The NCETM Podcast Episode 60

Using number lines in secondary maths

Hello and welcome to the NCETM podcast. My name's Steve McCormack [SM] from the NCETM's Communications Team, and we're recording this in the Easter holidays, actually just before the summer term 2022 gets going, and with me I have my colleague Becky Donaldson [BD] from the NCETM's Secondary Team. Becky's been with us at the NCETM for just over a year, before which she taught maths in secondary schools. Hi, Becky: how long did you teach maths in secondary schools before you came to us at the NCETM?

BD: So I started teaching in 2010, just over a decade, and my most recent role was in a MAT across primary and secondary, which gave me a bit more insight into the journey before secondary as well.

SM: OK. Well, that's interesting because today we're going to be talking about number lines, and specifically how they can be used in secondary-school classes. But you may remember if you were listening to this a few weeks ago, I did something very similar with another colleague of mine at the NCETM, Debbie Morgan, about how number lines can be used in primary school maths lessons. So this really is a continuation of that theme, and is it fair to say, Becky, that possibly in some secondary school maths departments number lines are underused or their power is probably not quite appreciated?

BD: Yeah, I think so. It's quite common to see a number line above the whiteboard. I'm not sure how often it's always referred to in lessons, and I've deliberately not chosen negative numbers as an example here, because I think probably teachers are relatively familiar with using them to explore negative numbers, but there are other uses of this representation that I think perhaps are underused and could really support students if they're explored a bit more.

SM: OK, so like the primary version of this podcast chat, we're going to look at five examples of how number lines might be used in secondary school maths lessons. Just five examples varying in complexity, varying in the sorts of classes you might use them in, and hopefully there will be one of these somewhere where you think, yeah, I could use that. Even if you're already using number lines a fair bit, maybe there's going to be something in the next 10 or 15 minutes which will give you another idea to use a number line with one of your maths classes. Now this is a podcast so you can't see what we're doing, but nevertheless, if it's safe, you might want to pause now and go and get a pen and paper and just draw them as we start to describe them. Or if it's not safe, use your imagination, which is what math teachers try to do all the time with students in their class using their imagination, and additionally the five number lines we're going to talk about will be on the page for this podcast on the NCETM website, so you can go and actually see a representation there, print it off, and maybe that will help you to get more out of our discussion if you listen to it a second time. So let's get started, Becky, with the first number line that we're going to talk about: can you describe it to me please?

BD: Sure. It's a number line, I would say, starting at 0, but it doesn't really start at 0 because it's an arrow to the left of the 0 pointing towards the negative numbers to show that this number line continues forever, and it's got ten marks on it. So we're going from 0 to a mystery number, not 10. A box on the 10th line, and again an arrow after our box to show that the number line continues infinitely on from there, so 0 to something with 10 marks.

SM: Let's just deal with the arrows point generically, it's important that once we get into secondary education that - well, why is it important? You tell me that every number line has an arrow left and right.

BD: I think it's important that children get this sense of the number system being infinite in both directions, and I'm sure there's plenty of secondary teachers listening who are very particular about graphs, for example in the axes having arrows on to indicate that. So when we're working with number lines at secondary, often we're zooming in on a part of it. And so it's important that you understand this is zooming in. And there is stuff that goes on beyond the bit that we're focusing on.

SM: So the first number line we had in the primary discussion had exactly this sort of line, which we've got in our heads or on our sheets right at the moment. But the number in the box was 10, but now you've left the number in the box blank, and you're saying a teacher could put any number in that box for any purpose. So what would be the first one you might think about putting in that?

BD: Obviously any number could go there and it would tell you how students can work with number and separate it and divide it. But I think in this for this particular intention, I've put the number one in first to explore students' understanding of the decimal number system and whether they understand tenths building up to 1. So you'd want to see them putting 0.1, 0.2 or 1 tenth, 2 tenths written as fractions all the way up to 0.99 tenths. And then one.

SM: Let's say that goes smoothly. How might this be developed?

BD: I think children are generally quite comfortable with tenths. Obviously there'll always be some who come less confident, but actually children first experienced tenths on the number line like this back in Year 3, which is something that surprised me quite a lot when I started working in primary, how early that comes in. So once you're sure that they understand tenths, you'd want to explore whether they understand the infinity of the number system, in a zooming-in sense. Do they understand that we can zoom in between 0 and 0 .1 for example? And then again, between 0 and 0.01? So you're looking to see where they understand hundredths, thousandths, whether they're understanding that place value can be applied to this representation of the number line, so the next number I'd put in would be 0.1 to see if they understand that we'd have 0.01, 0.02 and so on, up to that 0. 1.

SM: That that could go on forever, couldn't it? The number of noughts before the digit 1 in these decimals. But what about a way of just shaking them a little bit and getting rid of the digit 1 in that box, putting a different digit in that box?

BD: You could continue exploring it and seeing if they go into thousandths and 10 thousandths. But I think the number 0.5 would be an interesting one to put there next. So playing around a little bit, do they understand the use of tenths and hundredths simultaneously? Do they understand if I'm putting 0.5 there that I've got 5/10 but the tenths are every other one of those markings, and in between them would be 5 hundredths, 15 hundredths and so on? So I think that would be a really useful assessment point, because where students are seeing the 1, the 0.1 the 0.01 all the way through, they are continuing a pattern which is the pattern that Debbie described when she talked about putting the teen numbers over. You know it's that same relationship, but if you put 0.5 there, you can really probe and explore whether they have fully understood how it is connected.

SM: OK, let's move on to the second example now. And if you see the sheet on the website, there's actually three versions of the second example, but the simplest one, the blank one, so to speak, is a segment of a number line, and we've got a box without anything in on the left, a box without anything in on the right and the midpoint mark in the middle. So where's this one going, Becky?

BD: This is very much a continuation of the understanding for the first number line of that decimal number system, but specifically looking at rounding. And this is a really useful representation to explore rounding. The rule that children often come to secondary school knowing that above 5 you round up, below 5you round down and 5 itself rounds up, perhaps without being able to place this to any number in context. So the idea here is that you can zoom in on a bit of the line and explore two numbers, their midpoint and the decimals either side of that midpoint. As to whether it rounds up or down, the example that I put there is between 3 and 4. Do they understand that 3.5 is the midpoint of those two numbers and then you could start just suggesting decimals? How about 3.2? Where would that be on that number line? Would that round to 3 or to 4? How about 3.7? You could introduce hundredths there as well. What about 3.92? So exploring whether they can place those decimals on that number line and then connect it to the rules of rounding that they might already have learned.

SM: You mentioned the word ‘midpoint’, and that's a key principle and a key piece of understanding which is useful to children right from Year 3/4/5 right up to and through secondary school and beyond, isn't it?

BD: Absolutely. When I listened to Debbie's podcast talking about Year 1 and 2, I was really struck by how those skills are still so fundamental in Key Stage 3 and Key Stage 4 - and Key Stage 2 as well. So that sense of the midpoint is absolutely crucial to this understanding of it. And it's the same skill, really, that you're asking them to apply to it. Just that now we have this infinite number system, this sense that between any of our numbers there will be more numbers that we're applying to it. Not just to the integers that have worked with early primary.

SM: And what about the question which some students would pose to a teacher about rounding? ‘OK, miss, what about if something is exactly in the middle?’ So for example three and four and right in the middle 3.5 most students know that that is exactly in the middle. It's not any nearer 4 than it is any nearer 3. ‘So why miss? Why do we always round that up?’

BD: I always just tell my children that it's the convention and it's the convention that we use. Obviously later on you can get into more detail when you're looking at bounds, but I think it's important they understand that we've made decisions, as mathematicians, to do things certain ways, and, yes, that is exactly in the middle, but we treat it as rounding up.

SM: OK, let's move on to the third example, which leads on nicely from the second. There's a bit of progression here from our first two examples and for the third example I've got a line in front of me here with arrows at the end, and there's three boxes without numbers in at the moment, one at the left-hand extremity, one at the right-hand extremity, and one right at the midpoint in the middle. But also we've got the midpoints between each of the boxes there so it's aligned with three boxes. Nothing in the boxes at the moment, midpoint between Box 1 and Box 2 marked, midpoint between Box 2 and Box 3 also marked, and we're going to put some numbers in these boxes to investigate - what are we going to investigate? Bounds. So how might this lead us on to bounds, Becky?

BD: So I was really struck when I was thinking about this, about how we probably explore rounding and decimals early in secondary and late on in primary and then in Year 10/11, we explore these bounds, and possibly there's not been much continuity between them of this representation, but actually exactly the same representation that we've used for rounding is really useful to represent. bounds. So the question that I was thinking of here was a really simple one. A tank has a capacity of 80,000 litres to the nearest thousand litres. What is the maximum and minimum capacity of the tank? So 80,000 goes in the middle - that is our capacity - and you're asking children to think about what is the previous thousand and what is the next thousand. So on the left-hand end they would hopefully write 79,000 and at the right-hand end they'd write 81,000, and then we've got the midpoints between each of those thousands to consider. So again, that same skill that they've been doing with their rounding, and back in Key Stage 1 with their number lines, asking children to identify 79,500 and 80,500. And if you can connect that back to their work on rounding, they will understand that any number to the right of that 79,500 would round up to 80,000 and any number to the left of the 80,500 would round down to 80,000. So they're exploring and experiencing bounds in a really visual way.

SM: And that's the point, isn't it? Because I think most secondary teachers listening to our conversation would acknowledge that bounds often is found tricky by Year 10 and Year 11 students who otherwise are finding a lot of the curriculum reasonably comfortable, but bounds sometimes trip kids up, doesn't it? But this visual way of helping them understand it is perhaps going to help. Perhaps teachers don't think about the visual aid when doing bounds, but perhaps using the visual aid will help lots of students get over hurdles in their heads.

BD: Yeah, I think so. I think some of the things really challenging with bounds is when you're then asked to calculate and select whether the upper or the lower bound will create a certain answer. And I think having spent a bit more time exploring bounds at the beginning might give students a little more foundation to be able to do that. I also think there's quite a big gap between the last time they've explored rounding in great depth. It's very often done at the beginning of Key Stage 3 because it is also in Key Stage 2 curriculum. And then there's this massive gap until you explore bounds in Key Stage 4 and yes, they will have been rounding as part of their mathematics all the way through, but have they stopped to consider it again with the representation? To really have got to the depths of their understanding there to be able to apply it to the bounds. I think an interesting next step might be to take exactly the same numbers, but change what the right has been rounded to. So I've sketched next to that what it would look like if it wasn't to the nearest thousand and look to the nearest 100. And I think that's a really useful bit of variation that you could do there if you're going to spend a bit more time exploring bounds before you go on to calculate with them. Maybe keep the numbers the same but change the limits of accuracy so that they can really get to grips with what the 100 or 1,000 or whatever the nearest accuracy is before and after, and then explore those midpoints again.

SM: The final two examples, four and five, that we're going to talk about are not connected in any way. They're separate, but nevertheless equally useful. So I'm looking at number four now and it's a very simple number line with 0 on the left, a box with nothing in on the right, arrows left and right as always. And underneath you've written where is a quarter. So what would you be trying to do with the class there?

BD: So when we deal with fractions, they can trip students up. I think a lot of teachers will put their hands up and say that that fractions are not always the easiest thing to teach. And I think part of that problem is that they are both proportions and they are values in their own right. So this is about exploring them as values in their own right - ¼ as a position in the infinite number system. And it's a really simple concept. The idea is that you just vary what number you put in the right-hand box and ask students to estimate, and I use estimate really deliberately there. It's not about precision, it's about their number sense. Can they estimate where ¼ is? So if you start with one, can they approximately work out where ¼ is? How about if you change that to 2? Do they move their quarter so it's halfway between where their old quarter was and 0? How about 5 or 10? A really interesting bit of variation is to go into the fraction itself. Do they know where ¼ is in relation to ½ in relation to $^1/_3$. How about $^1/_5$? Do they realise that ¼ will go the other side of that that missing box? So it's really about exploring their number sense, and I think number sense is something that is perhaps less explored with fractions but pays dividends for when you are calculating with fractions as you go through secondary if they can make sense of and reason whether the answers are about the right size, because they have that sense of what size a fraction is.

SM: Would you, as a teacher, put an eighth in that box as sort of a cheaty question?

BD: I can see like if I was in the classroom currently, I could see myself using this as a sort of a relatively regular starter, just sticking it up on the board and exploring. And yeah, if they'd seen this a couple of times, I'd definitely put an $^1/_8$ there and expect them to extend the line and put a quarter exactly the same distance on again. I'd also expect them to start to try and trick me a little bit - can you put a number there? Challenge me to put to a quarter in. And there's lots of things you could do with this. I think there's a sense of it building over time. It's not something I do as a one-off, it's something that I'd come back to, I think, to build their number sense, not just assess it.

SM: And that would just start to instil familiarity in students' minds – 'Oh yes, I've seen that before. I remember what miss did last time and that's familiar to me now. I'm now going to revisit my understanding there and just have another bit of a play with the same representation'.

BD: Absolutely. That's the reason there is a journey. But for the first three number lines, I think familiarity and repetition are really important. If you're going to use this representation, it's not a one-off: it's something that students are used to seeing.

SM: Final number line. We're going to talk about is a double number line, and we must make the point here that we're just going to dip our toe in the water of the territory that is double number lines here and give you one example. A few thoughts, but we may at some stage do a whole podcast about double number lines. Can you describe the one you've chosen to us please, Becky? What's this looking like?

BD: So I've chosen a relationship that will be familiar to teachers, of kilometres and miles. So crucially, you've got a single number line with numbers above and below, and crucially, you've got zero lined up. So 0 kilometres on the top and 0 miles on the bottom, and then along the number line you come to a marking and there's eight on the top, where the kilometres are, and five underneath it, where the miles are. So eight and five are related and you continue on not quite so far. And there's 12 on the kilometres side and a missing box on the miles side. And obviously an arrow to show that it goes on forever as well.

SM: Obviously the first question a teacher might ask is, what number's going to be in the missing box underneath?

BD: I'm not sure that would be the first question actually.

SM: OK.

BD: I probably would ask, what do you know if you know eight matches to five, what else do you know? And you might not choose this relationship if it's your first experience with the class because you want them to be able to just spot other relationships. So they might say, well, I know that four is going to be equal to 2.5 before they think about what 12 is equal to, or they might double the numbers they've been given and say I know that 16 is going to be equal to 10. From those numbers, whichever direction they've gone in, that's when they might be able to answer the question of what is in the box. I think there's something about slowing children down and not pushing them straight to the answer, but asking them to explore other things they know. And I think that they're probably the two most likely approaches with the particular relationship that I've chosen. With other relationships children might do other things, and they might find a quarter of one of the numbers or work out what the value of 1 is. The key point with the number lines is that it's really giving students ownership of exploring that relationship, rather than giving them a method like a unitising method and say you've got to find out what one is worth so you can find out anything else. They can use that visualisation as a relationship to find the most natural pairs for them, really.

SM: I'm thinking about days gone by when I taught maths, over a decade ago, and when I learned maths (goodness knows how long ago) and I'm thinking of similar questions about the relationship between miles and kilometres. Would it be right to say that teachers might have demonstrated, students might have actually done those, without the visual representation, just playing around with the numbers in their head, so to speak, without this representation?

BD: Yeah, absolutely. And I chose this one as it particularly used to bother me at school myself. That's really quite tricky. I think you can end up learning lots of processes for essentially the same multiplicative relationship. Now I've just sketched down here that this could be mph or metres per second, rather than kilometres to miles. It could be a conversion. It could be euros and pounds. It could be the cost of chocolate bars. So eight chocolate bars cost £5. Any relationship where there's something per something or something for every something could be represented on a double number line, and certainly rather than memorising lots of different methods or processes. You've got one visualisation of a relationship and from that you can explore and use your own reasoning to work out what the solution will be. And I think it simplifies things for students that have a representation of visualisation rather than lots of processes to learn.

SM: OK, great. Well, thanks very much indeed Becky. And there are five examples there. Thank you for planting these thoughts in my mind and in the minds of our listeners. Thank you for listening. I hope there's been something for you out there that you might use sometime during the next few weeks of your maths lessons. So that's the end of this edition of the NCETM podcast. If you've liked it, please let other friends know, recommend it, subscribe to the NCETM podcast so that you get automatically notified when the next one comes along.