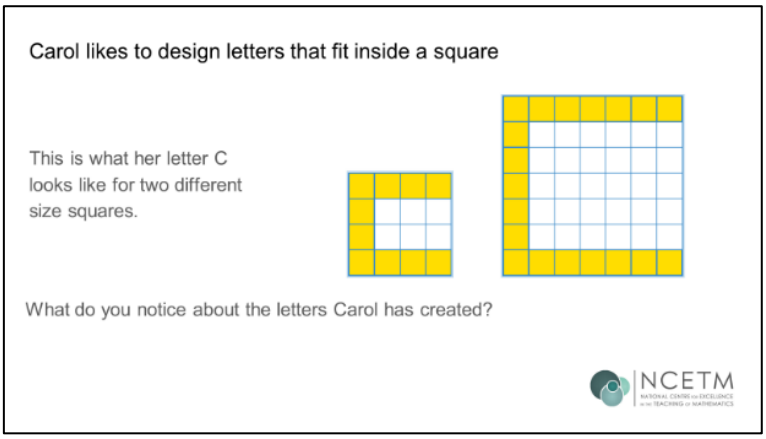
Letters

This sequence of tasks is designed to encourage pupils to generalise about shaded squares making letters on a grid and to express their generalisations in different ways; for example, by creating further shapes that follow the rule or by means of a rule expressed verbally or with numbers. The aim of this resource is to compare these generalisations and to show that they are equivalent.

Activity overview

The tasks involve creating letters by shading small squares within a square grid. Pupils are encouraged first to explore the letters, to draw them for themselves and then to create generalisations to predict the number of small squares which would be shaded in different sized squares. The tasks then progress to look at different letters and pupils develop the initial generalisation for the letter, for the other letters presented, and for letters they design themselves.



Rationale

The aim of this sequence of activities is to explore patterns, describe them, and come up with generalisations. The discussions should support pupils to see the structure of the letters and to use that structure to form generalisations. This thinking will lead to the use of more formal algebraic representations but this is not the focus of these activities.

If pupils are already confident with using letters to represent variables, then they may naturally go to that recording, but it is ok if they don't at this stage.

The focus of pupil learning and thinking is to describe the way that the letters have been constructed and use this to support generalisations about how many shaded squares will be needed to create the letters within larger squares and then within a square of any size.

Language and representations

Pupils are encouraged to create the shapes within squares marked out on squared paper.

Handouts of the letters within squares are available in Worksheets 1, 2 and 3.

Transition issues

Whilst the use of symbols and letters to represent unknowns and variables is mentioned in the Year 6 National Curriculum, and pupils have been familiar with the use of 'empty boxes' in calculations, they will not be used to using letters and symbols when creating generalisations. The aim of these activities is not to teach this, but to allow pupils to develop the thinking which will support understanding of this in the future.

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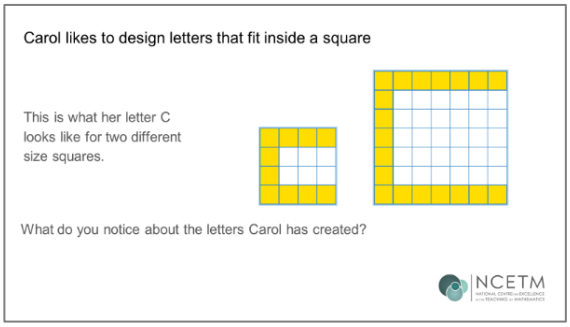
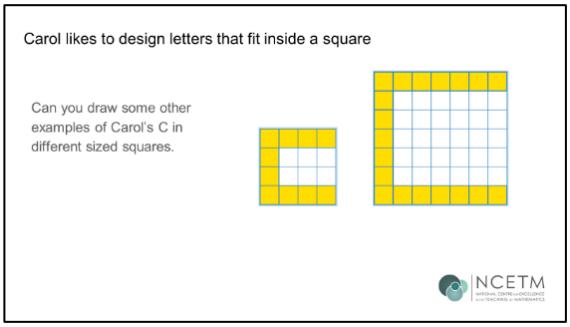
Task 1: Carol's C

Share the pictures of Carol's C shapes in squares and ask pupils what they notice about the letters. **Worksheet 1** has copies of the C shapes for pupils to use.

Slides 3 and 4	Commentary
Slide 3  	Introduce pupils to the notation of 4 by 4 or 4 × 4 to describe the size of the squares they are working with. Introduce stem sentences to support pupils in articulating their thinking: In the 4 × 4 square there are ___ yellow-shaded squares. In the 7 × 7 square there are ___ yellow-shaded squares. Pupils may make reference to the pictures they have created to explain how they know the number of shaded squares. Repeating this language will support them when it comes to Task 2, which encourages generalisation.
Slide 4  	The 4 × 4 and 7 × 7 starting squares have been chosen deliberately to encourage pupils to think about the structure of the shapes rather than to think of a sequence. Slide 4 invites them to draw other examples of Carol's square and think about the number of yellow-shaded squares needed in each case. The following stem sentence can be adapted for this slide: In the _ × _ square there are ___ yellow-shaded squares.

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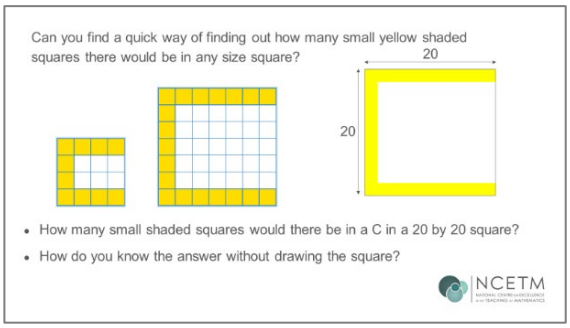
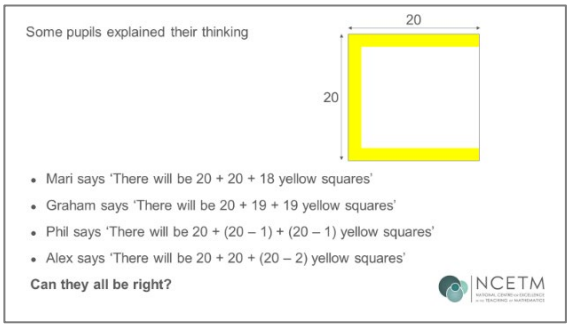
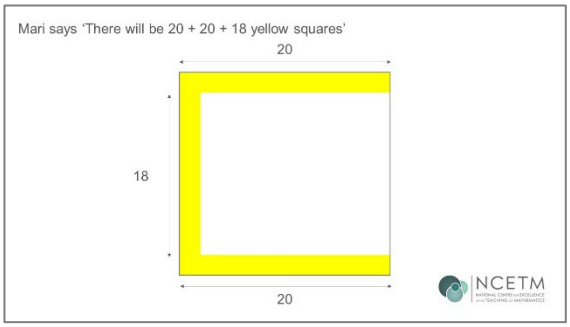
Task 2: Finding the rule

The aim of this task is to encourage pupils to extend their thinking from the first activity and think of a way of working out how many shaded squares are needed without drawing and then counting.

Worksheet 2 has images of the unmarked 20 × 20 square for pupils to annotate.

Slides 6–17	Commentary
Slide 6  	This task starts by looking at a 20 by 20 square but the image has no small squares showing. Ask pupils to picture the bigger square: *'How many yellow squares will you need to shade to make the C shape in a 20 × 20 square? Explain how you know.'*
Slide 7  	The second part of this task involves comparing different explanations of how many yellow squares are shaded in the 20 × 20 square. Four explanations are presented on slide 7. *'Can all the explanations be correct?'*
Slides 8, 9, 11 and 12  	Slides 8, 9, 11 and 12 present the four explanations. Slide 8 is shown here as an exmple. • Mari and Graham's explanations relate directly to the 20 by 20 square. • Alex and Phil present the explanation in a way that can be adapted to other sizes of squares, as the '20' can be substituted for other values. This will be explored later in the task when we consider a 100 by 100 square. Another strategy that might be mentioned in discussion involves subtracting the unshaded area from the whole square. Slides 10 and 13 are hidden but show how Alex and Phil have cut up the C to come up with their generalisation.

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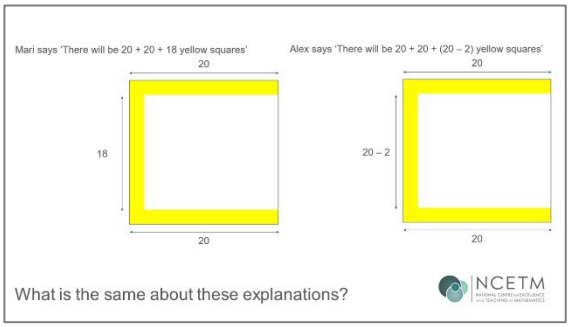
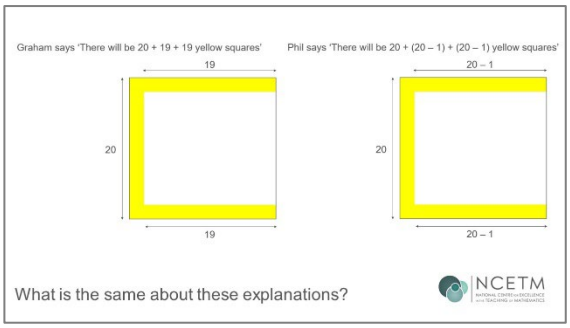
Slides 14 and 15



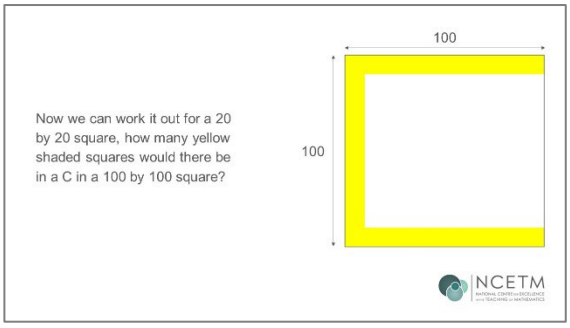


These slides invite discussion about the similarities and differences between the explanations by Mari and Alex and then Phil and Graham.

Pupils should be encouraged to see that Mari and Alex's explanations are the same … [Slide 14] … and that Graham and Phil's are also linked [Slide 15].

This thinking will lead into representing the variable size of the square with a letter, but that is not essential in this activity. If it is suggested by the pupils, then it can of course be explored.

Slide 16



Now that pupils have looked at the 20 by 20 square, ask them to think about a 100 by 100 square.

'How many yellow squares would be needed to make the C shape?'

'Can you use the way you worked out the number of yellow squares for 20 to help you?'

'If you have found a different way, could you use the way of thinking about the 20 by 20 square to work out the number of yellow squares needed for a C shape in a 1000 by 1000 square?'

Allow pupils time to think about this. They may be able to extend the generalisation for 20 by replacing the 20 with 100.

Don't be surprised if they come up with a new generalisation. This may indicate that they aren't yet aware that having one generalisation to suit all criteria for this shape is useful and desirable.

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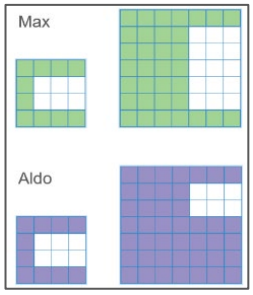
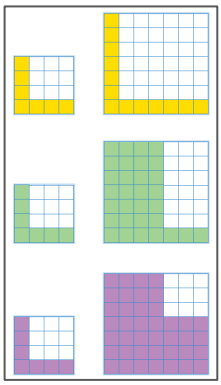
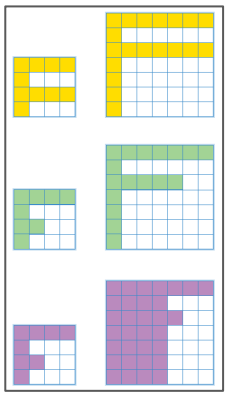
Task 3: Exploring other letters in squares

At this point we want to encourage pupils to apply the thinking they have explored with the C in a square to other versions of the C shape and then to other letters. Pupils may want to design their own letters and create rules for them in the same way as they did with the first C pattern.

Slides 19 and 20	Commentary
Appendices 3 and 4   	Ask: *'What is the same and what is different in the letter shapes?'* Encourage pupils to explore the new letter shapes in squares and for each answer the following questions: • How many small shaded squares would there be in a 20 by 20 square? • How many small shaded squares would there be in a 100 by 100 square? • How many small shaded squares would there be in any size of square? Pupils may want to start by drawing the shapes. You could also give them print-outs of the shapes (Worksheet 3), which are included for pupils to annotate to support their thinking. The aim is for pupils to give a generalisation that would allow them to work out the number of shaded squares needed to create an L, F or alternative C for any size of larger square. Pupils may begin to refine these into algebraic expressions if they are familiar with the use of letters and brackets.

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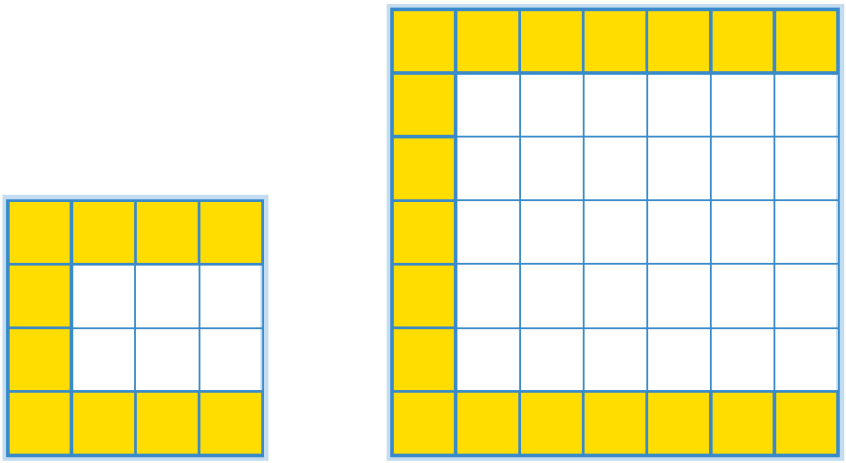
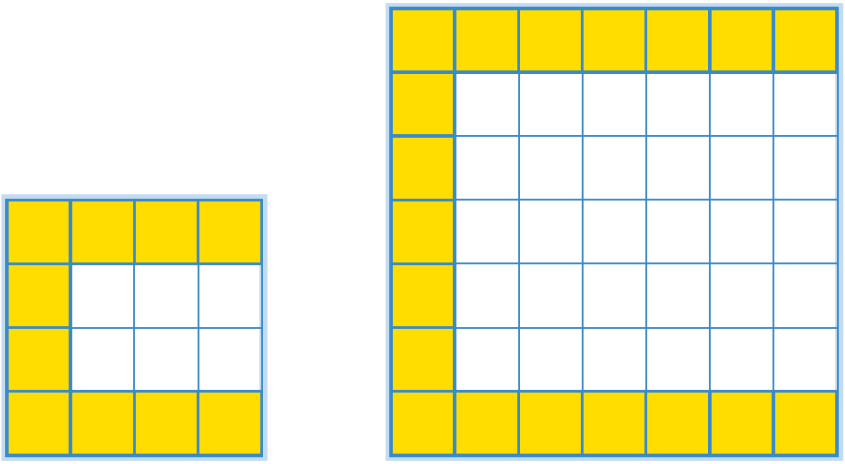
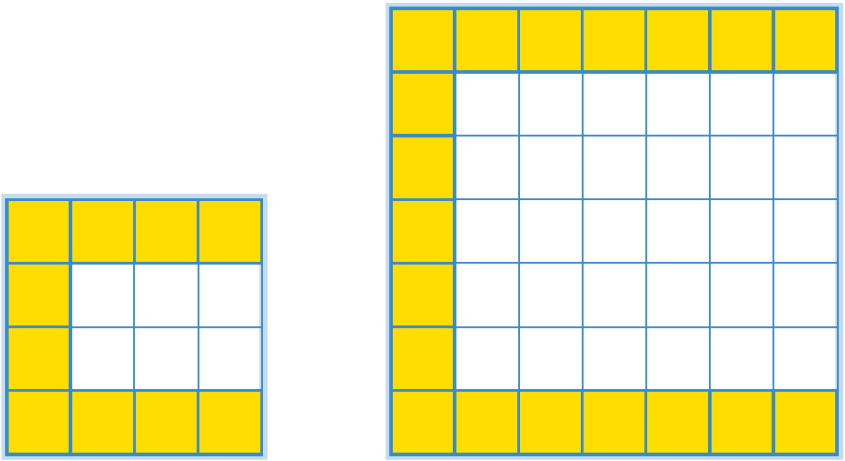
Worksheet 1







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Worksheet 2

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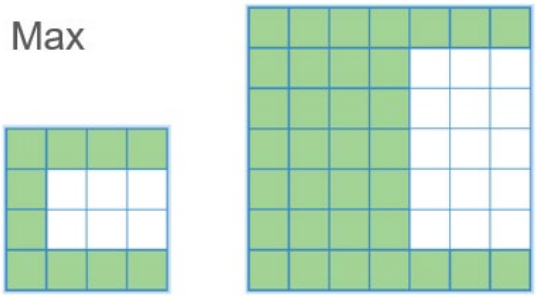
Worksheet 3 (2 pages)

Max



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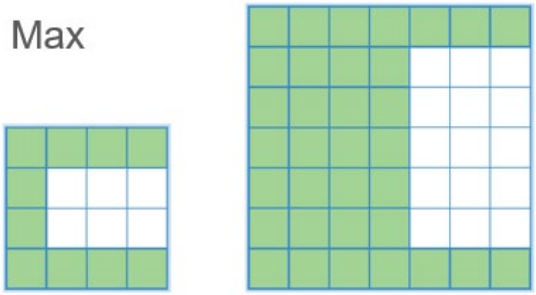

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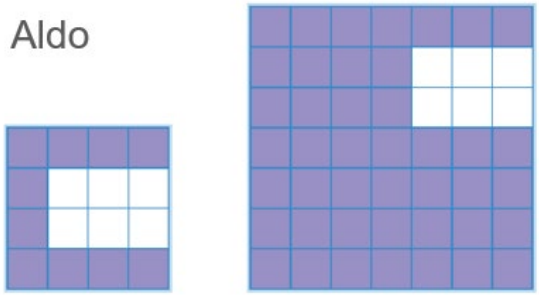

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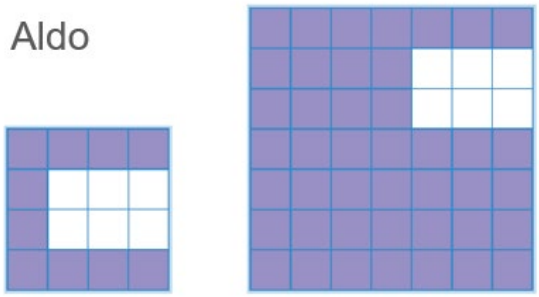

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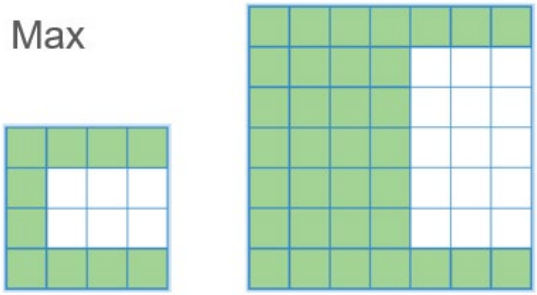

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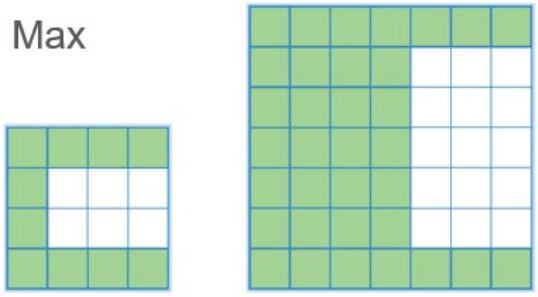

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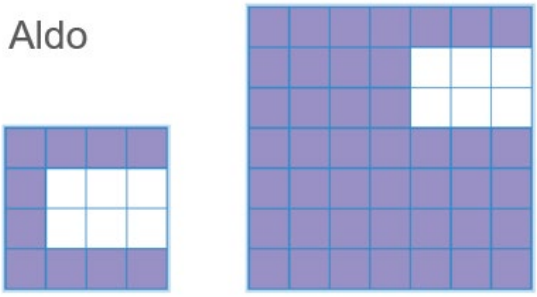

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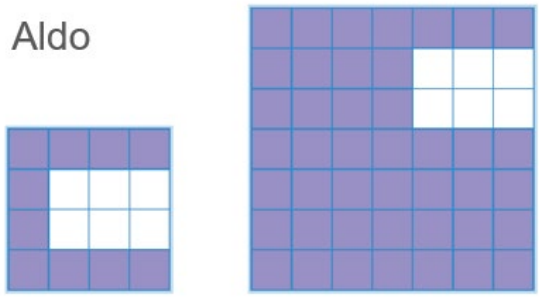

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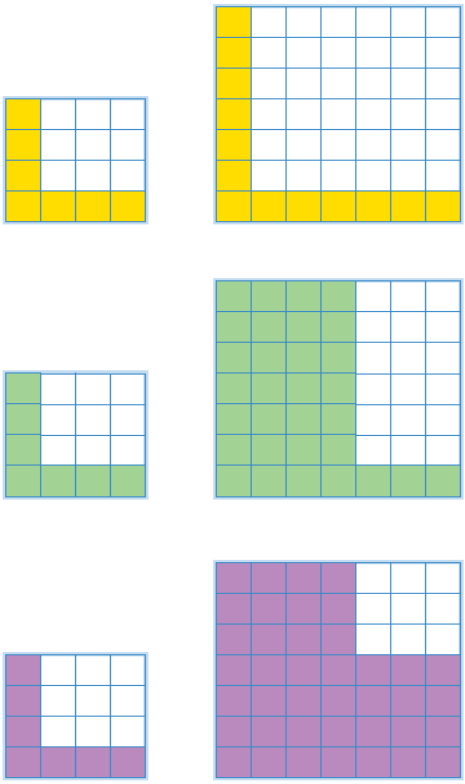
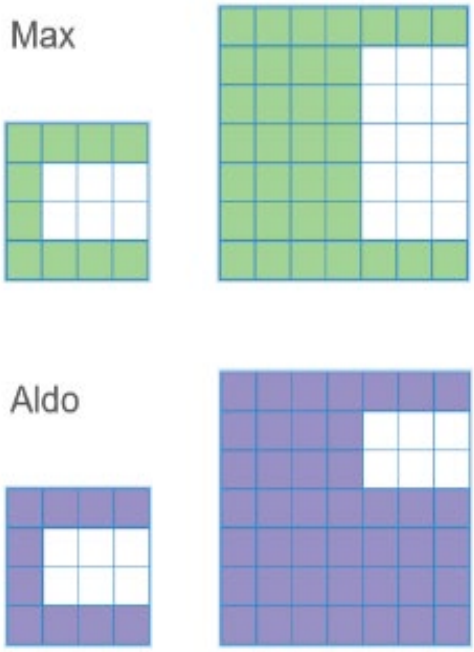
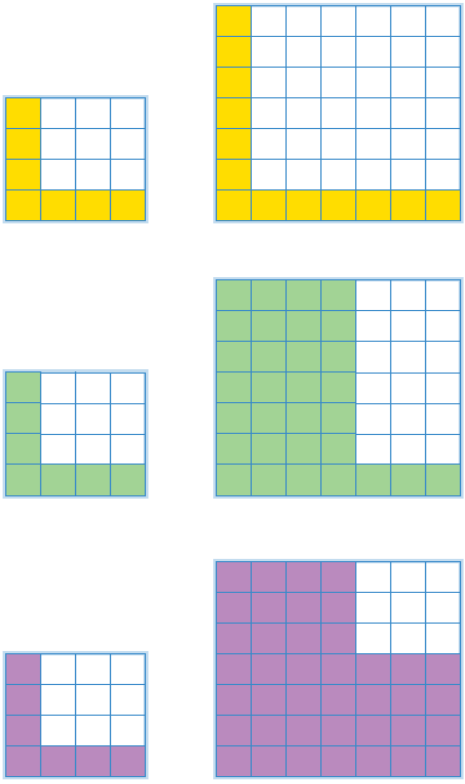
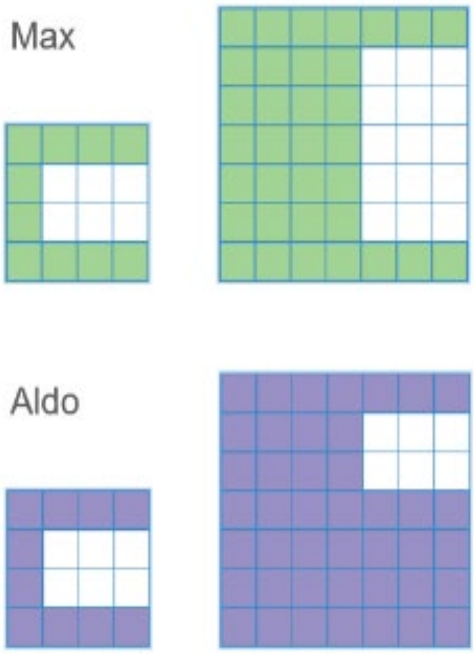
Worksheet 3









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