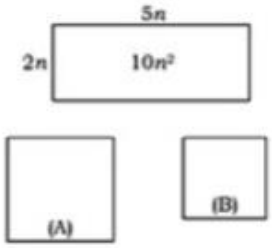
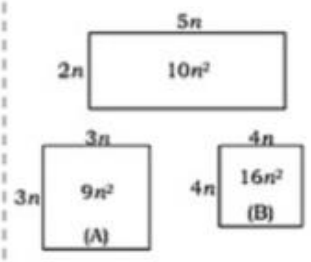
This information is about a rectangle and two squares.

The side-lengths of the rectangle are in the ratio 2 : 5

The area of square A is 90% of the area of the rectangle.

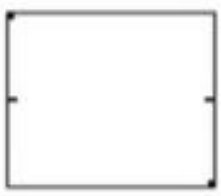
The side-length of square A is $\frac{3}{4}$ of the side-length of square B.

List C

Adding this mathematical knowledge / applying these mathematical procedures ...	... transforms this representation ...	... to this representation.
• because the ratio of the side-lengths of the rectangle is 2 : 5, if one side-length is $2n$, then the other is $5n$, for any value of n • the area of a rectangle is length × width		
• '90% of' is equivalent to '9 tenths of' • the side-length of a square is the square-root of its area • if $3n = \frac{3}{4}$ of x, then $x = 4/3$ of $3n$ • the area of a square is its side-length squared (side-length)²		

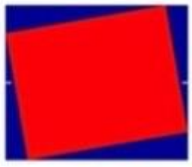
- a short discussion was generated by a teacher's comment that **students who can solve mathematical problems 'have the ability to think flexibly, make connections and spot patterns**. They know there are (often) many ways to solve a problem, and that the first way may not be the most efficient or 'best' … several teachers agreed that by engaging in 'quite small tasks, often repeated', students can learn to think flexibly, recalling and linking their existing knowledge … a contributor tweeted an example:

The teacher drew a rectangle, and marked the midpoints of one pair of opposite sides …

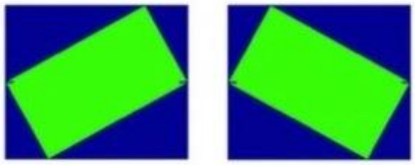


… and asked what shapes might be drawn inside it with all their corners on sides of the rectangle.

Student A: There could be smaller rectangles with all their corners on sides.

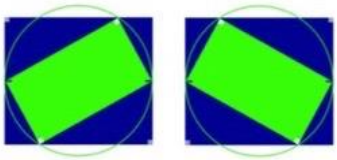


Student B: When two of the corners of a rectangle are on the marked midpoints there are only two possibilities.



Teacher: How did you know that there are only two possibilities?

Student B: The angle in a semi-circle is a right angle, and I pictured where the circle would cut the sides.



Student C: I can work out why these rectangles are reflections of each other!

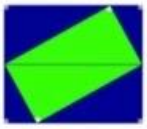
(Before saying this, what knowledge had student C recalled?)

Student D: If you split the diagram in two parts along a diagonal of the smaller rectangle you get two congruent trapeziums.

(On what knowledge did student D base this conclusion?)

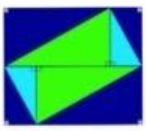
By calling-up from memory, and linking, more facts, students concluded that in each part the two smaller triangles are similar.

Student E: You could do the split along the other diagonal!



Later …

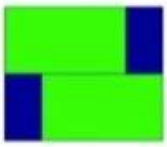
Student E: You can split both the top and bottom triangles into two smaller triangles using perpendiculars …



Student E: … the two smaller right-angled triangles that you make in each triangle are similar to each other, and they are identical to the corner triangles …

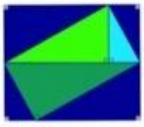
(What knowledge did student E use to conclude this?)

Student F: So there are similar rectangles too!



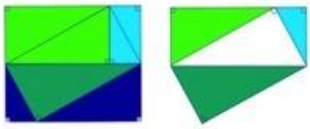
(What knowledge did student F use?)

Student G: The largest triangle is similar to the two smaller triangles …



(To reach this conclusion student G linked existing knowledge into a short chain of reasoning.)

Student G: … and so it's like Pythagoras' Theorem …



Student G: … one big triangle is on the hypotenuse of the other big triangle, and the other two triangles are on its other two sides. The three triangles are all similar! And the areas of the two smaller triangles add up to the area of the triangle on the hypotenuse!

- when asked **how teachers might 'build that 'safe' yet challenging culture in the classroom'**, a teacher responded with 'when questioning students verbally, ensuring that all students know that their contributions are valued whether correct or incorrect';
- teachers discussed the struggles students have when they are trying to solve mathematical problems … saying that **'struggle is an unavoidable part of problem solving**' … that 'struggle is what makes it interesting' and 'the trick is to have just enough struggle for challenge but not so much that it's dispiriting' … a teacher had written (for NRICH) 'about the struggles students have and how teachers need to plan for it' (link provided above) … this is how one teacher expressed a view:

the host asked whether teaching pupils to solve problems is different to teaching them to understand mathematical ideas and follow mathematical procedures:

- some teachers asked 'Is this not what the new Ofsted guidance says?' … these teachers tweeted about 'teaching from first principles' in order to build 'a toolbox from which students have to effectively choose and apply strategies for problem solving' … they mentioned that when they 'challenge the students to come up with as many facts as they can that pertain to a particular problem or diagram' they 'rebrand what students are doing as 'Anything Goes

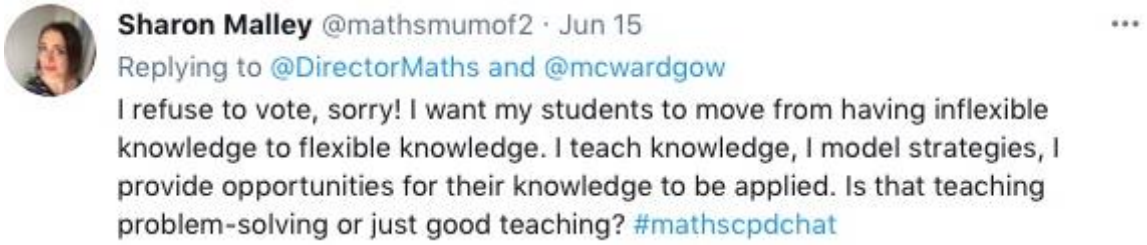
Maths', so as to **get away from the exasperation that some students feel when they're not being led step by step'**;

- other teachers reported that **what they do in 'problem solving lessons' 'isn't too far removed from what I would do in a skills lesson'** … to which one teacher responded by asking 'What on earth is the point of maths, if not problem-solving?';
- many teachers do not teach 'problem solving lessons', but instead **see problem solving 'as a natural part of the progression of a topic'**;

the host asked what teachers can do to help students become able to select appropriate 'methods' when faced with a mathematical problem:

- teachers mentioned that they intend to devise tasks in which students **compare worked examples** … 'showing two side-by-side ways of approaching the same problem and getting students to critique them' … other teachers present a problem and '**start by having a 'What might work?' discussion, collecting students' ideas, trying a few, and then asking 'What methods worked/ were more efficient?'**' … a teacher pointed out that 'Polya talks about the importance of reflecting on a problem afterwards to probe just that sort of issue';

the host tweeted a poll and asked if we can 'teach' problem solving … the consequent discussion is shown above in the sequence of screenshots of tweets, but there was also this significant tweet:

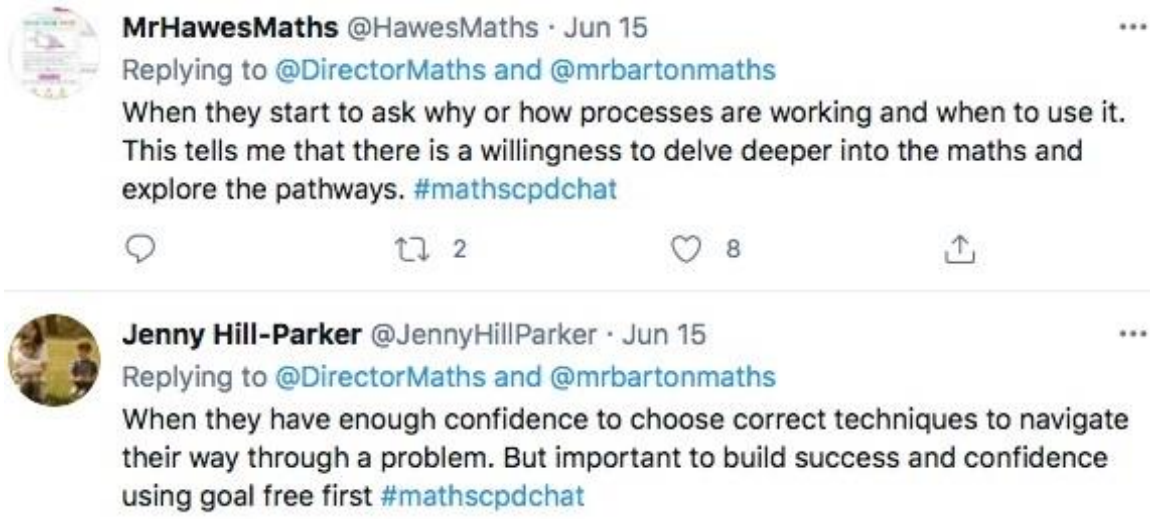


the host asked how, in order to help students develop problem-solving skills, they might/do use tasks of kinds that are described as 'Goal free problems', minimally different questions, and 'Same Surface, Different Depth' (SSDD) problems:

- a teacher pointed out that **'Different Surface, Same Depth' (DSSD) problems are 'useful in connection building so pupils can see how different problem types have the same underlying structure'** and are therefore 'possibly more useful than SSDD for teaching problem solving';
- a teacher commented that he had found that **after pupils had spent time working on 'Goal free problems' 'more pupils were willing to attempt more questions** whereas previously they'd left them blank';

the host asked teachers how they know that students are ready to start using their knowledge to solve problems:

- there were only two tweets in response to this question:



responses to the host's final question (Question 6) are show in the sequence above of screenshots of tweets.