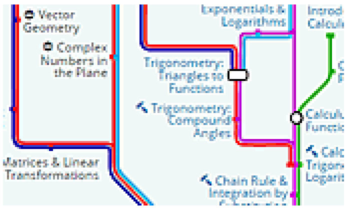
Secondary Magazine 137



National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 137 of the Secondary and FE Magazine

Welcome to the first Secondary Magazine of the new school year. In this issue, we begin to address those new GCSE topics with suggestions and ideas for the classroom (a feature we intend to continue in future editions), we offer some thoughts about what your new Y7 class might have experienced in the primary school maths classroom, and we tell you about a new, free A level resource package. We also review the summer exam results – offering a maths focus to those headline figures – and we take a look at recently published research into the effects on maths attainment, of teachers sending texts to parents.

Contents

nth terms (new on the Higher GCSE) may like to use the patterns or to 'read ahead' to understand where the topic goes beyond GCSE.

Qualifications and Curriculum
What did we learn from this summer's results? Here is an analysis of the maths specific results. How do they compare to yours? Does any of this surprise you? See what others thought by trawling through last week's Twitter chat. Add your thoughts to the comment box at the end.

Heads Up

There's an opportunity for secondary schools to participate in a free series of workshops to help maths departments improve the teaching of mathematical reasoning at Key Stage 3. Each Maths Hub is looking for between five and eight schools for this school year. More details here.

Want to know more about the strengths and weaknesses of your new Y7s? You can download a question level analysis of the KS2 SATs data for your own classes. If you missed it, you might also like to read our general advice on KS2 assessments - what secondary teachers need to know from July's Secondary Magazine.

Ofsted has updated its very succinct myth busting advice to schools (including specific advice on marking (point 5)), which prompted this tweet from Sean Harford, Ofsted National Director, Education:

> @HarfordSean Sep 6 At the start of this year, please all remember: no preferred style of teaching, marking, planning, data cutting/use #Ofstedmyths #HelpSean

Following a trial in March, Ofqual has confirmed that the first National Reference Test (NRT) will be held in February and March 2017. Each year a sample of students will take the same tests (in English and maths) so it will show, over time, if there is any change in how students perform at a national level. Results from the NRT will only be used to measure changes in performance nationally and these will be published. There will be no results for individual students or schools. An overview of the process is shown in this diagram. More details from gov.uk.

Grants of up to £500 towards delivering a maths activity listed in the online STEM Directories are being offered by the Royal Institution, funded by The Clothworkers' Foundation. The scheme is designed to help integrate maths enrichment and enhancement activities into school practice and to support teachers' professional development. The application deadline is **30 October**. For more information and to apply, visit the STEM Directories website, or contact stemdirectories@ri.ac.uk.

The link between music and numbers was explored by the well-known populariser of mathematics, Professor Marcus du Sautoy, on BBC Radio Four at the beginning of September. Listen to the programme on the BBC iPlayer.

Mathematics in Education and Industry (MEI) has added a new page to their website to support teachers in integrating technology into their post-16 schemes of work. Also available from MEI are the mobile apps Sumaze and Sumaze 2, which also have a desktop version.

The new term has seen the resumption of our weekly Twitter-based chats, which take place on Tuesday evenings 7-8pm, using the hashtag **#mathscpdchat**. You can find more information, including future discussion topics, and details of how to take part on the mathscpdchat webpage.

Forthcoming events: a selection of maths events coming up you might be interested in:

- 10 October: Cambridge Maths Hub Mastery Conference
- 17 October: GLOWMaths Hub Maths, Mindset and Mastery Conference.

Email us if you have an event that you think we should include here.

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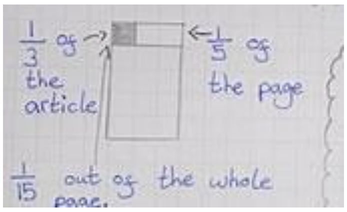


Classroom View
What Do You *Really* Understand?

By *Dan Polak, a primary school teacher from Devon, offering thoughts to secondary colleagues*

Some children are easier to shift towards having a growth mindset than others. We've found that our highest achievers can be the hardest to convince, because they feel they have the least to gain and the most to lose. When children are used to quick success, the philosophy behind Carol Dweck's work on mindset can feel discordant to them.

We know that a flexible mathematician is likely to be more efficient. If learners know different routes to the answer, they can choose their methods as calculations change. They can also demonstrate a real and concrete understanding of the mathematics involved. When children feel successful without deepening their understanding, however, in my school, we have had to take different measures to ensure they value deep understanding and mastery of skills.

One method is the use of elicitation and application to bookend teaching sequences. Our elicitation task is presented at the beginning of a sequence of about three weeks of learning, we then plan based on the children's performance at this. At the end of a sequence they do the same task again so we have a clear indicator of progress. Last year, when we started to develop how this was going to look, I was struck by a Year 5 child who was below ARE (age-related expectations). One question which stumped her on the elicitation task was:

$$\frac{1}{3} \times \frac{1}{5}$$

She wrote in her book she couldn't solve it, but knew the answer was one fifteenth. It is interesting to say that you cannot solve a calculation when you know the answer, but our school grading system for these tasks reserves the top mark, 'E for exceeded', for answers which explain the answer. Pupils get the lower 'A for achieved' if they answer everything correctly without explanation.

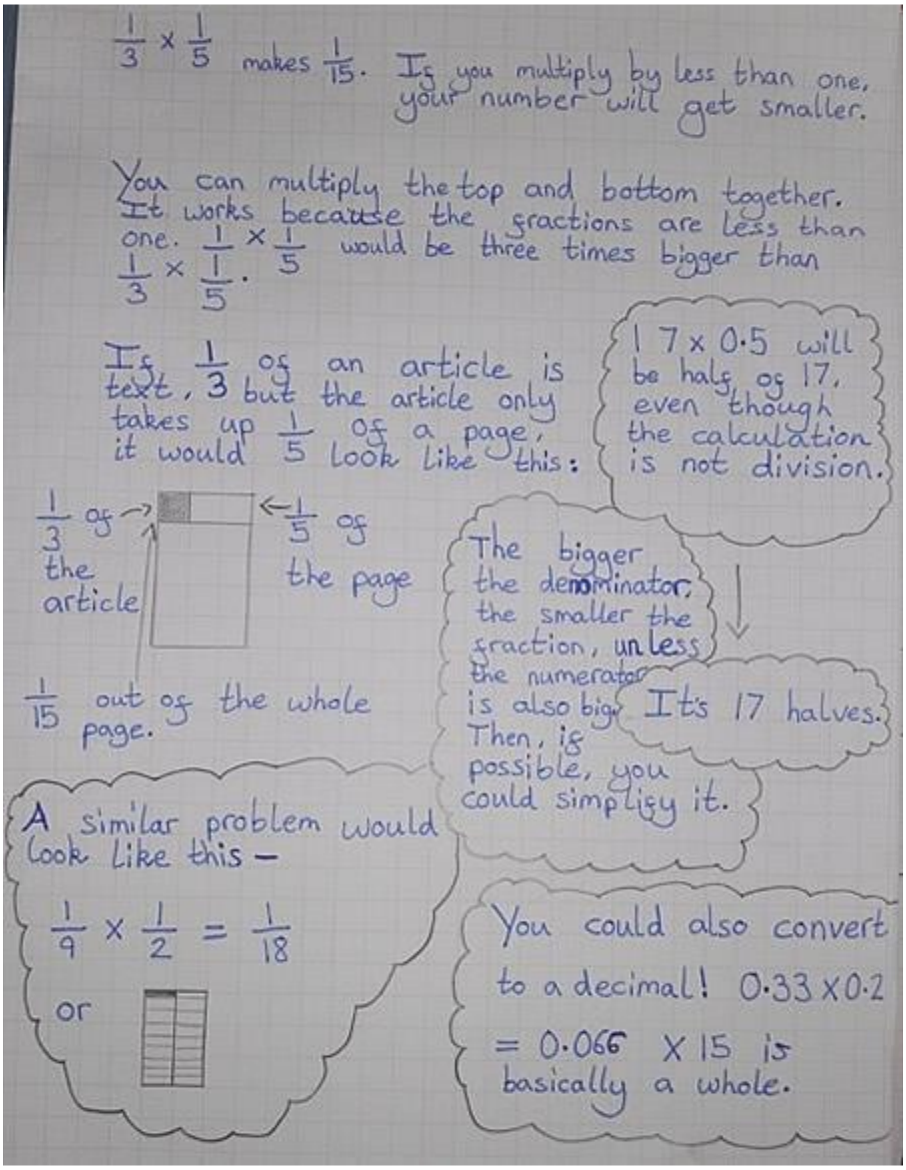
So this child wrote to me to explain what she didn't understand about her answer; this is the text from her elicitation:

> "I know to multiply the top and the bottom, but I don't know why. I know that you have a smaller result at the end but I thought multiplication makes things bigger, so this is confusing. I wouldn't know where to start if you wanted a picture of this - when might this happen in real life? I could make another question, $\frac{1}{4} \times \frac{1}{7}$ and know the answer is $\frac{1}{28}$, but this doesn't tell me why I got the first one right. I can't tell you why this works!"

Our highest achieving mathematicians reacted with a bit of shock when I spoke to the class about how brilliant I thought this response was. They started to understand that the answer was only part of the understanding. We could give this child an 'A for achieved' mark but she needed to understand much more to be considered a master of this skill of multiplying fractions. They started to give different options on how to solve this and represent the calculation in different ways. We had previously worked on fractions with magazine articles, so one child suggested that we used the 'magazine articles' context to show the fraction on a page. We came up with the question: "If an article is $\frac{1}{3}$ image but takes up $\frac{1}{5}$ of a page, how much of the page is image?" This served to visualise the fractions' multiplication.

Our children have been so immersed in the principles of mastery that by the time they reach Year 7, they might not just give an answer to a problem, but will explain the full extent of their understanding. This deeper understanding will improve their fluency and ability to apply mathematics in unfamiliar contexts.

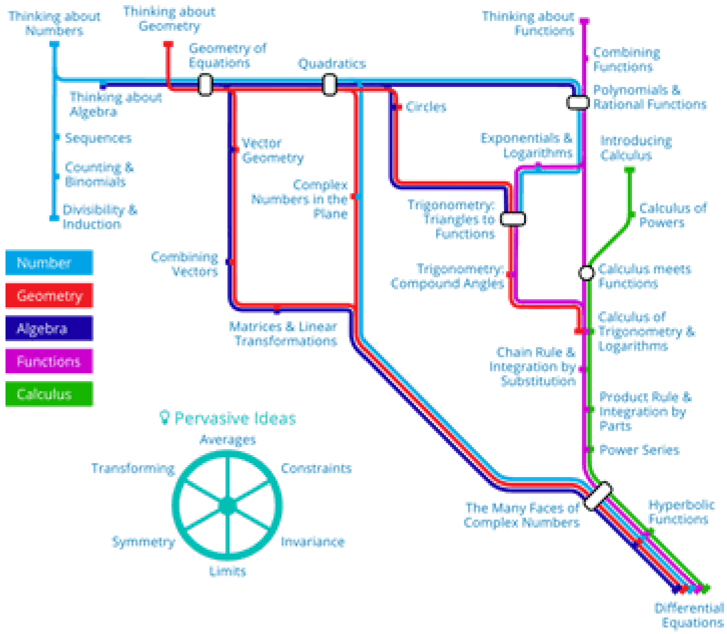
Here is the quality of her application task, given the same question:



Footnote: as Dan suggests, depth of understanding is important for all children, not only high achievers. Those with good understanding should be encouraged to go deeper still. For clarification, see the NCETM Primary National Curriculum Assessment Materials (particularly useful to secondary teachers for diagnostic assessment of their pupils' deep, conceptual understanding of primary topics).

Sixteen Plus
Underground Mathematics

"Underground Mathematics" – the name suggests something a little more subversive than a new A level maths resource from the University of Cambridge and funded by the Department for Education! But this network map explains more…



Click map for larger version

… A level topics usefully displayed to show their connectivity, helping teachers to draw out those links between topics that are too often parcelled up into isolated chunks.

Underground Mathematics is a new A level maths website that has been developed to provide rich, interactive, free resources for students and teachers in anticipation of the new A level for first teaching in 2017. The resources highlight important learning points, promote reasoning skills, and encourage deep mathematical thinking. Resources come with support materials to help teachers to maximise student learning and make mathematical links across multiple lessons and between topics in A level maths. Each resource is housed at a 'station' on one or more mathematical 'tube lines', and many resources include suggestions for 'looking forward' and 'looking back'. The 'Discuss' facility on the website provides teachers with a platform to talk about how they are using the resources – and adapting them for use in their own contexts. There is also a personal user area provided for bookmarking resources.

Katie Binks, a teacher at Long Road Sixth Form College, Cambridge, has been involved in trialling and developing Underground Mathematics, and uses the resources regularly with her classes. She explains that the philosophy behind Underground Mathematics aligns well with her beliefs about how mathematics should be taught:

> *"It is important that students learn to really think about the challenges they are solving, rather than just remembering quick tricks and standard responses to answer standard questions."*

So Underground Mathematics does not represent a new approach for her, but it provides ready access to resources that she really feels she can trust. Katie says she doesn't have those 'I wasn't expecting that', coming-unstuck moments with Underground Mathematics. The resources are carefully designed and many are classroom-trialled so that they can be expected to elicit the mathematics intended – for example, card-sort exercises are designed to avoid the possibility of being matched by unintended pattern-spotting or elimination.

> *"The tasks have been thought out so well - so that students have to engage with the maths that you want them to engage with. Also, I want students to learn to articulate their maths and justify their answers and justify their ideas and their problem-solving – Underground Mathematics resources are designed for that."*

Students often struggle with the sheer volume and pace of learning at A level. In the pressure to cover content, it is easy to overlook the need to dedicate time to learning to think mathematically. Katie explains that time invested in developing mathematical thinkers is time that is clawed back later when students are in a better position to take on larger volumes of content because of their increased capacity to understand what they are learning.

> *"By using resources that make them think in different ways, use their logical processing and see all the links between different areas of maths, that actually speeds up their learning anyway. So you might lose time in some ways, and gain it in others."*

She uses the example of the resource Teddy bear – a resource for matching equations with circles. "The important thing is not to double the effort in an attempt to fit in activities. They don't need to do a whole exercise (on circles), because they are doing that proficiency and practice, while they are doing the Teddy bear activity. But they are also honing thinking skills at the same time." If her students do need extra routine practice, Katie sets this in their own time. She also points out that not all the activities are designed to take large chunks of lesson time. For example, she uses Powerful Quadratics as a starter with all her classes: working in groups, it can be a very quick activity.

Katie is positive about how students have responded to the Underground Mathematics approach. However, she cautions that teachers should not expect a quick fix – students and teachers need to become accustomed to the more activity-based approach. Often students opt for maths at A level because they enjoy the security of feeling that there is one correct answer to any problem they are given, and such students may have had a very didactic experience of maths teaching prior to A level. Underground Mathematics resources challenge this way of thinking and require students to communicate and discuss their reasoning in ways that they may not always be familiar with. Katie observes that the students that are most resistant to such an approach are often those that need it the most – she says their feedback often recognises this in retrospect. Whilst some students find the activities challenging, others welcome the support offered by the discursive approach of some of the resources. Teachers need to be prepared to take a step back, allowing space and time for mistakes and peer correction. They also need to be ready to ask questions to assist students in figuring things out for themselves - Katie finds the teacher notes a real support for this.

Katie has some favourite Underground Mathematics activities:

- A tangent is… a resource Katie used last week, that encourages students to really examine their understanding of the definition of a tangent, good for introducing calculus
- Gradient match is another early calculus resource thinking about matching pairs of functions so that one is the gradient function of the other. The resource can also be used in multiple other ways

- Can you find…cubic edition: looking for cubics that fit various definitions – e.g. a cubic with no stationary points…making students think "backwards" compared to many standard questions
- Two-way calculus examines the properties of functions, such as stationary points, students have to work out how functions are similar and create other functions with matching qualities
- Building catenaries is a video explaining why engineers use catenaries.

Visit undergroundmathematics.org to see more Underground Mathematics resources and find out more about the project.

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From the Library

A new report, Texting Parents, from the Education Endowment Foundation (EEF), has shown that regular text messages to parents can improve students' results in maths by a small but significant amount. This intervention is likely to be particularly attractive to schools due to its simple, low-cost nature and because many schools already have the necessary technology.

The research, funded by the EEF and the Nominet Trust, and carried out by teams from the University of Bristol and Harvard University, shows that, over the course of the research (carried out over the school year 2014/15), pupils whose parents received the texts made on average, one month's additional progress in maths compared to the control group. Previous research has indicated the importance of parental engagement in the schooling process but comparatively little has been said about how this can be achieved. The new research begins to address this. Dr Raj Chande, project lead, explains that the rationale behind the idea was a recognition that schools are investing a lot in communicating results with parents after the event (such as report cards), but less in communicating things in advance. Added to the lack of daily school-run contact that secondary parents have, and the stereotypical lack of communication from teenagers, this can often make parents feel out of touch with their child's education.

The texting trial involved over 15 000 pupils in 36 English secondary schools, in years 7, 9 and 11. Parents were sent an average of 30 texts over the course of the year, with messages equally split between Maths, English and Science departments. There were three types of text message used in this trial:

- scheduled texts (e.g. departments can submit timetables for scheduled tests in advance)
- automated texts triggered by teachers updating the school's databases (e.g. 'your child has not submitted his/her homework')
- texts to the parents of a class (e.g. conversation prompts – 'today your child has been measuring the height of trees using trigonometry').

The software is also capable of sending texts to individual parents though this was not trialled.

Although the improved maths attainment impact was relatively small, interviews with participants suggested that the effect of improving communication between schools, parents and their children was a positive outcome in itself. A mid-year parent survey received positive feedback from 90% of parents:

> "I feel connected with my daughter and her school when receiving these texts"
>
> "I LOVE receiving the texts cos it keeps me informed and it takes no time to read them.......PLEASE NEVER stop sending the texts to the parents, as without them children would not go into the detail or tell us as much of what was happening at school!!!! Thank you"
>
> "A really good concept to give parents the heads up. Keep up the good work. Sadly **** was grounded as a result. Oh dear. Then again, he did catch up with course work. He is not happy with you although I am."

Pupils were less gushing in their reaction to the project. Although they were generally in agreement that the project improved their parents relationship with the school, and did accept that this could be 'helpful', it was also often referred to as 'annoying' as some felt increased awareness led to 'nagging' from parents.

"They have more of a connection with the school, like because obviously they don't come to the school with us so it's ways of finding out what we're doing because otherwise like if you don't really talk to your parents they wouldn't find out."

There was also some frustration from both parents and pupils due to occasional inaccuracies in the messages:

"The texts are very informative, the only problem sometimes is that they are not given the test on the date it says on the text."

Researchers suggest that the maximum cost for this intervention is £6 per pupil but likely to be much less in most schools, where the technology is already installed. Schools involved embraced the intervention for its low cost but also for its immediacy, though there were some suggestions of the need for schools to allocate a dedicated coordinator to oversee the intervention. Many schools also found staff reluctant to take on an extra task. As one teacher said:

"…English, maths and science are core subjects, you're always being asked to do extras, this was just another one on the pile, so it's not a lot to do, and I know it sounds like a lot, but fitting it in somewhere and finding something like a good conversational topic was tricky actually and then trying to make sure that the prompt and statement all lined up on the dates for all the classes and things like that was, in the evenings as well."

But other staff found a workload reduction:

"It's made life a little easier around the fact that low level stuff you're not on the phone constantly all night trying to get hold of parents. I can usually say, yes I've contacted that parent by just clicking a button. And that is, when you've got a hundred other things to do, it's really quick, it's really easy. It allows us to tick a box to say, yeah, we've done that bit. And I've really appreciated it, definitely."

And there was widespread recognition from schools that the project had increased communication in a positive way:

"I think my staff would say that they very definitely appreciate having parents on their side as well, having a way of communicating with parents, adult-to-adult, so that there is a team around the child, a team around the student, trying to get the best out of them."

The EEF was set up in 2011, an initiative of the DfE, with the intention of breaking the link between family income and educational achievement by promoting evidence-based projects aimed at tackling the attainment gap. As such, the EEF intends to allocate £200m of grants over its 15 year life, researching interventions in education by any organisation working in schools and other educational settings. The results of these projects are reported publicly and feed into the Teaching and Learning Toolkit – a catalogue of possible interventions for school improvement with likely impact and cost clearly given. Full details on how to apply for project funding can be found here. Schools and teachers interested in becoming involved in existing projects can find details here.

Full details of the texting project can be found in the independent evaluation report.

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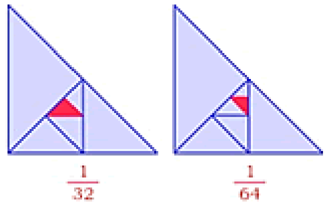


National Centre for Excellence in the Teaching of Mathematics

It Stands to Reason

A double bill this month!

First, with our eyes on the unfamiliar topics in the new GCSE specification, we offer a nice puzzle that involves completing the square to find turning points.

Then, we suggest some visually breath-taking ways to introduce geometric sequences and series, with a particularly enticing visual demonstration of the formula for the sum-to-infinity. Although much of the material here is for use at A level, GCSE teachers introducing the idea of geometric sequences and their nth terms (new on the Higher GCSE) may like to use the patterns or to 'read ahead' to understand where the topic goes beyond GCSE.

Completing the Square

Completing the square has long been one of those 'sticky' GCSE topics whose method is taught long before its use becomes clear. It has been a topic that is difficult to teach and that students despair of. Context-free exam questions of the form:

"Express $y = x^2 + 6x + 1$ in the form $y = (x – p)^2 + q$ where p and q are integers to be found"

reinforced the idea that this was simply an algebraic manipulation to be memorised with little to recommend it. It is hard to teach an difficult algorithm for something when the answer to the perennial student question 'What is the point of this?' is

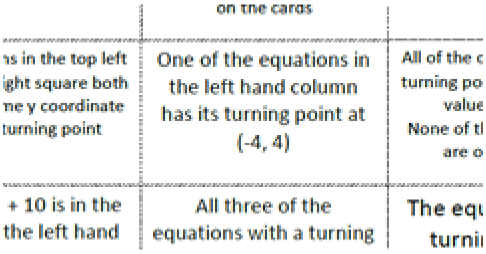
'Well, it will be useful if you continue studying maths next year'

What exactly was the point of completing the square? Yes, it could be used to solve a quadratic equation, but where was the advantage in that over factorising or using the formula? The real power of completed square form was lost on GCSE students. Now, with the new GCSE content at higher level, students are exposed to some of the power of the completed square format – its ability to quickly give co-ordinates of the turning point of a quadratic curve, allowing its position on a Cartesian grid to be accurately sketched and understood. Whereas previously students could determine where a quadratic curve crossed the x and y axis (by letting x=0 to solve for y, and y=0 to solve the quadratic for 2 values of x), now they are expected to be able to determine the turning point of the curve.

The quadratic curve: $y = ax^2 + bx + c$, expressed in the form: $y = (x - p)^2 + q$, has a turning point with co-ordinates (p,q) on a Cartesian grid

This exposes one of the uses of completing the square, and gives a peek into A level work ahead, where completing the square is used in the transformation of graphs.

The following activity has been designed to give students routine practice using the algorithm to complete the square, and then to use logical deduction and reasoning to solve a problem.


on the cards

ns in the top left ight square both ne y coordinate turning point	One of the equations in the left hand column has its turning point at (-4, 4)	All of the c turning po value None of tl are o
+ 10 is in the the left hand	All three of the equations with a turning	The equ turnii

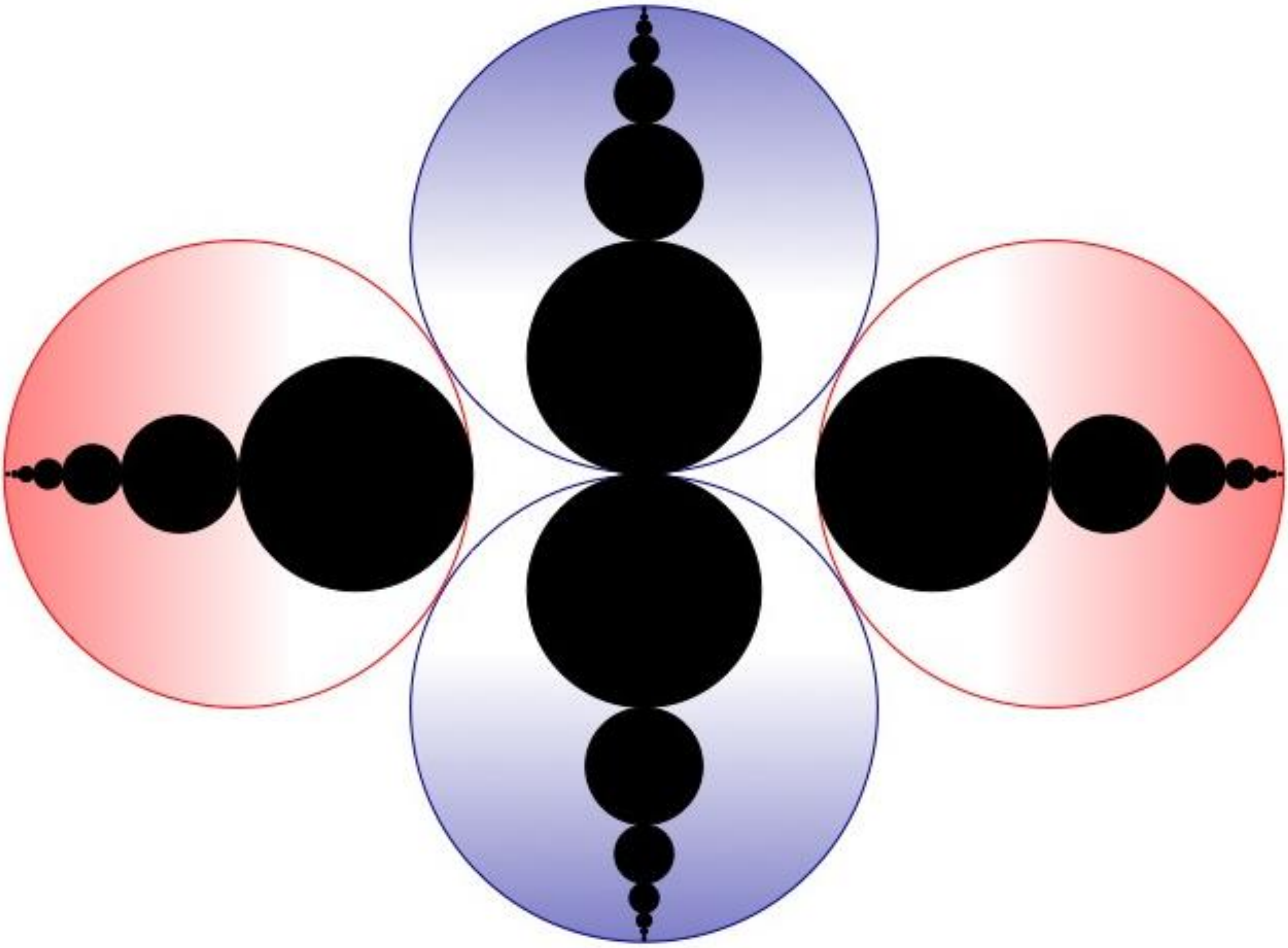
A PDF of the activity, so that cards can be cut out is here.

Crucially, the activity also requires students to understand **how to determine a turning point from the completed square format**. It also brings this together with knowledge of the equations of vertical and horizontal lines, and how to determine whether a point is on a line, given the equation of the line. The exercise can be further broadened if students are encouraged to sketch the curves on a Cartesian grid, along with the relevant straight lines. We suggest that this activity would be best used with students working in small groups. They could then share out completing the square for 10 equations, checking each other's work and sketching the curves on a shared Cartesian grid. They could then use the cards (cut out) to solve the problem given.

The task Which Parabola? designed for the new A level website Underground Maths asks students to find the equations of different parabolas. Whilst there are various ways of solving the problem, completing the square is likely to be most efficient.

Geometric Series: Sum to Infinity

We start with this image …



… and a question …

What fraction of each large circle is black?

You might like to work on this question with your pupils after they have had opportunities, of the kind suggested below, to develop their understanding of ***Simple Geometric Progressions***. You may need to assure them that the diameter of the largest black circle is equal to the radius of the surrounding circle, and that the diameter of each black circle (as they get progressively smaller) equals the radius of the previous one. An example of how a examinee might present reasoning that leads to the solution of this problem is here.

Pupils who have deduced how to sum-to-infinity a ***geometric sequence*** by previously exploring other examples of 'works-of-art' in which it is possible to see ***geometric sequences*** should be able to reason to the correct answer: this is because they will have reached useful and general mathematical conclusions about ***Simple Geometric Progressions***. (A link to a solution is given below.) Therefore, since most pupils enjoy the visual patterns in mathematical art, the recent ***introduction of 'Simple Geometric Progressions'***

into the content of GCSE mathematics seems to be a good move.

The following examples suggest how reasoning about some maths-art images can support and confirm conventional reasoning to a general formula for the sum to infinity, S, of a ***simple geometric progression***. Such geometrical reasoning may enable pupils to understand the conventional general (algebraic) reasoning when, without 'seeing it geometrically', they would otherwise have struggled.

Teaching Geometric Sequences provides an excellent opportunity to start a new topic by simply presenting an image, and then inviting pupils to comment on anything that they notice about it.

You might start by using an example to focus pupils' thinking on ***arithmetic sequences*** which we assume they will already have met. Invite pupils to comment on this image.

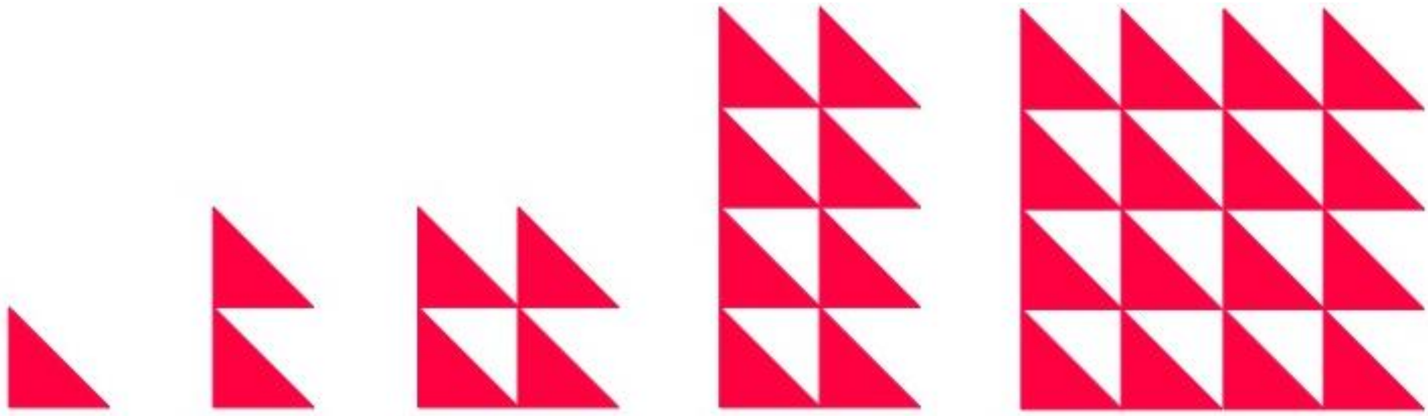
If necessary prompt pupils to focus on this question:

What would be the next image in this sequence?
How do you know?

Establish that:

- the number of red triangles in each part of the image forms the sequence, 1, 3, 5, 7, …,
- the same number, 2, is added to each term in the sequence to make the next term,
- the general, n^{th}, term of the sequence is $2n - 1$

Now display a related image in which it is possible to discern a sequence that is ***geometric***, rather than ***arithmetic***, such as this …



… and ask:

What is the same and what is different *about this image compared with the last one?*

After pupils have had plenty of time to think-about and discuss their answers to this question, establish that:

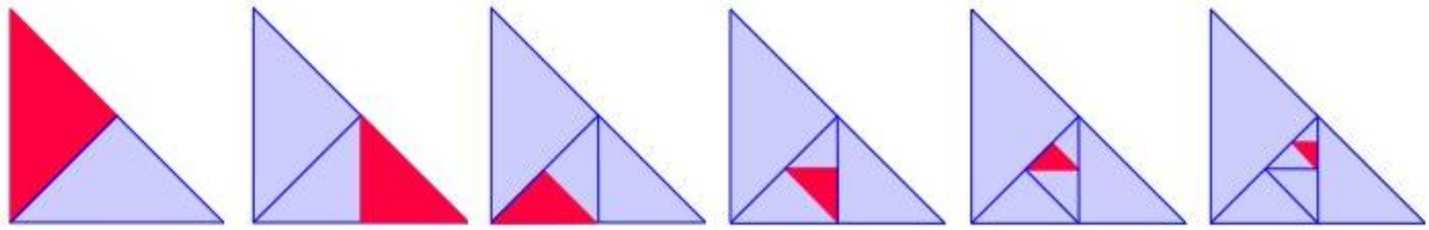
- the number of red triangles in each part of the image forms the sequence, 1, 2, 4, 8, 16 …,
- each term in the sequence is multiplied by the same number, 2, to make the next term,
- the general, n^{th}, term of the sequence is 2^{n-1}.

At this stage you might tell pupils that any sequence in which each term is multiplied by the same particular number (called the ***constant ratio***) to obtain the next number in the sequence is described as a ***geometric sequence***.

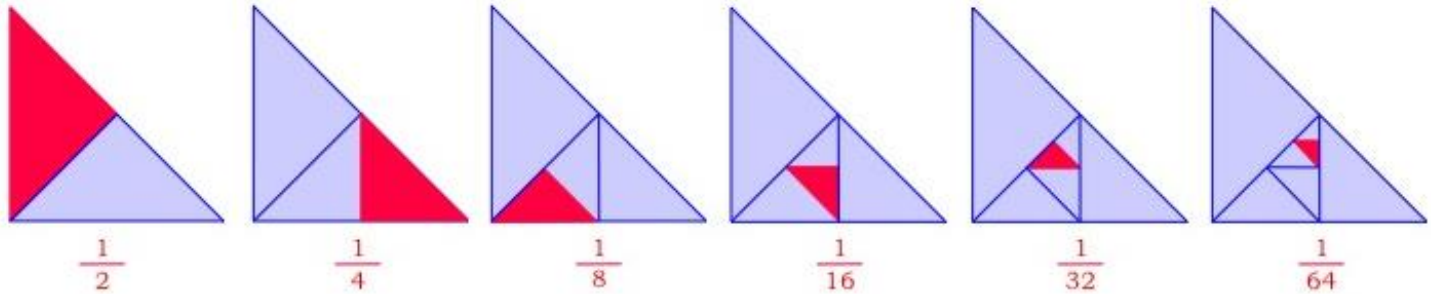
This would be a good opportunity to use the 'another-and-another' strategy; challenge pupils to think of their own example of a ***geometric sequence***, and as they offer their examples keep asking for another example, and another, and another … .

Now you can start to provide opportunities for pupils to investigate geometric sequences more deeply – for them to appreciate the difference between a sequence and a series (a 'series' is when each term of the sequence is added together), and to use their natural reasoning to reach general conclusions (using interesting geometric images) about the sum of a geometric series, both to a finite number of terms and to infinity.

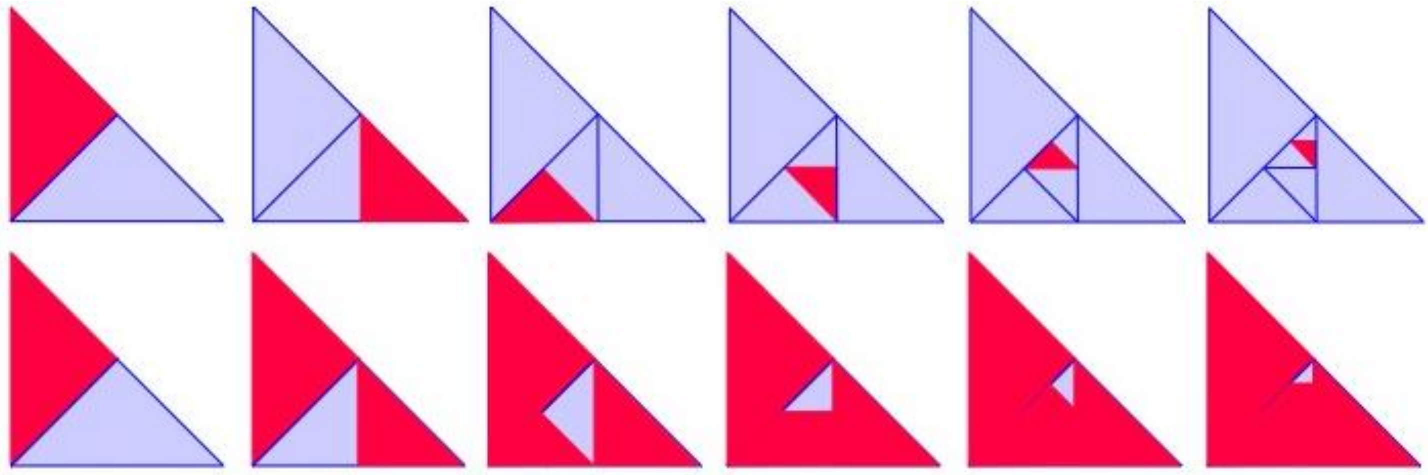
Ask pupils what they see in, and notice about the following image of a sequence of triangles. (Confirm the assumption that the 'whole' triangles are identical right-angled isosceles triangles, and that ALL the triangles are right-angled.)



There are many facts and relationships to notice, all of which should be acknowledged. But the 'pattern' that you need to 'pick-up' from what pupils say is the sequence formed by writing the red-area as a fraction of the area of the whole triangle, for each triangle going from left to right.



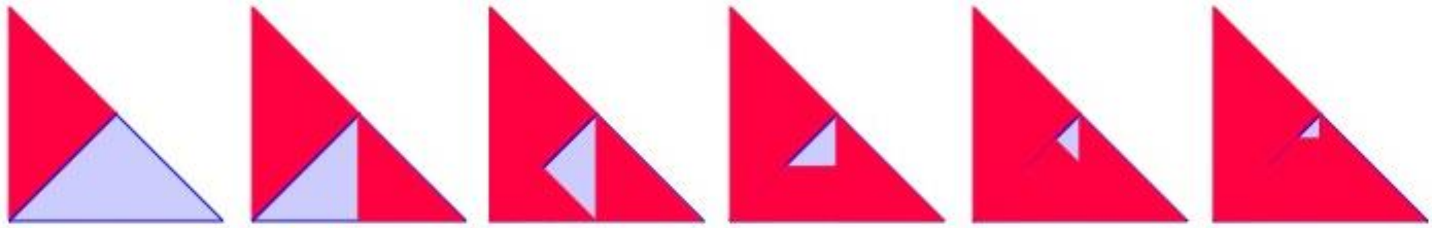
Challenge pupils to describe the numerical sequence. Establish that each term is obtained by multiplying the previous term by ½; ask them to explain why this is (by thinking about what is 'going-on' in the image).
Now show together these two related images of sequences of triangles …



… and ask …

- How are these images related?
- In the lower image, for each whole triangle what fraction of the area is now red?
- If this 'pattern-of-triangles' continued for ever, what would happen?

By discussing answers to these questions (preferably in pairs or small groups, and then as a whole class) pupils are likely to try to explain to each other how the number sequences below are related to the image …

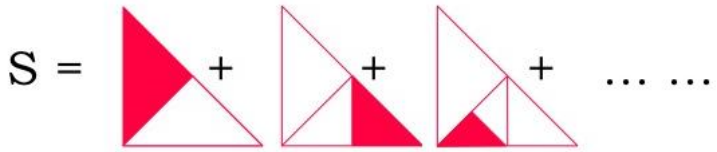


$\frac{1}{2}$	$\frac{1}{2}+\frac{1}{4}$	$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$			
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{31}{32}$	$\frac{63}{64}$
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$

They have opportunities to reach their own conclusions; for example they may make conjectures about what happens to the sum of the first few terms of the geometric sequence, ½, ¼, ⅛, … as more and more terms are included. What would be the sum of ALL the terms of this infinite sequence? (That is, what would be the sum to infinity of the ***geometric series*** ½ + ¼ + ⅛ + … .) The image gives a strong visual 'clue'!

Pupils may conjecture that the total area of ALL the red triangles in the original sequence (if the sequence continues for ever) will be equal to one 'whole' triangle. Ask what this implies numerically. (That the sum to infinity of the geometric series ½ + ¼ + ⅛ + … is 1.)

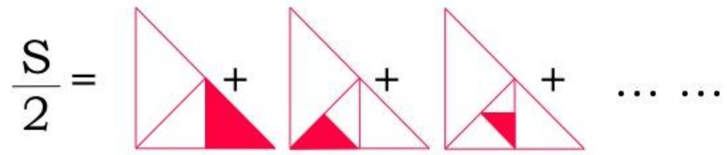
How can they TEST this conjecture by thinking about the sequence of images?
Challenge pupils to visualise the sum, S, of all the red areas:



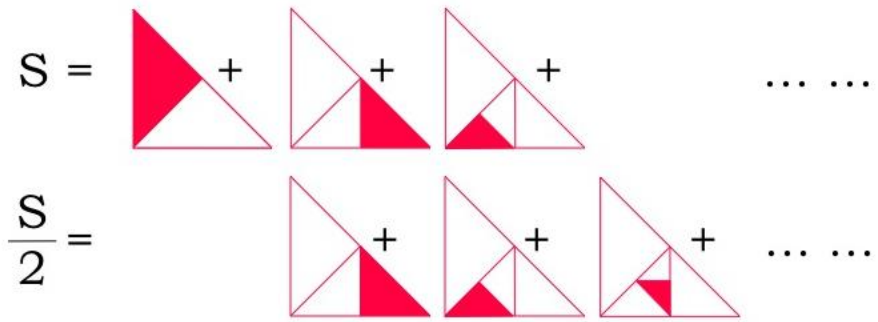
Ask how they could imagine halving this sum. Allow plenty of time for discussion. Focus on pupils' observations that:

- 'you could halve the red area at each stage (in each 'whole' triangle)
- 'at each stage you could halve the 'other half' (which is white in this sketch)

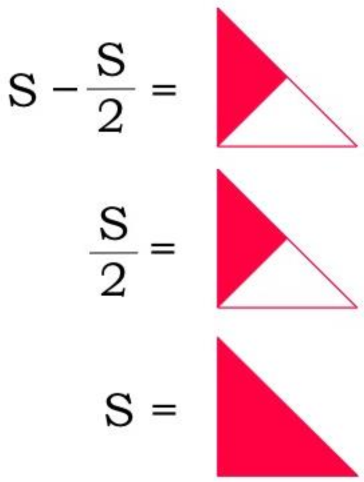
Eventually draw out, or hopefully focus on pupils' suggestions, that you could visualise half of S in this way:



Pupils then only need to visualise the whole-triangles in this image of half-the-sum-to-infinity shunted to the right …



… to see (literally) …



The conventional reasoning to a general statement of the relationship between the sum-to-infinity, S, the first term, a, and the common ratio, r, of a simple geometric progression, that the geometrical 'way-of-seeing' above suggests is …

$$S = a + ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6 + \dots + ar^n + \dots$$

$$rS = ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6 + \dots + ar^n + ar^{n+1} + \dots \dots$$

$$S - rS = a$$

$$S(1 - r) = a$$

$$S = a/(1 - r)$$

… which, for the geometric sequence ½, ¼, ⅛, … …, could be presented as …

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \dots \dots$$

$$\frac{S}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \dots \dots$$

$$S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \dots \dots$$

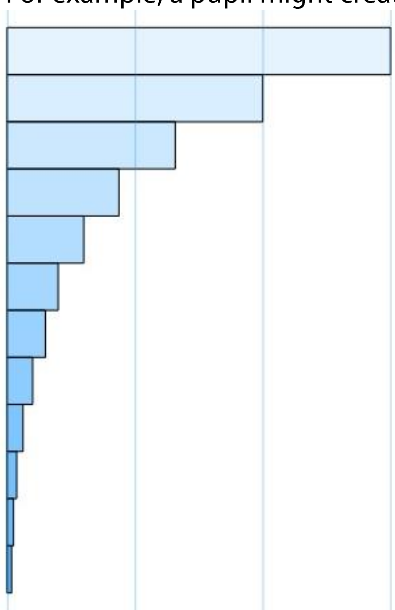
$$\frac{S}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \dots \dots$$

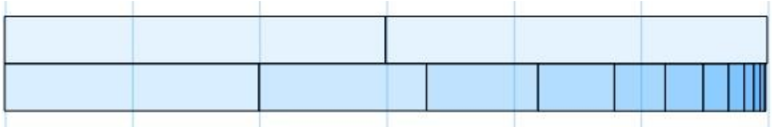
$$S - \frac{S}{2} = \frac{1}{2}$$

$$\frac{S}{2} = \frac{1}{2}$$

$$S = 1$$

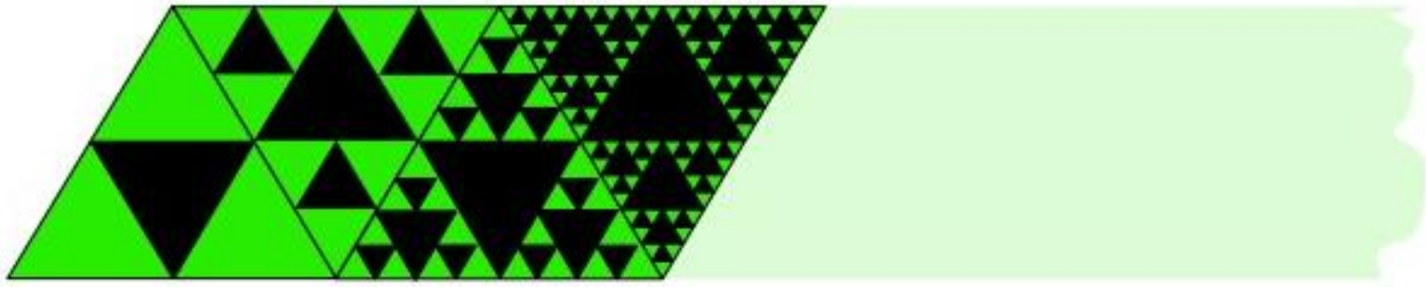
Pupils could create their own mathematical artefacts to show some facts about geometric progressions that they have observed or deduced. For example, a pupil might create these images …





… to illustrate their finding that the sum-to-infinity of the geometric sequence with first term 2/3 and with common-difference 2/3 is 2.

Many other mathematical artefacts, of various kinds, reveal geometric sequences. For example, the image below shows the left-hand end of a frieze in which we can imagine that the pattern-process continues in the right-hand direction for ever.

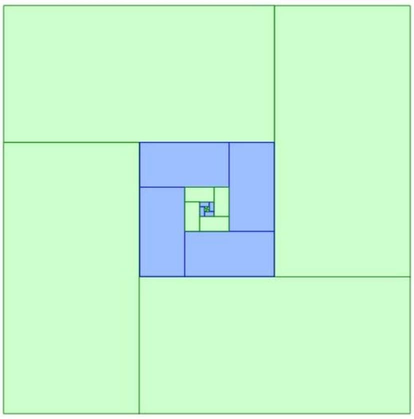


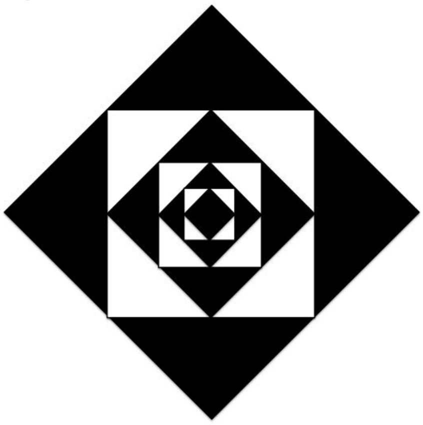
A question about this frieze that can be answered if one has investigated, and thereby developed understanding of, ***geometric sequences***, is:

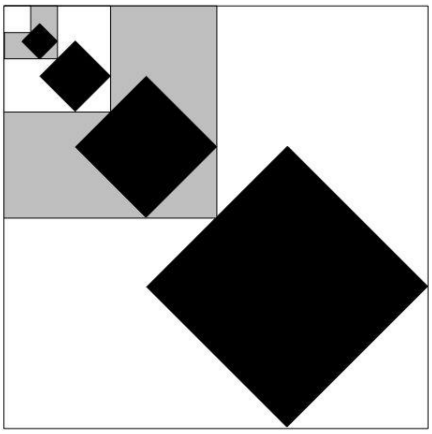
If T is the area of the largest triangle,
what (in terms of T) will be the total-area-to-infinity of the parts of the frieze that are green?

The answer is surprising! It is here (with an explanation).

Challenge pupils to choose some other 'maths-art' images such as these …







… and create questions for each-other to try to answer, that can be answered using reasoning and their understanding of geometric sequences.

Pupils might like to search on the internet for examples of mathematical art that reveal geometric sequences. Here are some links to help you get started. The Bridges Organization, which is described …

'The goal of the Bridges Organization is to foster research, practice, and new interest in mathematical connections to art, music, architecture, education and culture'

… provides links to mathematical artefacts of all kinds. You can see all the artwork from their conferences and the contents of a Virtual Museum.

On the American Mathematical Society's Mathematical Imagery site you can see images of many maths-art creations, including a useful essay on Mathematics and Art. Wikipedia's list of mathematical artists may be useful, and Wolfram Mathworld's Fractal page has some very relevant images.

Pupils could also look at web pages devoted to particular mathematical artists, such as:

- Frank Stella born 1936,
- Gene Lang,
- Julio Le Parc born 1928,
- Eusebio Sempere

You can find previous *It Stands to Reason* features here

Qualifications and Curriculum
What we learned from this summer's maths results…

GCSE

- **Headline Maths grades A*-C figure: 61.5%** (down from 64.1% last year). This drop of 2.6% compares to the drop for all subjects of 2.2% (66.6% A*-C , down from 68.8% last year)

  GCSE Maths Results 2016 for England (2015 figures in brackets)

Gender	Number Sat	% of Total Sat	PERCENTAGES by Grade A*	A	B	C	D	E	F	G	U
Male	343941	14.5	6.0	10.4	16.8	28.6	18.6	8.1	4.4	3.2	3.9
	(339533)	(14.3)	(6.8)	(10.8)	(17.4)	(29.7)	(17.1)	(7.1)	(4.1)	(3.2)	(3.8)
Female	357958	14.6	5.3	10.3	16.8	28.8	19.6	8.5	4.5	3.0	3.2
	(352318)	(14.3)	(5.6)	(10.2)	(17.3)	(30.4)	(18.5)	(7.6)	(4.0)	(3.0)	(3.4)
Male &	701899	14.6	5.7	10.3	16.8	28.7	19.1	8.3	4.4	3.1	3.6
Female	691851	(14.3)	(6.2)	(10.5)	(17.4)	(30.0)	(17.8)	(7.4)	(4.0)	(3.1)	(3.6)

  Click to enlarge

- **New entry patterns** had a big effect on the national A*-C figures. This reflects the increase in post-16 entries (now over 20% of total entries for maths) and also the reduction in early entry (Y10 and below). This is a result of the new requirement for post-16 students to continue to study until they achieve a C grade, and the new measure that allows only first-time entries to count in school statistics. These policies particularly affect maths entries. The picture for 16-year olds actually follows the upwards trajectory seen for many years:

  GCSE maths results 2016 (breakdown by age)

Cumulative percentage at grade

	2015			2016		
	Pre-16	16-year old	Post-16	Pre-16	16-year old	Post-16
A*	16.9	6.7	1.0	15.0	7.0	0.8
A	30.6	18.6	3.1	30.0	19.7	2.4
C	66.6	69.1	35.8	66.7	70.5	29.5

  A*-C figure for 16 year olds = (up 1.4%)
  A*/A figure for 16 year olds = (up 1.1%)

- The **fall in the A*-C figure for post-16 students** reflects the higher number of entries (due to requirement to study maths post-16 if grade C not achieved in Y11). The absolute numbers achieving A*-C post-16 increased, from 46 890 to 51 200.
- It appears schools are now waiting until students have the best chance of maximising their grade rather than expecting students to sit the exam repeatedly using **early entry**. Early entries dropped from 33 484 to 13 209. Despite this, the A*-C rate for early entry candidates is only 66.7%
- Maths is the only GCSE subject where **boys outperform girls** (based on numbers of A*/A grades and A*-C grades)
- **London students** do significantly better than others: 70.1% A*-C compared to 66.6% (for all subjects)
- Entries to STEM subjects by students aged 16 have increased by 6.4% compared to 2015.

A level

- **Headline figures:** percentage of entrants achieving each grade:

  A level Maths (England)

  A* = 17.7% A = 23.9% B = 22.2% C = 16.1% D = 11% E = 6.2% U = 2.9%

  - these are very similar to last year, as are the Further Maths figures (see below):

Gender	*Number Sat*	*% of Total Sat*	*PERCENTAGES by Grade*						
			A*	**A**	**B**	**C**	**D**	**E**	**U**
Male	52390	15.2	19.2	23.2	21.5	15.8	10.9	6.3	3.1
	(52578)	(14.9)	(19.3)	(23.0)	(20.9)	(15.9)	(11.1)	(6.8)	(3.0)
Female	32678	7.7	15.3	24.9	23.5	16.7	11.0	6.0	2.6
	(33070)	(7.7)	(15.4)	(25.0)	(22.9)	(16.7)	(11.3)	(6.3)	(2.4)
Male &	85068	11.0	17..7	23.9	22.2	16.1	11.0	6.2	2.9
Female	85648	(10.9)	(17.8)	(23.7)	(21.8)	(16.1)	(11.2)	(6.6)	(2.8)

Click to enlarge

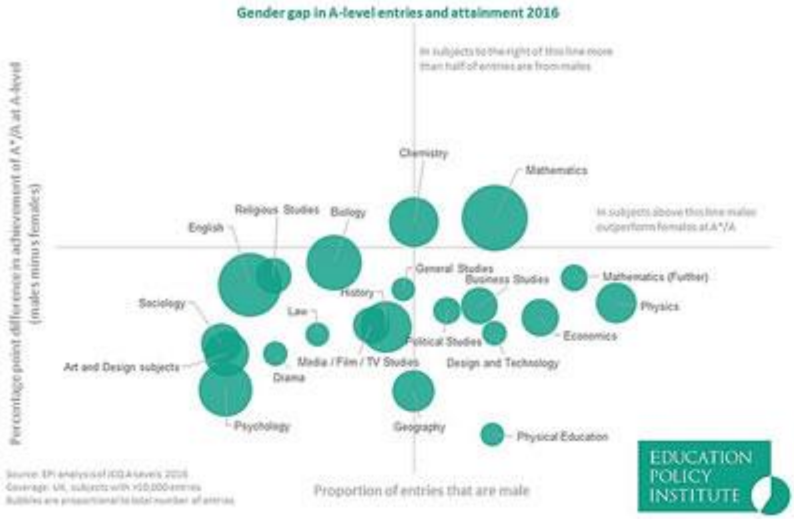
A level Further Maths (England)

Gender	*Number Sat*	*% of Total Sat*	*PERCENTAGES by Grade*						
			A*	**A**	**B**	**C**	**D**	**E**	**U**
Male	10583	3.1	28.4	26.8	20.9	11.6	6.6	3.6	2.1
	(10316)	(2.9)	(28.6)	(26.8)	(20.4)	(11.4)	(6.9)	(3.5)	(2.4)
Female	3983	0.9	27.2	30.3	19.9	11.2	6.6	3.2	1.6
	(3982)	(0.9)	(26.9)	(30.4)	(20.0)	(11.1)	(6.4)	(3.3)	(1.9)
Male &	14566	1.9	28.0	27.8	20.7	11.4	6.6	3.6	1.9
Female	14298	(1.8)	(28.1)	(27.9)	(20.2)	(11.3)	(6.8)	(3.5)	(2.2)

Click to enlarge

- The proportion of students getting A*/A grades in Maths (41.6%) and Further Maths (55.8%) well exceeds the proportion getting the top grade in other subjects (all subjects A*/A figure = 25.8%)
- The number of entrants for A level maths is very similar to last year
- Mathematics remains the most popular subject at A level. 11.1% of all A level entries were for Maths. Next most popular is English at 10.1%. Further Maths takes 1.8%. The picture is slightly different for girls where Maths entries represent only 7.7% of total female entries, making English (13.6%), Psychology (10.3%) and Biology (8.2%) more popular
- The proportion of students studying STEM subjects (including Maths and Further Maths) at A level remains stable
- There is a significant gender gap in the numbers opting to study Maths and Further Maths (Maths: boys 52 390/ girls 32 678, Further Maths: boys 10 583/ girls 3983) – the only other subjects showing large gender gaps are Physics (more boys) and Psychology (more girls)
- Boys get more A*s than girls. They also get more Es and Us. Girls get more As, Bs, and Cs. The picture is similar in further maths but not so clear-cut.

We offer this diagram (from the Education Policy Institute, with kind permission - click to enlarge) for use with your statistics students:



Core Maths Level 3 Qualification

- The total entry figure was only 2931, since this was the first year the exam was taken, and, so far, only small numbers of pilot schools/colleges are involved.
- 82% secured a pass grade
- Almost half (47%) achieved grade C or above and 10% achieved grade A.

	Entry	A	B	C	D	E	U
No. of Candidates	2931	307	461	613	562	459	528
Percentages of entries		10	16	21	19	16	18

These are aggregated results from all boards – Core Maths qualifications under various names. Board-specific data, grade boundary marks and examiners' reports can be found on the examination board websites:

- AQA Mathematical Studies
- OCR Quantitative Reasoning (MEI) and Quantitative Problem Solving (MEI)
- Edexcel Mathematics in Context.

Further discussion on the summer's results can be found on the #mathscpdchat archive if you missed it.

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