MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.22 Combining multiplication with addition and subtraction

Teacher guide | Year 5

Teaching point 1:

Multiplication can be combined with addition and subtraction; when there are no brackets, multiplication is completed before addition or subtraction; when there are brackets, the calculation within the brackets is completed first.

Teaching point 2:

When adding or subtracting multiplication expressions that have a common factor, the distributive law can be applied.

Overview of learning

In this segment children will:

- identify opportunities to group items and use multiplication to complete calculations, rather than using repeated addition or subtraction
- learn that when multiplication is combined with addition or subtraction, the multiplication must be completed first
- use brackets to change the order of operations in a calculation
- build on knowledge of the distributive law, introduced in segment *2.10 Connecting multiplication and division, and the distributive law*, extending it to include the use of brackets.

By the end of this segment, children should be able to apply their understanding of multiplication to problems involving more than one step where the products will be combined or compared using addition or subtraction. Brackets will be introduced and used for the first time. The segment then moves on to use the distributive law to solve calculations where two expressions have a common factor.

In *Teaching point 1* children explore problems involving groups of different sizes, which can be re-organised into multiplication expressions. Children are encouraged to move away from using repeated addition when combining groups (e.g. $6 + 8 + 6 + 8 + 8$) and instead to use multiplication combined with addition (i.e. $2 \times 6 + 3 \times 8$). It is important that children are able to identify groups and create their own multiplication expressions from a variety of contexts. To begin with, children may need to draw a simple representation to help them visualise the problem. However, children should be encouraged to work towards writing their own expression without the aid of a drawing.

The same process is then applied to problems involving subtraction rather than addition (e.g. $20 \times 2 - 10 \times 3$). Children should be reminded that multiplication must be carried out before addition or subtraction (when there are no brackets).

In step *1:3* brackets are introduced, which is the key learning for this segment. Children will learn that brackets can be used to change the order of operations in a calculation, because the calculation within brackets is always completed first. It is essential that children are confident in the use of brackets, as this is built upon in the next teaching point.

The distributive law was introduced in segment *2.10* and is revisited here, in *Teaching point 2*. Children are reminded that when adding or subtracting two multiplication expressions that have a common factor, the distributive law can be applied. The key extension is the use of brackets in the context of the distributive law. Although it is possible to work out solutions without using brackets, children should be reminded to look for the most efficient way to find the solution. This will prepare them for more complex calculations in the future.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Multiplication can be combined with addition and subtraction; when there are no brackets, multiplication is completed before addition or subtraction; when there are brackets, the calculation within the brackets is completed first.

Steps in learning

	Guidance	Representations
1:1	This teaching point explores combining multiplication with addition and subtraction. This will prepare children for further learning on the distributive law, which was introduced in segment *2.10 Connecting multiplication and division, and the distributive law* and will be developed in *Teaching point 2*. Start by adding two sets of equal groups, thinking about why it is efficient to look for patterns and to group numbers rather than adding all the numbers individually. Begin with a simple context, such as the children in boats shown opposite, using very simple numbers to allow children to focus on the structure of the calculation. Ask children to describe the picture. Encourage them to notice the two different groups (there are three children in some boats and five children in other boats). Ask how we could calculate the total number of children in all the boats: '*Do we need to count all of the children individually?*' Encourage children to move away from a structure of repeated addition, and towards a multiplicative structure by helping them to identify groups. Ask children how we can rearrange the boats to help make the calculation easier. Draw out the answer that we can group the four boats with five children together and the two boats with three children together. Ask	Rearranging groups: • *'How many children are there altogether?'*  5 5 3  5 3 5  4×5  2×3 $4 \times 5 + 2 \times 3 = 20 + 6 = 26$

children to write expressions to match each part of the image.

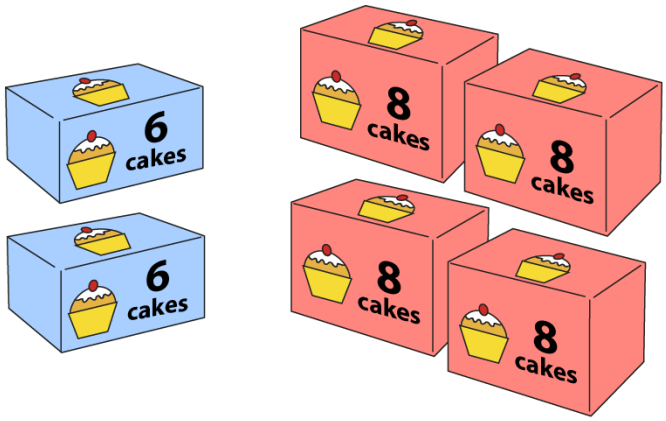
Now ask children to suggest a way to calculate the total number of children in the boats. Reach the conclusion that the two separate products must be calculated first; then they can be added together to work out the total number of children in the boats.

Next move on to contexts where the numbers are already organised into two groups and the representations are more abstract. To encourage children to visualise the problem, show them the written description before showing them the representation. Then ask them to write an expression to summarise the problem before moving on to calculate the answer.

Provide children with practice solving problems that combine multiplication and addition. Encourage them to draw a simple representation of the calculation until they are secure in their understanding. Some examples are shown opposite and on the next page. For the problem involving pencils some children may be able to progress straight to writing an equation, without needing a representation.

When children are confident, move on to more complex problems, including dòng nǎo jīn problems. Encourage children to look for groups in the data, and to use these groups to write equations.

- *'There are some boxes of cakes. Two of the boxes contain six cakes; four of the boxes contain eight cakes. How many cakes are there altogether?'*



$2 \times 6 + 4 \times 8$

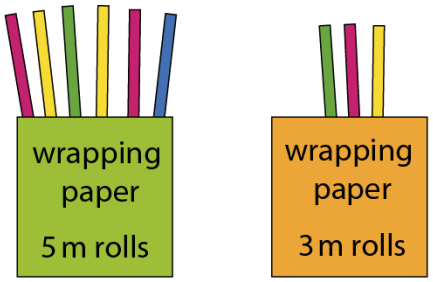
$12 + 32 = 44$

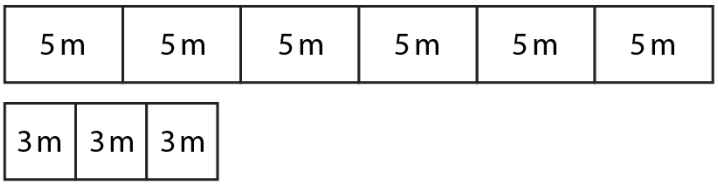
This could also be written as:

$2 \times 6 + 4 \times 8 = 12 + 32 = 44$

Practice word problems:

- *'I buy six 5 m rolls of wrapping paper and three 3 m rolls of wrapping paper. How much paper will I have altogether?'*

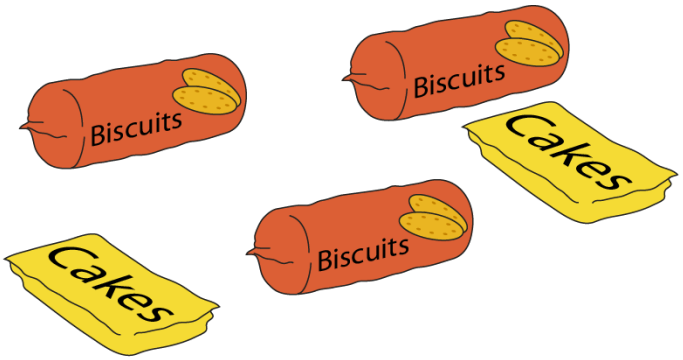




- *'I buy nine pencils for 50 p each and four erasers for 75 p each. How much have I spent altogether?'*

- *'A class is organised into three teams of five children and two teams of four children. How many children are in the class?'*

- *'A pack of biscuits weighs 260 g. A pack of cakes weighs half this amount. What is the total weight of this shopping?'*



- *'This table shows how long is spent in maths lessons each day. How many minutes are spent in maths lessons in total over the week?'*

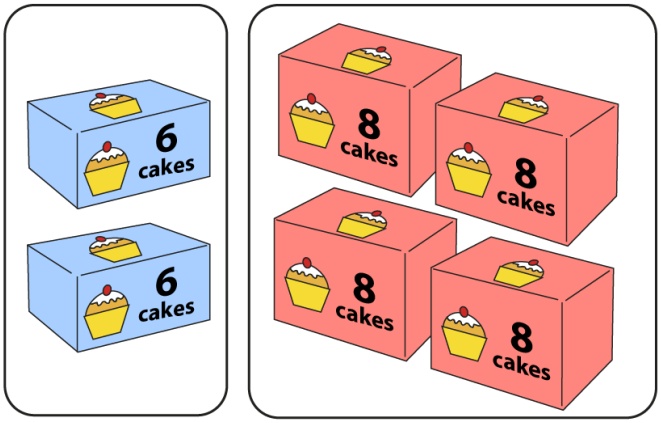
Day	Time spent
Monday	40 mins
Tuesday	55 mins
Wednesday	40 mins
Thursday	40 mins
Friday	55 mins

Dòng nǎo jīn:

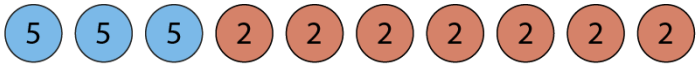
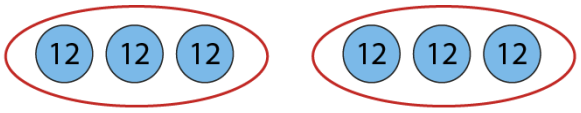
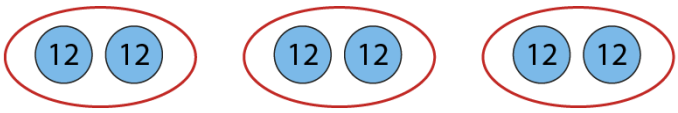
- *'I buy four cherry cakes and three chocolate cakes. I spend £6.20 in total. The cherry cakes cost 80 p each. How much does each chocolate cake cost?'*

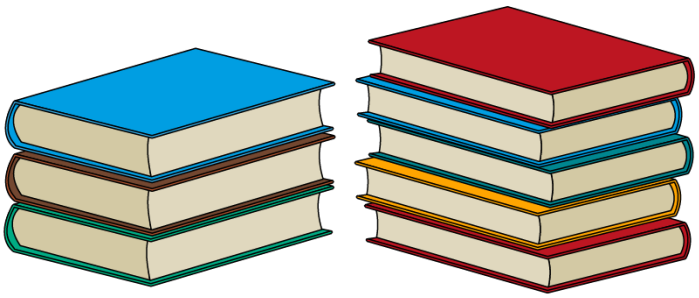
£6.20						
80 p	80 p	80 p	80 p	?	?	?

		• *'I have some 10 p coins and some 50 p coins in a bag. I have twelve coins in total, which have a total value of £2.80.'*
		  • *'How many 10 p coins and how many 50 p coins do I have?'*
		  • *'Is there more than one possible answer?'*

1:2	This step explores using subtraction to find the difference between two sets of equal groups. Representations such as the ones opposite can be used initially to support children in understanding the structure of the calculations. Encourage them to write down the steps of their working to build their confidence in writing equations. When secure in their understanding, children can solve problems by just writing equations, without the need for representations.	• *'I have three packs of ten stickers. You have two packs of twenty stickers. How many more stickers do you have?'*
		my stickers: 10 10 10
		your stickers: 20 20
		$20 \times 2 - 10 \times 3 = 40 - 30 = 10$
		  • *'You have ten more stickers than me.'*
		
		• *'Miss Grey runs 10 km a day for five days. Mr Green runs 8 km a day for seven days. Who has run further and by how much?'*
		10 km | 10 km | 10 km | 10 km | 10 km 10 km × 5
		8 km | 8 km | 8 km | 8 km | 8 km | 8 km | 8 km 8 km × 7
		$10 \times 5 = 50$
		$8 \times 7 = 56$
		$56 - 50 = 6$
		  • *'Mr Green has run 6 km more than Miss Grey.'*
		
		• *'Who has the most cakes, and by how many?'*
		Tom's cakes **Anya's cakes**
		 

		$6 \times 2 = 12$ $\quad$ $8 \times 4 = 32$ $32 - 12 = 20$ • *'Anya has twenty more cakes than Tom.'*
		• *'Carly has four £2 coins. She buys eight stamps costing 90 p each. How much money does she have left?'* $4 \times 200\text{ p} = 800\text{ p}$ $8 \times 90\text{ p} = 720\text{ p}$ $800\text{ p} - 720\text{ p} = 80\text{ p}$ • *'Carly has 80 p left.'* Dòng nǎo jīn: *'Aarav has a rectangular pond which is 7 m by 2 m. Harvey has a square pond with sides of 4 m. Whose pond has the largest area, and by how much?'*
1:3	Next, introduce children to contexts where addition needs to be done *before* multiplication and therefore brackets are required. Use a problem such as the following: *'Mrs Brown needs to order some textbooks and workbooks for her class. Textbooks cost £1 each and workbooks cost £2 each. There are thirty children in the class. How much will it cost in total?'* Look at the two possible answers below and ask children: *'Who has done the correct calculation?'* • Danielle: 30 × £1 + £2 30 × £1 = £30 £30 + £2 = £32 • Sanj: 30 × £1 + £2 £1 + £2 = £3 30 × £3 = £90 Discuss which calculation is correct and why. Explain that multiplication is normally carried out before addition, and so if we want to do addition first	

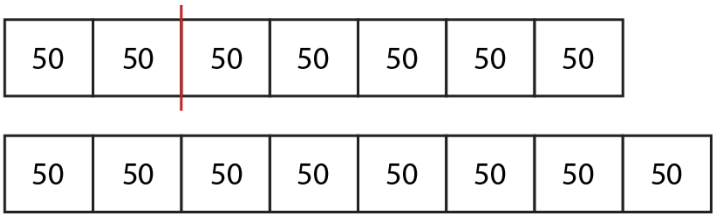
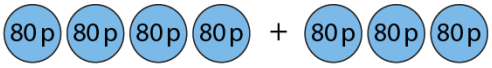
	then we need to put brackets around the addition part of the calculation: $30 \times (£1 + £2)$ If there are no brackets then we know to do the multiplication first. If there are brackets then we do the calculation in the brackets first. In this example Sanj is correct because he did the addition first, but he should have used brackets in his working out.	
1:4	Next, children will develop the knowledge gained in steps *1:1–1:3* by learning to solve abstract problems that require them to combine multiplication with addition or subtraction. The focus in this step is the use of brackets and the use of the mathematical rule stating that multiplication is carried out *before* addition and subtraction when working with abstract equations, unless there are brackets. Show children an abstract problem such as the one opposite (do not provide a context) and discuss the possible solutions. Ask them to consider how each of the solutions has been found. You could use grouped counters to illustrate the different methods. Remind children that, when there are no brackets, multiplication must be completed before addition or subtraction. Using this rule, ask children which solution is correct. Now ask children to place brackets to indicate how the other calculations could have been completed. Return to the example in step *1:3* for support if needed. By the end of this step, children should be comfortable with the following generalisation: ***'When there are no brackets, multiplication is completed before addition and subtraction.'***	*'Solve $3 \times 5 + 7 \times 2$.'* • Joseph says: $3 \times 5 + 7 \times 2 = 15 + 14 = 29$  • Megan says: $3 \times 5 + 7 \times 2 = 36 \times 2 = 72$  • Becca says: $3 \times 5 + 7 \times 2 = 3 \times 24 = 72$  • *'Joseph is correct, because he has multiplied before adding as there are no brackets.'* Dòng nǎo jīn: *'By inserting brackets, how many different solutions can you find using these numbers and operations, keeping the order the same?'* $3 \times 6 + 7 \times 5 =$

1:5	To finish this teaching point, provide children with varied practice combining multiplication with addition or subtraction. Include both contextual and abstract examples. • *'A restaurant has six large tables with ten chairs each, and four small tables with four chairs each. How many chairs are there in total?'* • *'Another restaurant opens on the same street with twice as many large tables and the same number of small tables. How many more chairs does the second restaurant have?'* Remind children of the generalised statement in the previous step to ensure they are carrying out the calculations in the correct order.	• *'Complete the calculations.'* $3 \times 4 + 8 \times 2 = \square$ $6 \times 3 + 13 \times 1 = \square$ $8 \times 4 - 6 \times 5 = \square$ $12 \times 6 - 7 \times 8 = \square$ • *'Fill in the missing numbers.'* $6 \times 3 + 8 \times \square = 26$ $40 \times 3 - 5 \times \square = 100$ $70 \times \square - 2 \times 5 = 200$ • *'Each of Danny's books have fifty pages. Each of Sophie's books have thirty pages. Whose books have the most pages in total, and by how many?'* Danny Sophie 

Teaching point 2:

When adding or subtracting multiplication expressions that have a common factor, the distributive law can be applied.

Steps in learning

	Guidance	Representations
2:1	To begin this teaching point, compare expressions with common factors. This will help children to develop estimation strategies, and will also reinforce the learning from the previous teaching point that multiplication must be completed before addition or subtraction. Provide children with various missing number problems using 'greater than', 'equal to' or 'less than' options. Bar models can be used to help children visualise whether the expressions are greater than, equal to or less than each other. Continue to use the generalised statement from step *1:4*: ***'When there are no brackets, multiplication is completed before addition and subtraction.'***	• *'Fill in the missing symbol (<, > or =).'* $50 \times 2 + 50 \times 5$ ◯ 50×8  • *'Fill in the missing numbers. There may be more than one possible solution.'* ☐ $\times 20 - 3 \times 20 = 4 \times 20$ $20 \times 5 + 20 \times$ ☐ $> 20 \times 10$ $17 \times 6 < 17 \times 8 - 17 \times$ ☐
2:2	Next recap the distributive law introduced in segment *2.10 Connecting multiplication and division, and the distributive law*: we can first add the two factors that are different, then multiply them by the common factor. The distributive law can be represented algebraically as: $a \times c + b \times c = (a + b) \times c$ Children do not need to see the algebraic representation; this is for the teacher's information only. To begin, show children a problem in which the group size remains the same and the number of groups changes,	*'Apples cost 80 p each. Amelia buys four apples at first. Then she goes back and buys three more apples.'*  *'How could you work out how much money Amelia has spent in total?'* 80 p × 4 + 80 p × 3 • Method 1: *'Charlotte wrote this to find the answer:'* 80 p × 4 = £3.20 80 p × 3 = £2.40 So the total spent is £3.20 + £2.40 = £5.60

	such as the one opposite. Use counters to help with visualisation. Before showing children the solutions, ask questions to encourage them to consider how they would approach the problem: *'How could you work out how much money Amelia has spent in total?'* Next show children the two different methods that could be used to solve the problem. Encourage children to explain why both methods produce the correct answer. Ensure they are comfortable using brackets in Method 2. Recap the learning in step *1:3* if needed. Work through a similar problem, such as the one below, using the two different methods. Start by writing the problem into an expression, using counters for support to begin with. *'A running track is 800 m long. Jamir runs eight laps of the running track then stops for a rest. He then runs another two laps. How far has he run in total?'* After working through both methods ask children: • *'Which method do you prefer? Why?'* • *'Is one more efficient than the other?'* Guide children towards the answer that using the distributive law (Method 2) is more efficient.	• Method 2: *'Daniel wrote this to find the answer:'* 80 p × (4 + 3) = 80 p × 7 = £5.60 *'Who is correct? Explain your answer.'*
2:3	Now move on to examples where the group size changes and the number of groups stays the same. Ensure children are aware that the distributive law can be applied to any multiplication equation with a common factor, irrespective of whether the common factor represents the number of groups or the group size. Discuss a problem such as the one on the next page, using counters for support. Make sure children know what each row represents:	

- The first row represents the six pencils for 30 p each.
- The second row represents the six erasers for 40 p each.
- The third row represents the six notebooks for 80 p each.

Referring back to step *2:2*, ask children to consider two different ways they could calculate the total amount spent.

First work through the problem using Method 1, finding the amount spent on each type of item and then adding these three totals together.

Then work through the problem using Method 2, finding the total for each party bag and multiplying the answer by six.

Again, ask children:

- *'Which method do you prefer? Why?'*
- *'Is one more efficient than the other?'*

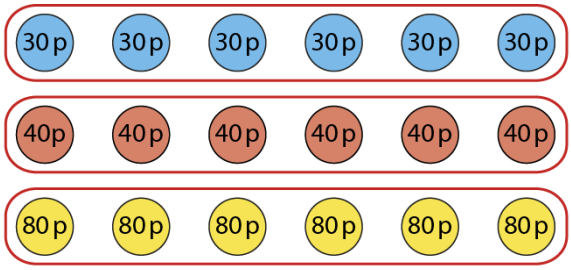
Reinforce the idea that using the distributive law (Method 2) is more efficient.

Provide children with practice of using the distributive law to calculate the answer to questions such as those given below.

- *'In a junior triathlon, competitors swim 50 m, cycle 800 m and run 600 m. Ten competitors finish the triathlon. What is the total distance travelled by all ten competitors in the event?'*
- Dòng nǎo jīn:
  'Hannah is making drinks for a party. For each drink she uses 100 ml of orange juice, 80 ml of cranberry juice and 150 ml of lemonade. She wants to pour all the ingredients needed for eight drinks into a 3 litre jug. Will it fit? Explain your answer.'

'Robyn needs to make six party bags. She buys six pencils costing 30 p each, six erasers costing 40 p each and six notebooks costing 80 p each. How much has she spent altogether?'

- Method 1:



30 p × 6 = £1.80

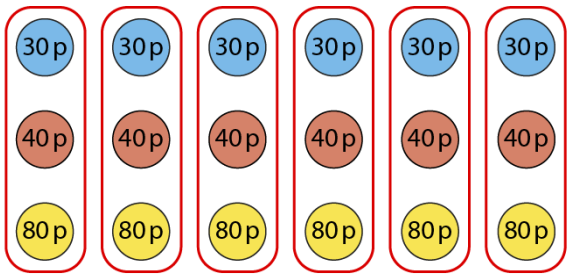
40 p × 6 = £2.40

80 p × 6 = £4.80

'The total amount spent is:'

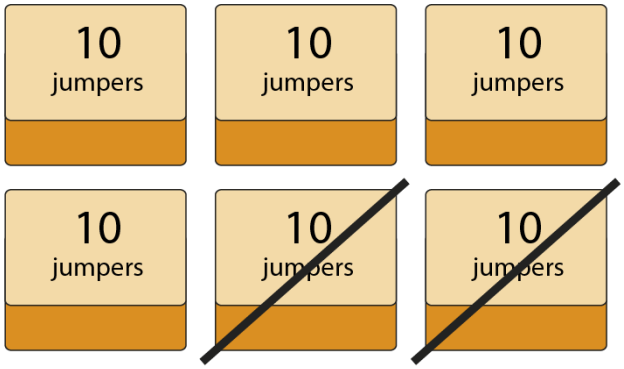
£1.80 + £2.40 + £4.80 = £9

- Method 2:



- *'The total amount spent on each party bag is:'*
  30 p + 40 p + 80 p = £1.50

- *'There are six party bags, so the total amount spent is:'*
  £1.50 × 6 = £9

2:4	In this step we use the same approach in order to solve problems that require subtraction rather than addition. We will look at problems that require finding the difference between two expressions when one factor is common to both expressions. This could be represented algebraically as: $a \times c - b \times c = (a - b) \times c$ Provide children with problems such as those shown opposite. Use simple numbers to begin with while they become familiar with the structure. Show children how the problems can be solved using the distributive law. You could use bar models to help them visualise the problems.	• *'There are six boxes of jumpers in the school office with ten jumpers in each box. Two of the boxes are sold. How many jumpers are left?'*  $10 \times 6 - 10 \times 2 =$ $10 \times (6 - 2) =$ $10 \times 4 = 40$ • *'Easter eggs cost £1.80 each. Katya buys five Easter eggs. After Easter, the same eggs are reduced to £1.50 each. How much would she have saved if she had bought the eggs after Easter?'* 5 × £1.80 5 × £1.50 | ? £1.80 × 5 – £1.50 × 5 = (£1.80 – £1.50) × 5 = 30 p × 5 = £1.50
2:5	Provide children with varied practice applying the distributive law to problems involving addition and subtraction. • *'Daniel has four children. He buys each of them a hat for £5, a scarf for £4 and a pair of gloves for £3. How much does he spend altogether?'* • *'Sarah is making ten portions of fruit salad. For each portion she uses ten blueberries, five strawberries and two apples. How many pieces of fruit does she need in total?'*	

	• *'Normally cinema tickets cost £10. However, on Mondays the tickets are £7. Three friends buy cinema tickets on a Tuesday. How much money would they have saved if they had gone on a Monday?'* • *'There are twelve bookshelves, each with six books on. Two of the bookshelves are taken away. How many books are left?'*	