NCETM
NATIONAL CENTRE FOR EXCELLENCE IN THE TEACHING OF MATHEMATICS

Core concept 2.1: Properties of number

This document is part of a set that forms the subject knowledge content audit for Key Stage 3 maths. The audit is based on the NCETM Secondary Professional Development materials and there is one document for each of the 17 core concepts. Each document contains audit questions with check boxes you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications and explanations, and further support links. At the end of each document there is space to type reflections, targets and notes. The document can then be saved for your records.

2.1.1 Arithmetic procedures

How confident are you that you understand standard written methods for the addition and subtraction with integers and decimals?

1 ☐ 2 ☐ 3 ☐ 4 ☐

How confident are you that you can explain how to add and subtract negative numbers?

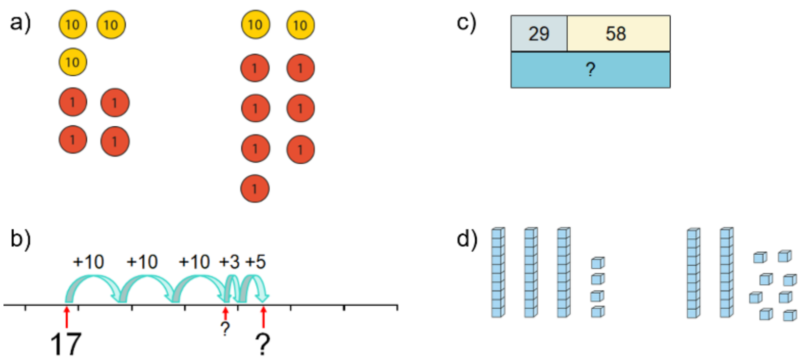
1 ☐ 2 ☐ 3 ☐ 4 ☐

The focus in Key Stage 3 is on deeply understanding the structures underpinning the standard columnar format and generalising fully to decimals.

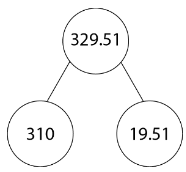
A key idea is that of 'unitising' – adding quantities of the same 'unit'. For example, the standard columnar method with decimal numbers exploits the idea that hundreds can be added to hundreds, tens to tens, ones to ones, tenths to tenths, hundredths to hundredths, etc., and this gives meaning to why decimals need to be aligned as they do in the standard method.

Addition and subtraction calculations can be represented using place value counters, number lines, Dienes, Gattengo charts and bar models.

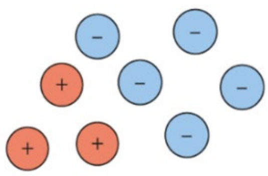
What calculations do these represent?



Partitioning is important for learning to understand the structures underpinning standard algorithms for addition and subtraction. For example, 329.51 can be written as 300 + 20 + 9 + 0.5 + 0.01. It can also be written as 320 + 9 + 0.5 + 0.01 or illustrated as a part–part–whole model (cherry diagram).



For addition and subtraction of negative numbers, modelling using partitioning with 'zero pairs' (where a positive and negative counter combine to give a value of zero) can support students' understanding of calculation.What number is shown here?



© Crown Copyright 2021

Further support links

- NCETM Secondary Professional Development materials: 2.1 Arithmetic procedures, pages 14–23
- NCETM: Using mathematical representations at KS3: Dienes (and place value counters), The Gattegno chart
- NRICH: Adding and Subtracting Positive and Negative Numbers (article): https://nrich.maths.org/5947

2.1.2 Understand and use the structures that underpin multiplication and division strategies

How confident are you that you understand written multiplication and division methods with integers and decimals?

1 ☐ 2 ☐ 3 ☐ 4 ☐

How confident are you that you can explain how to factorise with powers of 10 to simplify multiplications with both integers and decimals?

1 ☐ 2 ☐ 3 ☐ 4 ☐

A key feature of the standard algorithm for the multiplication of integers is that it involves sequences of multiplications of single-digit numbers. Place-value considerations and the lining up of columns ensure that the product is of the correct order of magnitude.

When using the method with decimals, it is important that the underlying mathematical structure is thoroughly understood, e.g. $300 \times 7\,000$ can be considered as $3 \times 100 \times 7 \times 1\,000 = 3 \times 7 \times 100 \times 1\,000$. This awareness supports informal calculation methods and underpins the columnar methods. When multiplying decimals, it is important to understand, for example, that $0.3 \times 0.007 = 3 \times 7 \times 0.1 \times 0.001$ and, therefore, how 3×7 and 0.3×0.007 are connected.

When dividing one decimal by another it is important to understand how multiplying and dividing the dividend and the divisor by 10, 100, etc., changes the quotient. For example:

- $74 \div 3 = 7.4 \div 0.3 = 0.74 \div 0.03$
- $7.4 \div 3$ is ten times smaller than $74 \div 3$
- $74 \div 0.3$ is ten times bigger than $74 \div 3$
- $74 \div 0.003$ is one thousand times bigger than $74 \div 3$

These various awarenesses come together to give meaning to the idea that a calculation such as 3.14×5.6 can be calculated as $(314 \times 56) \div 1\,000$ and that $25.7 \div 0.32$ can be calculated as $2\,570 \div 32$.

Multiplication and division involving negative integers is also introduced in this set of key ideas. It is important to explore why the rules for combining positive and negative numbers work and to avoid rote learning of the rules without meaning. For example, the structure $-a \times 0 = -a \times (+b + -b)$ together with the application of the distributive law can be used to give meaning to the fact that the product of two negative numbers is a positive number.

We can use powers of ten to simplify problems, for example, if $5 \times 8 = 40$, which of the following is the solution for 0.05×80? Explain your reasoning.

a) 0.04
b) 0.4
c) 4
d) 40

Answer: c) 4.

Reasoning: If $5 \times 8 = 40$, then $0.5 \times 8 = 4$ and $0.05 \times 8 = 0.4$. Therefore $0.05 \times 80 = 4$.

Further support links

- NCETM Secondary Professional Development materials: 2.1 Arithmetic procedures, pages 24–28
- NCETM Secondary Professional Development materials: 2.1 Arithmetic procedures, pages 29–32

© Crown Copyright 2021

2.1.3 Know, understand and use fluently a range of calculation strategies for addition and subtraction of fractions

How confident are you that you understand and can explain how to add and subtract fractions, including mixed numbers?

1 ☐ 2 ☐ 3 ☐ 4 ☐

The focus in this set of key ideas is to use addition and subtraction of fractions to further expand the range of possible examples that students explore as their understanding of additive structures grows and matures.

Here, unitising is again a key idea and one that is particularly evident when working with fractions. For example, adding halves and thirds is not using the same 'unit'; however, converting both to sixths means that both have the same unit and the addition is relatively straightforward.

Students should develop an understanding of the additive structures underpinning the operations, as well as fluency with strategies for adding and subtracting a wide range of types of fractions (including improper fractions).

When adding mixed numbers together it is usual to deal with the whole numbers and the fraction part of the numbers separately. Just as we know 5 + 3 + 6 is the same as 3 + 6 + 5, when we are doing addition questions we can use the **Commutative Law** to help with this.

For example: $2 + \frac{2}{10} + 3 + \frac{5}{10} = 2 + 5 + \frac{2}{10} + \frac{5}{10}$

2.1.4 Know, understand and use fluently a range of calculation strategies for multiplication and division of fractions

How confident are you that you understand and can explain how to multiply and divide with fractions?

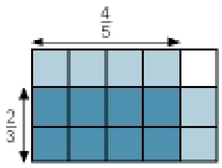
1 ☐ 2 ☐ 3 ☐ 4 ☐

There is a danger that students see the mathematics curriculum as a set of separate topics, each with its own set of rules and techniques. This unconnected view of the curriculum can result in an entirely instrumental and procedural approach to mathematics, with no sense of conceptual coherence. It is, therefore, important to see fractions or rational numbers as a part of a unified number system and that the operations on such numbers are related and connected to previously taught and learnt concepts for integers. For instance, the area model used for multiplication with integers can also be used for fractions.

Multiplication of fractions

This diagram shows that $\frac{2}{3}$ of $\frac{4}{5}$ is $\frac{8}{15}$

We can work this out as: $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$



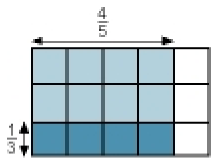
Mixed numbers can be multiplied by first changing them to improper fractions, and cancelling or simplifying, where appropriate.

Division of fractions

Division with fractions is best first considered through simple cases and the meaning of division:

$2 \div \frac{1}{4}$ means 'how many quarters are there in 2?'

Four-fifths of this rectangle is shaded light blue. The **light blue** area is divided into three equal parts by the horizontal lines.



So, four-fifths divided into three equal parts, or $\frac{4}{5} \div 3$, is the same as finding one-third of four fifths, or $\frac{4}{5} \times \frac{1}{3}$. This expression equals $\frac{4}{15}$.

© Crown Copyright 2021

Division of fractions can also be considered as multiplication by the reciprocal, and this gives meaning to some of the rules learnt by rote for these sorts of calculations. For example:

$4 \div 2$ is equivalent to $4 \times \frac{1}{2} = 2$.

$\frac{2}{3} \div 3$ is equivalent to $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

So:

$\frac{2}{3} \div \frac{3}{4}$ is equivalent to $\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

Further support links

- NCETM Secondary Professional Development materials: 2.1 Arithmetic procedures, pages 33–43

2.1.5 Use the laws and conventions of arithmetic to calculate efficiently

How confident are you that you understand the commutative associative and distributive laws and can use them to solve problems?

1 ☐ 2 ☐ 3 ☐ 4 ☐

How confident are you that you can explain how to calculate using the priority of operations?

1 ☐ 2 ☐ 3 ☐ 4 ☐

Students should both know and notice examples of the commutative [$ab = ba$, $a + b = b + a$], associative [$abc = (ab)c = a(bc)$; $a + b + c = (a + b) + c = a + (b + c)$] and distributive laws [$a(b + c) = ab + ac$] and need to be able to calculate fluently with the full range of different types of numbers in a wide range of contexts and problem-solving situations, exploiting these laws to increase the efficiency of calculation.

Calculating using priority of operations

When there are no brackets in an expression involving a combination of operations, do multiplication or division before addition or subtraction, for example:

$4 + 3 \times 7 = 4 + 21 = 25$

$4 + 12 \div 3 = 4 + 4 = 8$.

When there are brackets in an expression, do the operation inside the brackets first, for example:

$(4 + 3) \times 7 = 7 \times 7 = 49$.

When there are exponents in an expression, evaluate the exponents first, for example:

$3 + 2 \times 4^2 = 3 + 2 \times 16 = 3 + 32 = 35$

Further support links

- NCETM Secondary Professional Development materials: 2.1 Arithmetic procedures, pages 44–46

Notes

© Crown Copyright 2021