KS3 Progression Map: Algebra

This progression map expands upon the statements of subject content in the DfE document *Mathematics programmes of study: Key Stage 3* published September 2013. Suggested allocation of material to Years 7, 8 and 9 is given as starting points for writing schemes of work, but the implicit chronology is not intended to be prescriptive or restrictive; indeed, the programme of study is explicit that "Decisions about progression should be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content in preparation for key stage 4. Those who are not sufficiently fluent should consolidate their understanding, including through additional practice, before moving on". The NCETM fully endorses these principles, and will be developing further this progression map to help teachers achieve them.

Furthermore, although the map is organised by content, this is only for ease of reference and use. In the classroom, links between topics on the map, and between different maps, should be looked for and explored at every opportunity, so that "by the end of Key Stage 3, pupils … know, apply and understand the matters, skills and processes specified". Throughout Y7-9 pupils should have regular and opportunity and developmental feedback that helps them to **develop fluency**, to

- consolidate their numerical and mathematical capability from Key Stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]

- use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability and statistics;

to **reason mathematically**, to

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally;

and to **solve problems**, to

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

The NCETM will be developing further resources to support the development and embedding of these skills.

Year 7	Year 8	Year 9
Notation and vocabulary		
use and interpret algebraic notation, including: ab in place of $a \times b$ 3y in place of $y + y + y$ and $3 \times y$ a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$ $^a/_b$ in place of $a \div b$ brackets		use and interpret algebraic notation, including coefficients written as fractions rather than as decimals
substitute positive integer values into formulae and expressions, including scientific formulae	substitute integer values into formulae and expressions, including scientific formulae	substitute numerical values into formulae and expressions, including scientific formulae
understand the correct and incorrect use of '='; understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors	understand and use the concepts and vocabulary of expressions, equations, inequalities, terms, factors and correlation / covariation	understand and use the concepts and vocabulary of expressions, equations, inequalities, terms, factors, correlation / covariation and parameters
Manipulation		
simplify and manipulate algebraic expressions to maintain equivalence by: - collecting like terms - multiplying a single term over a bracket	simplify and manipulate algebraic expressions to maintain equivalence by: - taking out common factors - expanding products of two or more binomials	think about relational meanings before acting on expressions, such as recognise situations in which different ways of seeing the situation lead to equivalent expressions, and use manipulation and simplification to show that the expressions are equivalent (e.g. sequences of "dot patterns")
understand and use standard mathematical formulae	rearrange formulae to change the subject	recognise situations in which it is helpful to rearrange formulae to change the subject, and explain why it is helpful
use algebraic methods to solve linear equations	use algebraic methods to solve linear equations	use algebraic methods to solve linear equations

in one variable	in one variable (including all forms that require rearrangement)	in one variable (including all forms that require rearrangement) that express facts observed in situations, and interpret the solution
Expressing and exploring relations: functions and graphs		
work with coordinates in all four quadrants	understand how the position of a point changes if one or both of its coordinates are multiplied by ⁻1	know the relationship between the coordinates of two points when each point is the reflection of the other in the *y*-axis, the *x*-axis, the line $y = x$ or the line $y = -x$
model simple situations or procedures involving two variables by translating them into linear algebraic expressions or formulae and by using graphs	model situations or procedures by translating them into algebraic expressions or formulae and by using graphs	relate changes in situations or procedures to changes in algebraic expressions, formulae or graphs
produce graphs of linear functions of one variable with appropriate scaling, using equations in *x* and *y* and the Cartesian plane	recognise, sketch and produce graphs of linear functions of one variable with appropriate scaling, using equations in *x* and *y* and the Cartesian plane	recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in *x* and *y* and the Cartesian plane
interpret simple linear mathematical relationships, such as *y equals 5 times x* or *p is 3 more than twice q*, both algebraically and graphically	interpret linear mathematical relationships, such as *A plus 7 is 6 less than half of B* or *three-quarters of x is 3 times one more than half y*, both algebraically and graphically	interpret linear and quadratic mathematical relationships, such as *(P subtract 1) halved is 6 times Q plus 10* or *A equals the positive-square-root of (B plus 1)*, both algebraically and graphically
	reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically	reduce a linear equation that expresses a relationship between two variables in a situation to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of

	and algebraically	such linear equations numerically, graphically, algebraically and in the situation
use linear graphs to estimate values of *y* for given values of *x* and vice versa	use linear and quadratic graphs to estimate values of *y* for given values of *x* and vice versa and to find approximate solutions of simultaneous linear equations when at least one equation is of the form *y* = k or *x* = k	use linear and quadratic graphs to estimate values of *y* for given values of *x* and vice versa and to find approximate solutions of simultaneous linear equations
from given linear graphs find approximate answers to simple contextual questions	from given linear graphs find approximate solutions to contextual problems	find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs
Sequences		
generate terms of a sequence with a simple linear position-to-term rule (such as 'an expression for the value of the *n*th term is n + 2') from either the term-to-term or the position-to-term rule	generate terms of a sequence with a linear position-to-term rule from either the term-to-term or the position-to-term rule; begin to generate terms of a sequence from a quadratic position-to-term rule	generate terms of a sequence from either a term-to-term or a position-to-term rule
	recognise arithmetic sequences and find an expression for the value of the *n*th term	recognise geometric sequences and appreciate other sequences that arise