Teaching for mastery: inclusive by design

Exemplification to support mathematics learning for all in whole-class settings, including learners with SEND

Overview

Teaching for mastery is an inclusive pedagogy for the mathematical learning of all learners, including those with SEND. This resource is for teachers, SENDCOs and leaders, exemplifying how teaching for mastery can meet the needs of all learners.

The large majority of learners designated with SEND learn mathematics in whole-class settings as part of mainstream schools' Universal offer, within which 'commonly occurring needs that every school should be familiar with can be consistently met in mainstream education through adaptive teaching, calm environments, and enrichment opportunities'[1].

Depending on the complexity of learners' SEND, mainstream schools/colleges can offer increasingly intensive support for learning, and special schools/colleges have expertise working with learners with the most complex needs. This layered approach is shown in Figure 1.

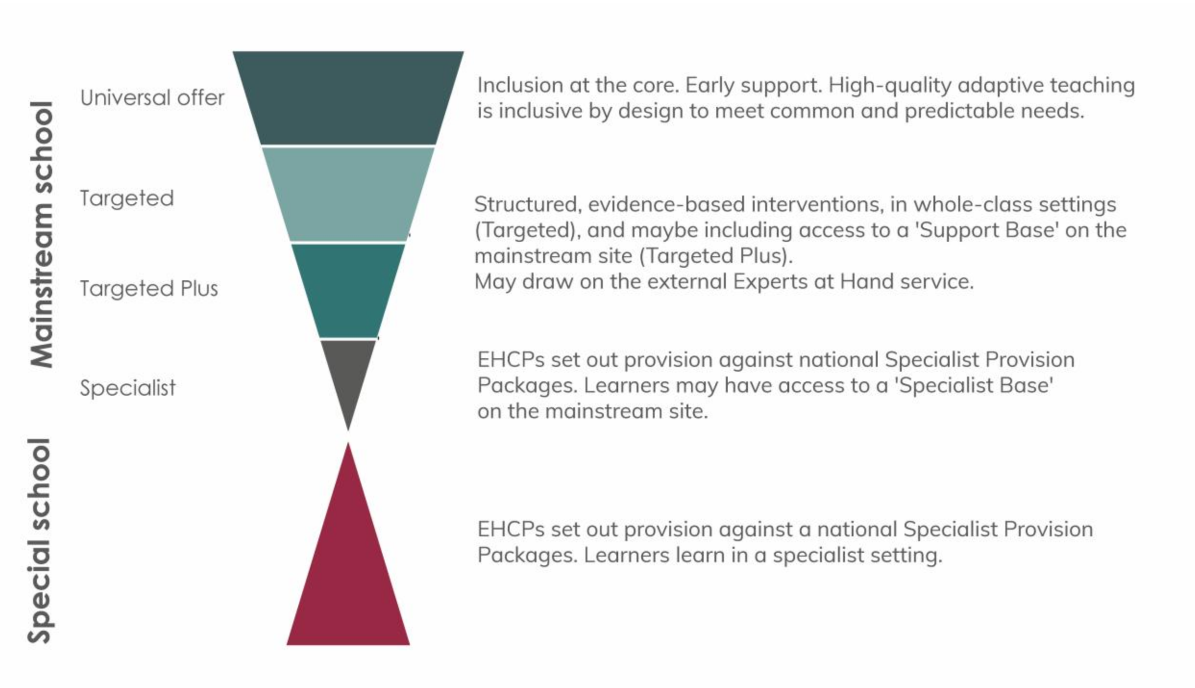


Figure 1: Layers of support. Adapted from the DfE Every Learner Achieving and Thriving white paper

Teaching for mastery is appropriate for mathematical learning at every layer of support in the school/college system.

[1] Every Learner Achieving and Thriving

This document supports the use of teaching for mastery for learners with SEND in whole-class settings, as part of schools'/colleges' Universal offer. Teachers' planning is inclusive by design – mathematical tasks and lessons are designed to anticipate and accommodate learners' needs.

Further guidance will exemplify the use of teaching for mastery at other layers of support.

Structure of this document

Detail is given for each of the Five Big Ideas in Teaching for Mastery for all, followed by an example of how teaching for mastery is an inclusive pedagogy for all learners, including learners with SEND in whole-class settings.

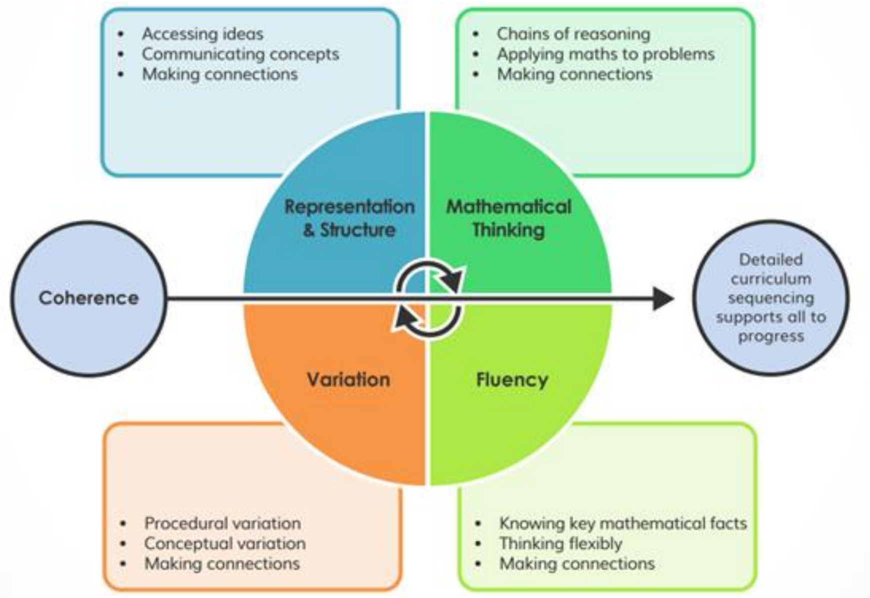


Figure 1: The Five Big Ideas in Teaching for Mastery

The exemplifications are structured as follows:

Know your learner

- What does the teacher know and understand about the learner's characteristics and their approaches to learning?
- What is known about the learner's mathematical understanding? How does this relate to the intended object of learning?

Planning and teaching with reference to one of the Five Big Ideas

- How can teachers plan inclusively, to meet the common and predictable needs of their learners, who they know well?
- How can a teacher build authentically on what the learner knows, and then design a learning experience so that the learner achieves a secure understanding of the intended object of learning?

The effect on the learner's learning of mathematics

- What is the impact of inclusion by design on the learner's learning of mathematics and that of the whole class?

The exemplifications describe common and predictable needs that appear in mainstream whole-class settings. Each exemplification considers a learner with SEND, who informs the starting point for teacher planning so that the lesson is inclusive by design. The section on '*Know your learner'* emphasises the importance of knowing exactly what this unique learner understands in maths and what this learner doesn't yet know.

The teacher in each exemplification considers how the Five Big Ideas in Teaching for Mastery can be used to design lessons that are inclusive, considering the starting point of the described learner, so that all learners engage in thinking about and learning mathematics. Each exemplification describes the impact on this learner's learning.

Teachers start by carefully assessing what the learner understands about the maths, not just what they can do in a test. This positive way of considering the learner's understanding is vital, so that the teacher can build from where the learner is at. Knowing in depth what a learner does understand, rather than focusing on what they do not, informs starting points and enables coherent, well-sequenced planning.

Who is this document for?

This resource is for teachers, SENDCOs and leaders, exemplifying how teaching for mastery can meet the needs of all learners. Leaders and SENDCOs work to embed a shared vision and culture across maths classrooms that enables inclusive teaching, where all learners think and communicate mathematically. Leaders and SENDCOs can use this guidance to shape their inclusion planning, considering the principles outlined and the detail of each exemplification. It's important that maths leads use their maths pedagogical expertise to support learners with SEND, and to liaise with the SENDCO about maths teaching that is inclusive by design.

The learners and intended learning have been drawn from particular year groups, with certain common and predictable educational needs. However, the exemplifications are not restricted to the particular year group, intended learning and educational need. Each exemplification illustrates how teachers' detailed knowledge of their learners and of the mathematics informs their approach to successfully meeting the learners' needs. Leaders might use this guidance as part of professional development for their teams, revisiting over time. Teachers can use these exemplifications as prompts to support them to develop inclusive planning, so that learners with SEND can learn and enjoy mathematics in whole-class settings.

Mathematical Thinking

Mathematical thinking is approached inclusively, with teachers providing tasks that are accessible to all learners while still promoting deep thinking. Teachers foster a classroom culture in which all learners feel safe to communicate their thinking, including both what they understand and what they don't yet understand, and learners are encouraged to ask questions about the maths that they are learning.

All learners can think mathematically. Mathematical thinking is threaded through every lesson and is not treated as a stand-alone skill. Teachers know exactly what it is that they want learners to think about in relation to the mathematical concept, and consistently focus on this without distraction. All learners are expected to engage in mathematical thinking. If all learners know that the aim in every lesson is to make sense of the maths, they will be looking for connections, spotting relationships and being mathematically observant. Teachers allocate time in every lesson to support learners to reflect on what they've been thinking about, so that all learners are aware of the focused thinking that they are engaging in, and the mathematical learning that has taken place as a result.

Teachers design lessons to focus on mathematical thinking and learning, rather than on task completion. Teacher questioning of learners prompts this thinking and learning. Learners' thinking is often supported by their active interaction with the representations and mathematical tasks that are made available. As learners think about the maths, they formulate their own questions about the concepts.

Teachers think carefully about the stages of reasoning through describing, explaining, convincing, justifying and proving. Teachers know that learners' capacity to reason is not fixed and can be improved through explicit teaching. This explicit teaching of reasoning benefits all learners and is particularly important for those who have had fewer opportunities to learn how to reason and practise reasoning both in maths lessons and in their wider learning. Learners lacking confidence in mathematics require more explicit teaching of reasoning, not less.

Oracy is crucial in the process of mathematical thinking. Learners do not automatically become adept at communicating mathematically. Teachers model how to communicate mathematical reasoning by articulating their own thought processes. Teachers consider how to introduce unfamiliar or new mathematical language, ensuring that both their own explanations and the language embedded within tasks are accessible to all learners. The teacher skilfully shapes the language used by some learners to support all learners to develop accurate, precise language. This then becomes part of the learner resource to draw upon in individual lessons and across lessons.

Teachers create structured opportunities for learners to describe and explain their thinking. All learner attempts at reasoning are valued, and learners know that the teacher wants to understand their ideas. Teachers are skilled at assessing learners' mathematical thinking, as expressed in many forms; including through talking, physical interaction with representations, or the use of gesture.

Teachers explain the decisions they make in each step of solving a problem. Learners' attention is drawn to the mathematical structure underpinning a problem. By consistently encouraging learners to notice patterns and mathematical structures, learners can decide how to approach a problem. Learners can then consider how to be systematic and to make informed decisions about how to use the information given. The teacher encourages the learners to compare strategies and to be critical of how they have approached each problem.

Scaffolding supports learners to reason and understand mathematical structure, rather than doing the thinking for the learner.

Exemplification: Mathematical Thinking

Know your learner

Ellie-Mai is a Year 5 learner, who has recently been diagnosed with DLD (Developmental Language Disorder). She has always found talking in maths lessons difficult, and can get quite anxious when she is asked to explain her thinking, particularly in front of other learners. She responds positively to visual support, and often communicates visually or physically rather than through spoken language. She frequently worries that she may not understand the teacher or know what to do.

Ellie-Mai can represent simple fractions, such as $\frac{1}{2}$ and $\frac{1}{4}$, by drawing shapes. She can count in fractional steps and can place these simple fractions on a number line. Her understanding is less secure with less common fractions, such as $\frac{5}{6}$. She finds it difficult to articulate the meaning of the denominator and numerator, although she has an idea that for fractions less than one, the numerator is smaller than the denominator.

Big Idea – Mathematical Thinking

Mr Klukowski, Ellie-Mai's class teacher, recognises that all learners need to think mathematically, and has cultivated a classroom culture in which they expect to talk about mathematics. He is aware, however, that mathematical communication must be taught explicitly. He is particularly mindful that Ellie-Mai finds communicating her thinking more challenging than many of her peers, and he plans accordingly.

He establishes clear norms: learners do not interrupt one another; they listen until a speaker has finished; and they learn to agree, disagree, or build on a peer's contribution. He intentionally facilitates questions from the learners, particularly *"what if...?"* questions. He frequently asks learners to share their partner's ideas to support them in developing listening to, and learning from, one another, as well as to improve confidence in mathematical thinking and talking.

The class is working on a sequence of lessons on equivalent fractions. The long-term intended learning is that:

> *When two fractions have different numerators and denominators to each other, but share the same numerical value, they are called 'equivalent fractions'.*

Before this, Mr Klukowski focuses on securing prior key concepts. He knows that he must provide opportunities for his learners to think about and represent unit fractions before moving on to thinking about fraction equivalence. Therefore, at this stage, he starts with the intended learning that:

> *Unit fractions can be compared and ordered by looking at the denominator. The greater the denominator, the smaller the fraction.*

He knows that Ellie-Mai needs opportunities to explore ordering fractions, so that she can develop understanding of the relationship between the numerator and denominator, and talk explicitly about it. He knows that this deep understanding is essential for later understanding of equivalence, and so decides to ensure all learners understand how a fraction is notated and what the denominator and numerator represent.

He thinks about:

- selecting representations as a tool for thinking, particularly choosing representations that encourage Ellie-Mai to notice the structure of fractions. He begins with familiar bar models, before introducing less-familiar shapes or linear models
- the vocabulary Ellie-Mai will need (denominator, numerator, whole, part, divided, quantity, equal, same value, worth more) and how to support her in understanding, learning and using it
- how to support Ellie-Mai to communicate her thinking, so that she can describe the mathematical structures that she notices
- how learners will interact with the representations, so that they can question and test their ideas
- how the learners will interact with one another to ensure that all the learners, including Ellie-Mai, can reason and articulate their mathematical thinking.

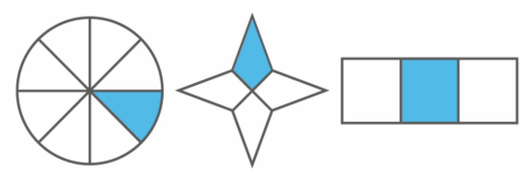
Teaching sequence

Mr Klukowski presents three images of $\frac{1}{2}$ represented in different shapes (a circle, a four-pointed star, and a rectangle).

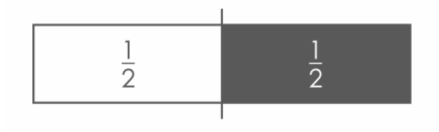
He provides learners time to talk about *"What is different?"*, before providing an opportunity for learners to share ideas with the class. He then asks, *"What is the same?"* again, allowing learners to discuss this in pairs. Having listened to the conversations, he sequences learners' contributions to whole-class discussion, encouraging learners to agree, disagree or build on what the previous learner has said. Ellie-Mai says, *"one is blue,"* and points to the coloured part. Another learner puts up her hand, *"I want to build on what Ellie-Mai has said. One part of two is coloured."* Mr Klukowski reintroduces a stem sentence that the learners have previously used, as he knows that this will reinforce key terms and clear language,

"The whole is divided into ___ equal parts and __ of those parts are shaded."

Mr Klukowski reinforces how to write the fraction with reference to the image, writing the denominator first as he reads out *"the whole has been divided into __ equal parts"*, before he writes the numerator to indicate the number of parts. He shows similar images with different fractions, and learners write the fraction on their whiteboards and repeat the stem sentence for each one.

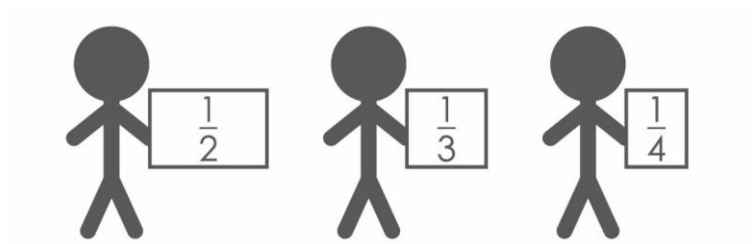


He then gives the learners a strip of paper with $\frac{1}{2}$ shaded.



He asks them to confirm that this is $\frac{1}{2}$ by cutting up the strip and comparing the pieces. He sticks the whole strip and a $\frac{1}{2}$ strip on the whiteboard. He then models how to fold a whole strip into equal parts and asks learners to choose whether to fold their strip into 3, 4, 6, or 8 equal parts (5 or 7 would be too difficult). He asks the learners to decide which unit fraction each equal piece represents and to write it down on the fraction strip, before cutting it.

He asks learners to hold one of their equal pieces and order themselves across the classroom.



He encourages the learners to talk to each other as they order themselves and listens carefully to their justifications, highlighting Ellie-Mai's explanation, *"Mine's bigger than yours because of the small bottom number."* He reinforces this reasoning and emphasises the vocabulary of the denominator.

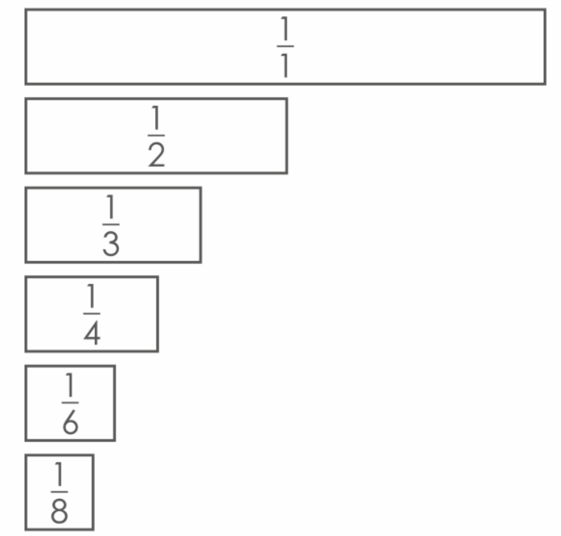
The effect on Ellie-Mai's learning

Ellie-Mai knows she will be expected to talk during the lesson, but she also knows that Mr Klukowski will try to understand what she is saying, even if it doesn't come out as she wants. He always lets her point or gesture if she needs to, and he gives her plenty of time to finish what she is saying. He values her thinking, so that she knows that thinking in maths lessons is important. She is comfortable with peers building on her ideas, because she recognises that her contribution initiated the discussion.

When she and her partner fold their strip into thirds, she notices that the pieces are larger than those of other learners and links this to the smaller denominator. Having both the physical strips and the written notation helps her to connect the concept to the symbol – something she would find difficult through verbal explanation alone.

She continues to find the word *denominator* difficult to remember and pronounce. Mr Kulkowski acknowledges this and supports the class in practising this. They have the word written in syllables on the board: *de-nom-i-na-tor* and sometimes clap out the syllables as they say it. Talking to peers while ordering the strips allows Ellie-Mai to communicate mathematically in a way that doesn't make her feel anxious.

Mr Kulkowski displays all the fraction strips with the correct notation on the board in the order that the learners have decided.



He asks learners what they notice about the fraction notation, which they discuss in pairs. He invites Ellie-Mai and her partner to the board so she can point at the board and use gesture and single words to express her thinking. He prompts the class toward a generalisation by using the sentence stem, *"The greater the denominator…"* checking understanding of greater, before asking them to complete the sentence. This supports Ellie-Mai to complete the sentence independently.

Representation and Structure

Teachers design lessons to be inclusive by using carefully-selected representations, to reveal exactly what they want all learners to understand about the mathematical structure. These representations are used consistently across the class and over time, with teachers making explicit connections between them, the underlying mathematical structures, and the language embedded within each task. Learners interact with the representations, which supports their reasoning. Teachers employ strategies to assess whether or not all learners have understood these connections, ensuring that learning is secure.

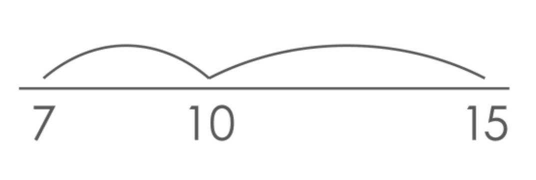
How maths is represented either prevents or provides access to unchanging mathematical structures. Concrete, pictorial and abstract (CPA) modes are different ways of representing mathematical ideas – it is not a hierarchical approach. Mathematical structure can also be expressed through language and gesture. The movement between the different representations is particularly important, allowing for explicit connections to be made.

Teachers select and use the same representation with all learners at the same time, building on the class's current understanding. Lessons are inclusive by design, using effective representations which reveal the structure for all.

Teachers do not use too many representations. The aim is not variety, but careful selection of representations to draw out mathematical structure.

Teachers deliberately choose simple numbers to allow learners to subitise easily, reduce cognitive load and make connections, before linking the same representation to more complex calculations.

Teachers plan for the withdrawal of representations. Representation should not be used to calculate, but to help learners see the mathematical structure. Teachers provide learners with sufficient time, repeated exposure to and opportunities to use the same representation, and teachers check learners' understanding of the concept within and across lessons, before removing a representation. Teachers consider developing and adapting representations to aid this withdrawal. For example, numbers other than 10 might be removed from a number line to discourage counting in ones.



Certain key representations persist throughout the curriculum, from Early Years to post-16, such as number lines and arrays. Teachers use these key representations consistently to connect early mathematical structures to later concepts.

Exemplification: Representation and Structure

Know your learner

Jayden is a Year 2 learner who displays several characteristics commonly associated with dyscalculia, although he has not received a formal diagnosis. He requires extensive practice to subitise and to secure number facts. He knows number bonds to 5, but has not yet developed fluency with number bonds to 10. Jayden lacks confidence in mathematics and often relies on counting in ones to calculate. He is enthusiastic about the subject and enjoys practical activities and talking about his learning.

Big Idea - Representation

Jayden's teacher, Mrs Shah, is teaching a sequence of lessons on adding three numbers. The intended learning for this lesson is:

> *The order in which addends (parts) are added or grouped does not change the sum (associative and commutative laws).*

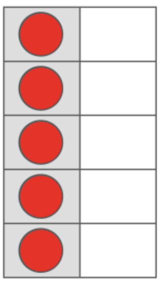
She begins with examples where the total is below 10 to focus on the commutative structure and the concept of '5 and a bit' (bridging 5). She plans to move on to bridging 10 in subsequent lessons.

Mrs Shah recognises that the **choice of representation** is crucial in supporting Jayden to understand the underlying mathematical structures. She thinks about:

- representations that Jayden is already familiar with
- how to reduce reliance on counting in ones
- how to represent the '5 and a bit' structure to support Jayden to bridge through 5
- how all learners will interact with the representation
- how learners need to know the composition of numbers within 5, in order to be able to bridge through 5. For example, if the learners are thinking about 2 + 1 + 3 in that order, they need to know that 3 is made of 2 and 1, if the '5 and a bit' structure is to be emphasised
- how to connect '5 and a bit' to '10 and a bit' using the same representation in future lessons
- the language and movement that will embody the mathematical ideas.

Teaching sequence

Mrs Shah chooses a 10-frame for this lesson. This enables learners to physically place counters themselves, and supports Jayden in seeing the structure of '5 and a bit'. She shades the columns of the 10-frame in different colours to encourage learners to complete the 5-column first, reducing the temptation to count in ones.



She teaches a downward sweeping gesture to indicate that the 5-column is full, accompanied by the phrase, *"I know there are 5 here."*

Mrs Shah begins with composition to 5 and then adds one more to make 6, helping Jayden make the conceptual link. She uses the language *"5 and __ more"* consistently.

To explore the composition of 6 and the commutative structure when adding three numbers, Mrs Shah asks learners to work in groups of three. To identify the three addends, she gives each of the three learners different-coloured counters. The first learner has 1 blue counter, the second learner has 2 yellow counters, and the third learner has 3 red counters. She monitors what the learners are doing, checking that they fill the 5-column first. She decides that physically changing the order of the numbers—by having learners swap places while holding their counters—is essential for making the structure visible.

Mrs Shah knows it's important for the learners to explore the effect of changing the order of the counters, and to make verbal generalisations about the structure by collaborating with their peers. Towards the end of the lesson, Mrs Shah talks through an equation that connects to the representation. This supports learners to explicitly connect the manipulatives, language and mathematical notation.

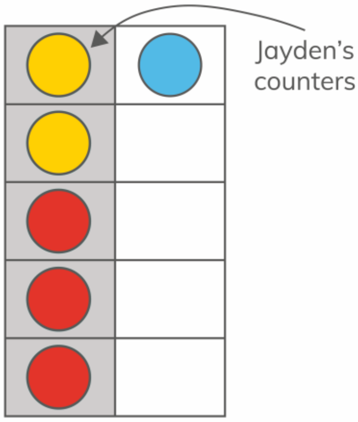
2 + 3 + 1 = 6

1 + 3 + 2 = 6

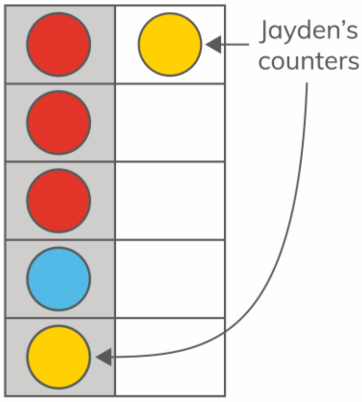
The different-coloured counters enable the learners to see that the order in which they have added their counters doesn't change the total.

The effect on Jayden's learning

Jayden is pleased to work with his peers and feels included in the lesson. By holding his two counters and placing them on the 10-frame himself, he engages more actively with the numbers. Once all counters are placed, he uses the downward gesture to show that the 5-column is full and says, *"I know there are 5 here. I can see 5 and one more."*



When the learners swap counters and repeat the activity in a different order, Jayden's peers have placed four counters in total before Jayden's turn. Jayden initially wants to place his two counters together in the next column. However, he remembers the gesture and separates them—placing one counter to complete the 5-column and the other in the next column. He again sees the structure as *"5 and one more"* and says this out loud.



Mrs Shah asks learners what they notice. Jayden explains that he had to separate his two counters so that one could complete the 5, leaving 'one more' in the next column to make 6. When asked to predict what would happen if the order changed again, Jayden suggests that they might need to separate the 3 next time, but it would still make '5 and one more, making 6.'

Repeating the same simple calculation with the same representation actively enables Jayden to see the mathematical structure clearly. He begins to understand both the '5 and one more' composition and the principle that changing the order of three addends does not affect the total.

Variation

Variation is not an end in itself. Variation is a means to highlight key features of mathematical structures and concepts.

Teachers design lessons to be inclusive, considering all learners' starting points and ensuring that the initial mathematical task is accessible to every learner. Subsequent conceptual steps are sequenced carefully, building upon and making explicit connections to prior learning. Teachers support all learners to communicate their understanding of these connections to deepen their understanding. This communication is facilitated through the use of appropriate strategies and resources, such as sentence stems, structured talk partner tasks, concrete manipulatives, and diagrams.

Teachers take a step back when designing lessons to think about learners' starting points, including learners with SEND. Teachers adapt planning to include all learners in the learning from the start, and learning is sequenced carefully to reveal mathematical structure. Teachers' knowledge supports them to select examples that 'point to' specific structures in a very explicit way.

One important plank of strong, explicit instruction is the use of non-examples. Non-examples test the limits and boundaries of a concept, as well as helping learners to think deeply about it. Teachers provide opportunities for all learners to compare examples and non-examples, and this variation supports learners to understand what a concept is and what it is not.

Teachers use purposeful variation to expose mathematical structure. Teachers can vary one or a small number of specific elements of examples and representations, so that learners are supported to notice key features and make connections. Learners learn to use variation to reveal structure when solving problems in unfamiliar contexts.

There can be a perception that the pace is too slow for some learners, but in fact, it benefits all learners to spend time on the detail of a concept. If key mathematical concepts are taught well the first time, less re-teaching is needed in the future. When key concepts are learned in depth, future learning on connected concepts is quicker.

Exemplification: Variation

Know your learner

Miquelle is a Year 10 learner who does not believe she can succeed in mathematics. She experiences significant anxiety in maths lessons, and is far more engaged in art, drama, and music. She regularly participates in school productions and hopes to pursue a performing arts course. She is aware that she needs a grade 4 in GCSE mathematics and frequently expresses doubt about achieving this. This contributes to heightened anxiety and often leads to disengagement during lessons. Miquelle has permission to sit her mathematics examinations in a separate room from her peers.

Miquelle feels that mathematics *"doesn't make sense."* She attempts to memorise procedures, but because she does not understand why they work, she finds them unreliable. For example, she can calculate the mean when data are presented as a simple list:

3, 2, 0, 0, 1, 4, 2, 1, 1, 1

However, she is unable to calculate the mean when the information is presented differently. For instance, she does not know how to begin a question such as:

The total number of goals scored by a football team in its first 10 games is 15. In the next two games, the goals scored are 4 and 2. Calculate the mean.

Big Idea – Variation

Mrs Smith, Miquelle's teacher, intends for all learners to:

Understand what the mean measures

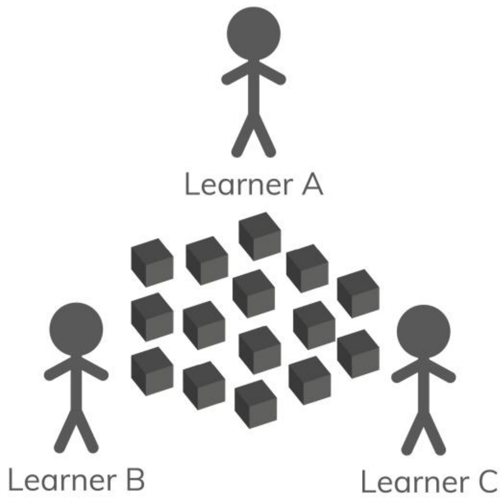
She anticipates that some learners may be able to perform the calculation without understanding its meaning. In this lesson, she designs the learning with the intention that all learners can both calculate the mean and understand what the calculation represents.

Mrs Smith thinks about:

- how to use concrete manipulatives to help learners explore what the mean represents (an equal share of the total across all elements of the set)
- how to use variation, so learners notice key characteristics of the mean (for example, if the total of a set of numbers remains constant, while individual values change, the mean remains unchanged)
- how to prompt deeper thinking about what the mean represents through carefully-planned questions for use in small-group and whole-class discussion
- how to group learners so that Miquelle will be supported to engage with the mathematics
- how to create time to listen to Miquelle's small-group contributions so she can be included in whole-class discussion where appropriate
- how to make Miquelle's learning visible to her, so she recognises what she has learned during the lesson.

Teaching sequence

Mrs Smith organises the class into groups of three, knowing that Miquelle is more likely to talk about the mathematics in a small, supportive group. Each group receives 15 cubes.



Each learner in the group is instructed to build a tower using some of the 15 cubes. Each person must have a different number of cubes in their tower. Mrs Smith says, *"This will mean each of your towers is a different height."* She exemplifies this task with cubes at the front of the classroom.

The group must then work together to redistribute the cubes from their towers so that all towers become the same height.

The group repeats this several times. Mrs Smith questions each group while they are working:

- What do you notice about the number of cubes each person has when the towers are the same height?
- What if one person starts with none? Does this change the number of cubes each person has in their tower when the towers are the same height?
- Why do you think the towers always end up with five cubes each, even though you all start with different amounts?

Mrs Smith notices that Miquelle shares some mathematical thinking in her group *("each person will always have five cubes, because fifteen divided by three is five")*. Mrs Smith tells Miquelle that she would like her to be ready to share this thinking with the whole class.

In the whole-class discussion that follows, Mrs Smith invites Miquelle to share the response that she gave in her group. Mrs Smith hopes that this helps Miquelle to know that her mathematical thinking in relation to the task is both valid and valued.

Mrs Smith then makes explicit links between the representation and the calculation for the mean, emphasising that the mean is the number of cubes each person has if the cubes are shared equally.

Mrs Smith then varies the task:

- Each group is given 3 additional cubes
- Now each group is given 6 additional cubes

Mrs Smith has a deep understanding of variation and so knows that she needs to use specific questions to prompt learners' thinking. She wants the learners to make the connection between the number of additional cubes and the change in the mean value.

She uses prompts such as:

"What do you think will happen to the mean number of cubes now that you have three more in total?"

"Why do you think this will happen?"

She provides time for the groups to explore these prompts, and then carefully sequences learners' responses in whole-class discussion, enabling her to explicitly connect the task, their observations, and the calculation for the mean.

To check understanding, she uses the following questions and prompts, chosen deliberately to highlight the intended learning:

Find the mean:

Question	**Prompt**
a) 6, 6, 6, 6	*"Is a calculation necessary? Why is the mean 6?"*
b) 6, 7, 5, 6	*"What has changed and what has stayed the same between parts a) and b)?"*
c) 12, 0, 0, 12	*"Why must the zeros be included in the calculation? What would the mean be if they were left out?"*
d) 4, 5, 7, 8	*"How do the numbers in part d) relate to the numbers in part a)?"*

She also asks questions such as, *"if you start with 6, 6, 6, 6, and add a value to one number and subtract the same value from another number, why doesn't the mean change?".*

The effect on Miquelle's learning

Working in a small group helps Miquelle feel supported, and reduces the pressure she associates with mathematics. Talking about the task feels less stressful than listening to an explanation and then attempting a method independently. As the lesson progresses, she becomes more confident, particularly as she begins to connect the calculation for the mean with the idea of equally distributing an amount.

She feels excited when she can visualise the numbers in parts (a) and (d) as towers, immediately recognising why both sets have a mean of 6. Miquelle leaves the lesson feeling that this is something she can do in mathematics, which helps her approach the next lesson with greater positivity.

Fluency

Teachers create a classroom culture where all learners know that thinking about the maths is valued over speed. Planning inclusively to support all learners to develop fluency requires both processing and thinking time to be embedded.

Fluency isn't about rote learning or speed; learners need to understand the ‘why’.

Learners must have a deep understanding of the mathematical structures that underpin mathematical facts. A learner demonstrates fluency when they combine both knowledge of mathematical structure and key facts to calculate efficiently.

Mathematical facts include part-part-whole relationships within 10, and the multiplication tables. Learners take time to achieve a deep understanding of these facts, so that they can recall them fluently. Knowing facts fluently frees up working memory, reducing cognitive load when learners are learning new concepts. Using those facts to ask, 'what do I know?' before beginning a problem supports learners to think about mathematical structure and connections, and leads to flexibility and efficient methods.

Talking about mathematical structure, seeing and manipulating representations, and gesturing can all help to embed facts in the long-term memory for all learners. Repetition contributes to learners' understanding and retention of mathematical facts, and practice needs to happen in both the short, medium and longer term.

When learners experience difficulty in learning mathematical facts to automaticity, teachers return to the conceptual understanding underpinning the fact. Teachers assess learners’ starting points and build conceptual understanding from these, supporting all to learn key facts to automaticity.

Learners are encouraged to derive facts from known facts, and to apply facts flexibly and in different contexts. Questioning prompts all learners to reflect on mathematical strategies and to notice mathematical structure, leading to efficient methods.

Exemplification: Fluency

Know your learner

Grace is a 17-year-old learner in an FE college, studying a Level 1 Travel and Tourism programme. She attends GCSE Mathematics as part of her study programme and is working towards a grade 4.

Grace enjoys the practical and vocational aspects of her course. She particularly enjoys learning about destinations and planning journeys, and she contributes confidently during discussions in her vocational lessons. However, she often describes herself as *"not a maths person"* and sees mathematics as something she has never been particularly successful at.

Grace has a slow processing speed and benefits from additional time to think about new ideas before responding. She can become overwhelmed when asked to hold too much information in her working memory at once, particularly when working mentally. She sometimes rushes because she worries about falling behind other learners, which can lead to errors and frustration.

Like many learners in the group, Grace has experienced repeated difficulties in mathematics. She believes that being good at mathematics means being quick, and often compares herself with other learners. As a result, she can focus on getting answers rather than understanding the mathematical structure that lies beneath them.

Grace can read, write and compare whole numbers and decimals. She can carry out calculations involving place value in familiar situations, but does not yet have a secure understanding of the multiplicative relationships between place-value columns. For example, she knows that 0.4 is greater than 0.35 but finds it difficult to explain why. She often focuses on the digits themselves rather than the value those digits represent.

Big Idea – Fluency

Perry, Grace's teacher, intends for all learners to:

> *"understand and use place value (e.g., when working with very large or very small numbers, and when calculating with decimals)"*

Perry is teaching a GCSE maths resit class. He knows that fluency is not about speed or rote learning. He wants all learners to develop a secure understanding of place value so that they can reason about numbers, make efficient decisions and apply their understanding across the GCSE curriculum.

Perry knows that many learners in the class, including Grace, can perform procedures without always understanding the mathematical structures that underpin them. He wants learners to see place value as a connected system rather than a set of rules to remember.

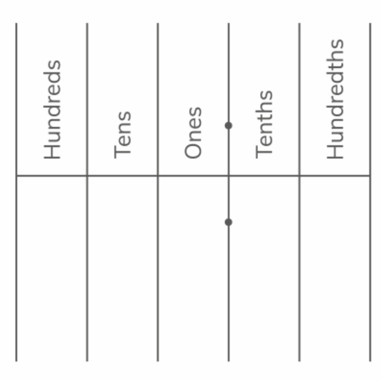
Perry thinks about:

- the multiplicative structure of the place-value system and how best to represent it
- what learners already understand about place value and where misconceptions may arise
- how to reveal mathematical structure through carefully chosen representations
- opportunities for learners to discuss and explain their thinking
- supporting learners' sense of magnitude and value
- how more secure learners can be supported to make connections with related areas of mathematics, such as standard form, measures and scale
- reducing cognitive load without reducing mathematical challenge

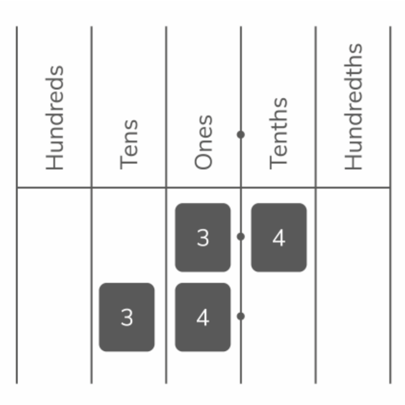
- creating a classroom culture where understanding is valued over speed
- ensuring every learner has sufficient thinking time to participate fully.

Teaching sequence

Perry gives each pair of learners a place-value chart and a set of digit cards.



He begins by asking learners to make the number 3.4 and then the number 34.



Learners discuss what has changed and what has stayed the same.

Perry asks:

"What is the value of the 3 in each number?"

and

"Why has the value changed even though the digit is the same?"

He provides time for learners to think individually before discussing their ideas with a partner. During whole-class discussion, he carefully sequences responses so that learners focus on the value of the digits rather than their position alone.

Learners then build and compare a series of numbers:

- 0.4 and 0.04

- 3.5 and 35
- 2.7 and 0.27
- 4.03 and 4.3

Perry asks learners to identify what has changed and what has stayed the same in each example.

He introduces and models stem sentences such as:

"The digit has stayed the same, but its value has changed because..."

and

"This digit is worth ten times as much because..."

Learners use the place-value charts to justify their responses and challenge each other's reasoning.

As understanding develops, Perry gradually removes the physical charts and encourages learners to visualise the structure of the number system. He revisits the representations whenever learners need support, ensuring they remain focused on the mathematical structure rather than memorised rules.

Towards the end of the lesson, Perry asks learners to predict the effect of multiplying and dividing by 10, 100 and 1000 before testing their ideas. He deliberately selects and discusses misconceptions, encouraging learners to explain why an answer makes sense. For learners who are secure with these ideas, Perry draws attention to how the same multiplicative structure underpins later work on standard form, measures and scale.

Throughout the lesson, Perry uses questioning to draw attention to structure, value and relationships within the number system. He is careful to value explanation and reasoning over speed of response.

The effect on Grace's learning

Grace benefits from the place-value chart because it makes the structure of the number system visible. The representation reduces the demand on her working memory and allows her to focus on understanding rather than remembering.

Working with a partner gives Grace time to rehearse and refine her thinking before contributing to whole-class discussion. The use of stem sentences helps her explain ideas that she previously struggled to articulate.

As the lesson progresses, Grace begins to focus less on getting answers quickly and more on understanding why they work. She starts to recognise that digits have different values depending on their position within the number.

By the end of the lesson, Grace can explain that the 4 in 0.4 is worth ten times as much as the 4 in 0.04 and can justify her reasoning using place-value language. She is beginning to see place value as a connected structure, rather than a collection of rules to remember.

The lesson design also benefits the wider group. Learners who could previously carry out procedures without explanation are supported to make connections between representations, language and notation. More secure learners are encouraged to see how the same place-value structure connects to later work on standard form, measures and scale. Perry continues to look for opportunities in future lessons to make these connections explicit, so that learners experience place value as a connected structure across the mathematics curriculum, rather than as a separate topic. The emphasis on understanding rather than speed helps more learners engage successfully with the mathematics and develop deeper fluency.

Coherence

Teachers know where learners have gaps in their learning and use this to plan lessons that are inclusive by design, so that all learners can access the learning and can then develop mathematical understanding further through carefully-sequenced steps.

The mathematics curriculum rests upon a small number of interlinked mathematical concepts that learners come to understand deeply over time. Coherence is about how the learning builds in ways that makes sense to learners.

Teachers know their learners well. They know what mathematical concepts and structures learners understand securely, and so plan lessons that are close and connected to this point of learning. Lessons are accessible to all learners, so all learners can engage and participate fully in making sense of the mathematics. Teachers scaffold the learning to enable all learners to understand and connect ideas together, by using consistent language and representations across the curriculum.

Teachers constantly check learners' understanding and slow down where necessary to ensure that every learner is ready to progress.

Teachers understand how today's learning is linked to the learning that has gone before and have an eye on where it is going. Teachers understand how to teach key concepts in at least the previous Key Stage.

Coherence is particularly important where learners may have incomplete understanding of mathematical structures. Any teaching that learners receive is sequenced and planned carefully, particularly when the content is outside their current year group's curriculum. Teachers also need to be aware when learners are completing calculations correctly, but do not have understanding. For instance, teachers notice when learners are counting in ones to subtract two-digit numbers and adjust their teaching sequence to address the key concepts needed.

Exemplification: Coherence

Know your learner

Hassan is a Year 7 learner who has been placed in a small class where his teacher, Ms Andrews, is using the Securing Foundations at Year 7 materials to ensure that all learners secure key mathematical concepts before progressing further. Hassan has some calculation skills, and he is proud of knowing his multiplication tables and being able to use formal written methods. However, Ms Andrews has identified significant gaps in his underlying understanding. Hassan is aware that he often struggles with reasoning questions, even when he can perform the calculations.

Using the assessment questions at the start of the Securing Foundations Deck 3, Ms Andrews now wants to find out more about what Hassan understands.

Assessment

a) Sally buys 12 cinema tickets. Each ticket costs £4. How much does Sally spend? Write the multiplication expression and calculate the cost.

b) Fill in the missing numbers.

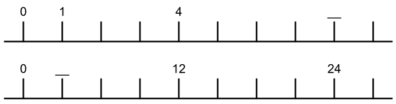
$7 \times 3 = \_$

$7 \times 30 = \_$

c) Miss Robinson asked Harry to get 80 apples from the kitchen. The apples come in bags of 10. How many bags does Harry need to get?

d) Joe want to use 54 paving slabs to make a patio. Each row needs to be 9 slabs wide. How many rows deep can Joe's patio be?

e) Fill in the missing numbers on the double number lines:



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She uses the **ready-to-progress criteria: Year 1 to Year 6** for the multiplication and division strand to track back into the primary curriculum, ensuring she has a clear understanding of the key concepts taught in Key Stage 1 and Key Stage 2.

She allocates part of the lesson to a diagnostic assessment with Hassan and a small group of learners. She provides them with a set of problems to solve collaboratively, listening carefully to their discussions and observing their approaches. At selected moments, she probes their thinking in greater depth.

Through this process, Ms Andrews learns that although Hassan knows his multiplication tables securely, he does not connect this knowledge to multiplicative word problems. When he encounters a problem that he is unsure how to solve, he tends to rely on step-counting or sharing items one by one. He is not unitising, and he does not attend to the unit as he step-counts or interprets word problems.

Big Idea – Coherence

Ms Andrews reflects carefully on the assessment of Hassan's learning. She recognises that the multiplicative structure is one of the main structures in mathematics and is a key concept within the Key Stage 3 curriculum. Identifying that Hassan has a significant gap related to understanding the unitised group and thinking multiplicatively, she decides to address this explicitly.

She knows that learners in her class understand different aspects of multiplicative structure, and that some already have some understanding of unitising. However, she also recognises that this is such a critical area, that most learners would benefit from consolidation and from making their knowledge more explicit. She is confident that once learners develop a deeper understanding of unitising, they will be able to build on this to learn the more complex concepts that follow later in the curriculum.

Ms Andrews therefore identifies the following intended learning:

> *When describing equally grouped objects, both the number of groups and the size of the groups must be defined.*

She has a secure understanding of how learners' knowledge of multiplication and division develops from Key Stage 1 onwards, and she considers the coherence of the curriculum carefully. Ms Andrews reflects on how learners might grasp different aspects of the multiplicative structure and how this is likely to affect Hassan and his peers. She thinks about:

- linking concepts in a connected way, so that Hassan is clear about the mathematics at every stage:
  - naming the unit being used (e.g., a bag) and talking about it with precision.
  - connecting the named unit to multiplication expressions and the double number line.
  - connecting a concrete example of a unitised group (such as a bag with marbles in it) to a more abstract one (such as a unit counter or an ill-defined pile of objects)
- building the concept of defining the group and relating this to multiplication and division facts.
- using a representation of the functional relationship (the relationship between the number of bags and total number of marbles) and so she selects a double number line
- checking Hassan's understanding at key points in the lesson, such as ensuring he understands what the double number line represents.
- how important this concept is for future learning, therefore justifying the additional time spent on it.

Teaching sequence

Ms Andrews considers the first section of Deck 3, which focuses on **unitising**. Instead of relying on the slides only, she decides to ensure that the learning is supported using concrete manipulatives. She decides to use familiar physical objects, ensuring that learners have a concrete unitised group they can name, handle, and reason with. She considers the mathematical language she wants learners to use and recognises the importance of encouraging explicit, precise explanations.

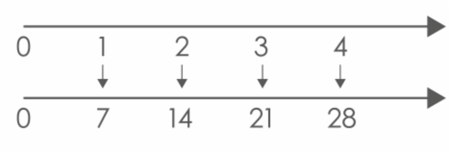
Ms Andrews decides to use a bag with marbles in it, and while one marble can be seen as a unit, in this lesson, she wants learners to understand that a bag with marbles in it, is the group unit. Ms Andrews can then support the learners to think about the relationship between the number of bags (the group unit) and the total number of marbles.

She gives each learner a bag containing seven marbles and asks them to describe the unit in front of them. Learners respond that they have *"a bag of marbles."* Ms Andrews introduces the stem sentence:

"One bag has 7 marbles, so __ bags will have __ marbles."

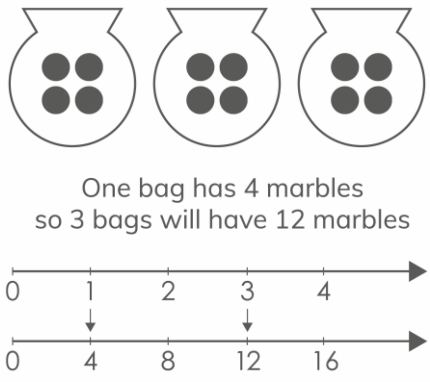
This directs learners' attention to the functional relationship rather than encouraging step-counting. She provides additional bags and marbles and encourages learners to repeat the stem sentence as they work.

She then represents this relationship on a double number line so that learners can see the functional relationship between the number of bags and the number of marbles.



Hassan is able to link his knowledge about times tables to the double number line.

Ms Andrews repeats the process using bags containing different numbers of marbles, each time emphasising the functional relationship and linking it to the corresponding written multiplication calculation.



She then uses the image of tennis balls from the slide deck and asks learners to identify the unit.

Unitising: Many as one

Mrs Bryce has put the tennis balls into equal groups.
How many equal groups does she have?

? How could you work out many tennis balls Mrs Bryce has?

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She uses the same stem-sentence structure to check their understanding of what constitutes a unit. She follows this by asking learners how many units are shown in the picture and represents this using unitised counters.

The effect on Hassan's learning

Hassan responds positively to the practical nature of the lesson. He can recognise the **unit** as the bag because he can hold it in his hand, name it, and refer to it with confidence. Previously, he was unsure what the 'group' represented or what exactly he was counting and only paid attention to the individual objects, rather than the unitised group. The concrete objects help him connect the idea of a unit to what he already knows about multiplication facts, enabling him to talk more clearly about units and groups. Hassan's thinking has shifted to recognise that this bag with marbles in it now operates as a unitised group.

Hassan frequently worries that he does not understand mathematics lessons and that he is falling behind his peers. He feels more confident because Ms Andrews listens carefully to what he says and encourages him to ask questions. She checks his understanding at each stage and does not accept vague responses such as *"I worked out that it was 14 in my head."* Hassan knows that she will provide multiple examples of word problems and will not move on until he has understood the concept securely.