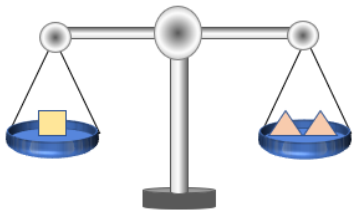
Balance Beam

This sequence of activities provides opportunities for pupils:

- **to develop their understanding of the equals symbol**
- **to engage with the notion of numerical variable**
- **to become more familiar with the Cartesian graph.**

Activity overview

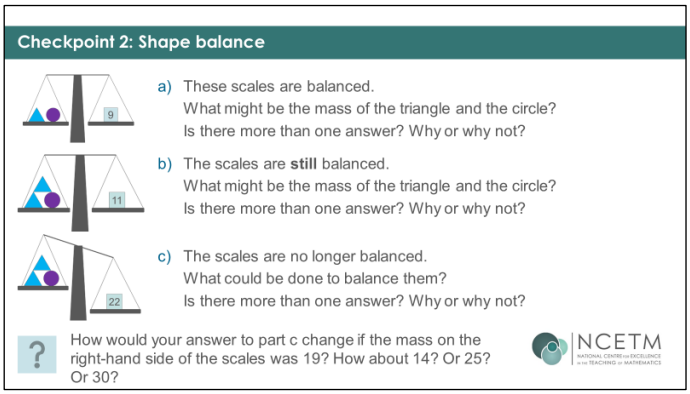


The tasks are set in the context of a balance beam with weights placed in the two pans, usually with one weight that is **known** in one pan and two weights that are **unknown** in the other pan. Pupils are asked to consider possible pairs of values for the unknown weights and to represent these as points on a graph. In these particular tasks, it turns out that the points lie on a straight line or in a region bounded by a straight line.

It will be important to check that pupils are familiar with balance beams and can relate them to reflecting the relative value of numbers. It is worth spending time discussing the initial slides to make sure that pupils are confident with the mathematics that the balances represent.

Inequality symbols are used to contrast with equality and this gives the opportunity to explore what equalities and inequalities look like when plotted on a graph. This is beyond KS2 expectations but is a good introduction to the use of line graphs to show numerical relationships as well as to display data.

This NCETM Checkpoints activity would be a good introduction to these resources:



Language and representations

As the focus of this activity is to develop a deeper understanding of equality, it is recommended that throughout the teaching sequence the phrases '*are balanced with*', '*balances*', '*is the same as*', and '*is equal to*' should be used. It is of value for this shift of language to happen not only during this sequence of lessons, but in every maths lesson, so pupils are consistently exposed to a deeper meaning of this symbol.

In addition, it would be helpful to draw attention to the balancing of the beam as indicating equilibrium, with sentences such as '*The beam is now balanced and both pans have the same value.*' The balance beam is a powerful representation of equality as there is an instant shift in the beam when it is balanced or imbalanced.

Rationale

The materials provide pupils with an opportunity to become more familiar with plotting points in the Cartesian plane and to seeing some of the ways in which a graph can show the structure of numerical

© Crown copyright 2023

relationships. We have decided not to use x and y on the axes in these slides, instead labelling them as □ and △ for ease of recognition for our pupils.

For a relation of the form x + y = c (where c is a specific number, e.g. 9), pupils get the chance to observe and think about phenomena such as these:

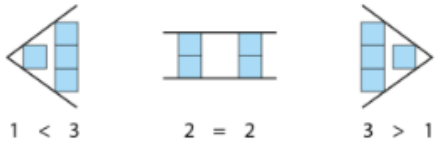
- the points representing pairs of values of x and y lie on a straight line
- symmetric values like 4,5 and 5,4 result in points that are symmetrical about a 45° line through the origin (y = x)
- the horizontal distance between a pair of points is the same as the vertical distance
- the lines for different values of c are parallel, and the larger the value of c the further the line is from the origin.

There is also the opportunity, in this or a subsequent lesson, for pupils to encounter the graphs of relations such as 9 = 2x + y and x = 9 + y. In terms of our balance model, these arise from adding further weights to the pans or moving weights from one pan to the other.

Focus of pupil learning and thinking

NCETM Primary Prof.essional Development Materials Spine 1: Number, Addition and Subtraction – Topic 1, Teaching Point 3.1

The image from this earliest section of the Primary PD Materials, illustrating the meaning of 'greater than', 'less than' and 'equals to', may prove useful for pupils who are finding it harder to move their understanding of the equals symbol from '*Hey presto, this is the answer*' towards '*is equal to*' or '*is the same as*'.



Misconceptions

Experience suggests that the equals symbol may be one that pupils see most often but have a limited understanding of. Through number sentences like 4 + 5 = 9, pupils tend to interpret the equals sign as '*makes*'. Here we attempt to shift the meaning from '*makes*' to '*is equal to*' or '*is the same as*' by using sentences like 9 = 4 + 5, 9 = □ + △ and, perhaps, 9 = x + y.

This activity seeks to address a second misconception around the plotting of points on a graph. By using axes that are already labelled with a square (x) and a triangle (y), pupils' attention can be drawn to the importance of accuracy and teachers can show that (2,7) is very different from (7,2). Pupils will be introduced to axes in Upper KS2. This is a chance to consolidate previous learning and correct any misconceptions. We recognise that it is important for pupils to learn how to create their own axes.

It is worth watching out for pupils who want to fix definite values to the shapes too soon and make assumptions based on the angle of the balance. Recording using the inequality symbols alongside the balance beams may help to address this.

Transition issues: new language, representations and the purpose of a representation

A limited understanding of equality is not exclusively limited to pupils in KS2/KS3 transition. Across primary education, schools are trying to move pupils to being able to express all eight number sentences associated with a particular additive or multiplicative fact. For example:

6 + 1 = 7	4 x 2 = 8
1 + 6 = 7	2 x 4 = 8
7 – 1 = 6	8 = 4 x 2
7 – 6 = 1	8 = 2 x 4
7 = 6 + 1	8 ÷ 4 = 2
7 = 1 + 6	8 ÷ 2 = 4
1 = 7 – 6	4 = 8 ÷ 2
6 = 7 – 1	2 = 8 ÷ 4

In this activity, pupils can be asked to explore an additive relationship, e.g. two triangles in a pan could be seen as △+△, or alternatively as 2△. This shift in thinking towards multiplicative relationships starts as pupils learn their times tables facts in Years 3 and 4, and develops across Years 5 and 6 with increasing depth.

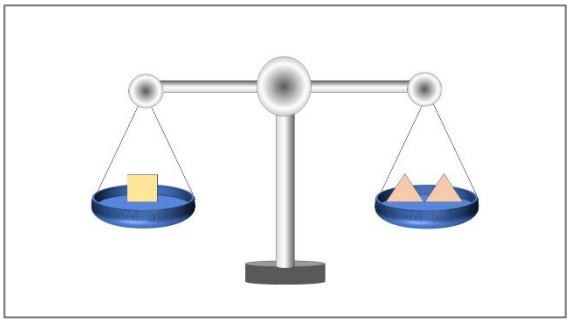
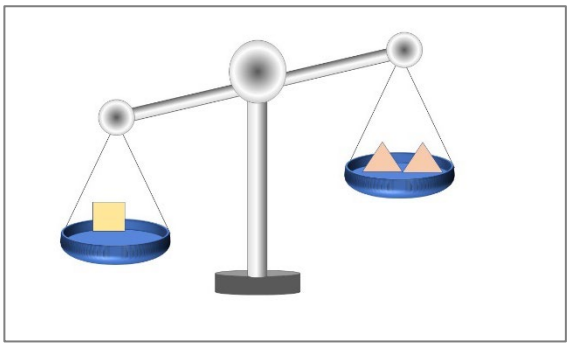
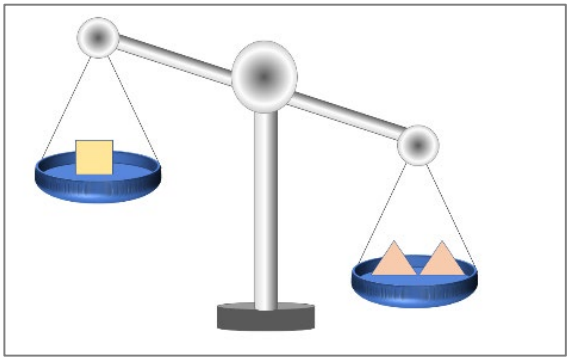
© Crown copyright 2023

Task 1: Introduction to the balance beam

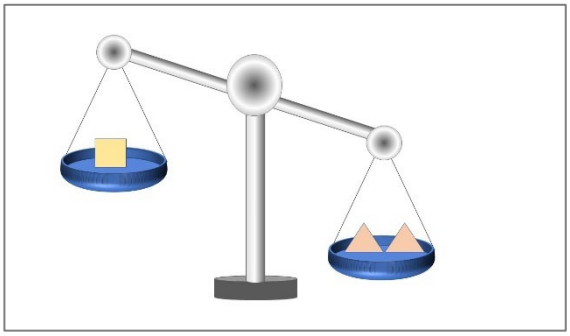
Remind pupils how a balance beam works, and how to read the various images of the balance.

- When the beam of the balance is horizontal, as on the first, the total weight in each pan is 'equal', or 'the same'.
- When the beam is slanting, as in the next three slides, this shows that the total weight in the lower pan is greater than in the other pan.

Rather than just showing the images, it would be best to start with an actual balance pan to model with, or include some images of old-fashioned kitchen scales.

Slides 3–6	Commentary
Slide 3 	*'What can you see?'* *'Could you give a value to the square?'* *'Could you give a value to the triangles?'* *'Do both triangles have to have the same value?'* Make pupils aware of another convention: identical shapes have the same value. For example: *'If one triangle weighs 3 units, the other triangle also weighs 3 units, and so in our current diagram the square would weigh 6 units.'* **Note:** whilst identical shapes have the same value, different shapes don't necessarily have different values, though they commonly will.
Slide 4 	*'What has changed?'* Here the square piece weighs more than the total weight of the triangular pieces. *'Do the values you assigned to the square and triangles still work? Why not?'* *'Do you have to change one or more of the values?'* To check pupils' understanding of the model, you might want to ask for values that fit, such as: $\square = 10$, $\triangle = 3$ or $\square = 100$, $\triangle = 49$ You could also ask pupils to form number sentences such as these: $10 > 3 + 3$, or $10 > 2 \times 3$ $100 > 49 + 49$, or $100 > 2 \times 49$ $\square > \triangle + \triangle$ or $\square > 2 \times \triangle$
Slide 5  Here the square piece weighs less than the total weight of the triangular pieces.	*What has changed?* *Why can the values you assigned to the square and triangles no longer work?* *If you keep the value of the square, what does the triangle need to become?* To check pupils' understanding of the model, you might want to ask for values that fit, for example: $\square = 10$, $\triangle = 7$ $\square = 100$, $\triangle = 51$ You could also ask pupils to form number sentences such as these: $10 < 7 + 7$ or $10 < 2 \times 7$ $100 < 51 + 51$, or $100 < 2 \times 51$ $\square < \triangle + \triangle$ or $\square < 2 \times \triangle$

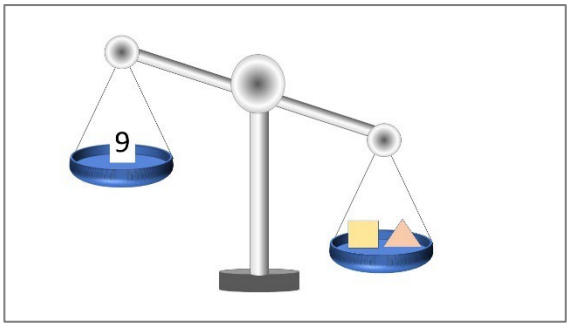
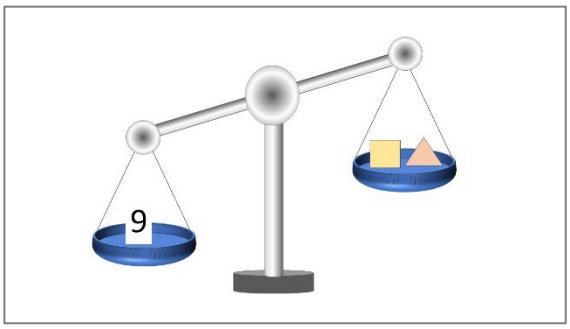
© Crown copyright 2023

Slide 6 This is an optional slide that allows you to consider the balance in greater depth. 	Pose questions like these: *'What happens to this balance if we:* • *move a triangular piece from the right-hand pan to the left-hand pan?* • *add a square piece to the left-hand pan?* • *add a square piece to both pans?'* We also touch on this issue in Slide 16, where we add a piece weighing 1 unit to each pan. You can make use of Slides 17–20 to extend these ideas.

© Crown copyright 2023

Task 2: Assign value to the square and triangle(s)

Here we know the weight of a shape in one of the pans and consider specific values for the weights of the shapes in the other pan.

Slides 8 and 9	Commentary
Slide 8 This slide is specifically designed to have the number 9 in the left-hand pan and the square and triangle in the right. This provides an opportunity to reinforce that the equation can be 9 = □ + △  	Discuss a few examples for the weights of the square and triangle, such as: • 2 and 8 • 2 and 80 • 2 and 7.1 You could look at this more deeply by: • thinking about the range of permissible values • discussing whether to allow fractional values, zero, and negative values. Pupils might notice that there are endless possibilities for the pairs of values, even if we only allow positive whole numbers. Some pupils may find it uncomfortable to see an equation written this way. By repeating the sentence as '9 is equal to 2 and 7', or '9 is made up of 2 and 7' and '9 is the same as 2 and 7' you can support a deeper understanding of the equals symbol. This could be a useful time to ensure that pupils are able to see how all eight equations can show the different additive relationships between the three numbers.
Slide 9  	This time, the total weight of the square and triangular shapes is less than 9 units. You can approach this slide in a similar way to the previous slide. Pupils might notice that if we only allow positive whole number values for the weights, there are only a few pairs of values that fit the situation this time. We will revisit this scenario a bit later in the lesson, where we represent pairs of values as points on a graph.

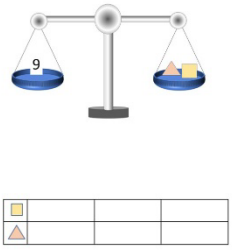
© Crown copyright 2023

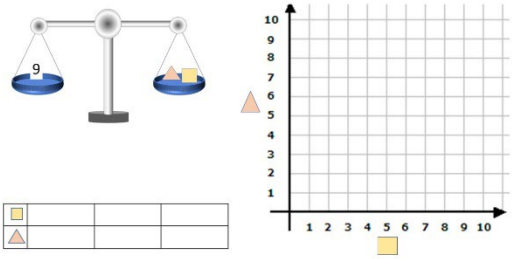
Task 3: Graphs

Slides 11–14

Slide 11

This slide contains a drawing of a beam balance, along with a table and graph that are initially hidden.





Commentary

Start with just the balance showing.

Ask pupils to

- describe the balance (e.g., the weights in the two pans balance, are the same, or are equal)
- suggest values for the weight of the square and triangular shape that fit the drawing.

Write two or three pairs as a number sentence such as 9 = 3 + 6, to show that the equals sign '=' does not simply mean 'makes'.

Here or later (see below), write the general statement 9 = □ + △ [or, perhaps, 9 = a + b, where a and b are number of grams or pounds, or whatever the unit is that the '9' represents].

Reveal the table.

Choose one of the pairs (e.g. 2,7 where the numbers are not too close together) and write it in the table.

Reveal the graph and say:

'I want to show our pair of values as a point on this graph. Where will it go?'

Plot the point.

Remind pupils that representing the pair 2,7 by a point that is '2 across' and '7 up' (rather than '2 up' and '7 across') is a convention. We **could** do it the other way round as long as everyone else agrees to do so!

Choose a second pair of values that fits the scenario, e.g. 5,4 (make sure it is not symmetric to the first pair, i.e. not 7,2).

Write the pair in the table and plot it as a point.

Spend time looking at the two points. Ask pupils to think about where other points that fit the scenario would go.

Choose a third pair of values that fits the scenario. This time choose a pair that **is** symmetric to one of the earlier pairs (i.e. 7,2 or 4,5). Write the pair in the table and plot it as a point on the graph.

Look at the three points. Discuss where pupils think other points might go, but don't plot them yet (to keep the drawing clear).

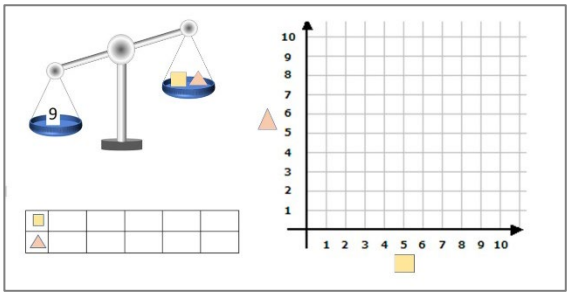
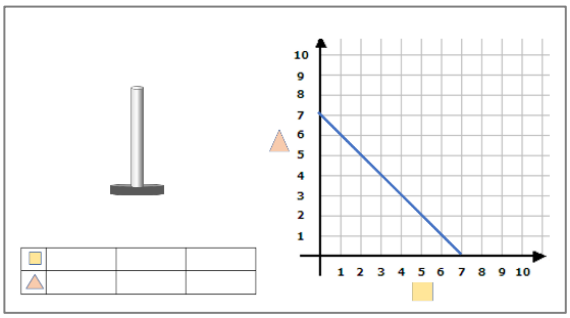
Discuss what sort of points to allow:

'Can we always put a point between two existing points – shall we allow fractional values?'

'What about a point on one of the axes, i.e. where one of the values is 0?'

'What about a point 'beyond' the axes (i.e. where one of the values is negative)?'

© Crown copyright 2023

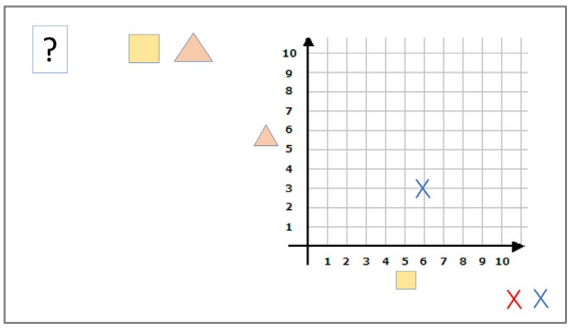
	Would it fit the balance scenario? Would it fit the relation 9 = □ + △? Discuss relations between points, for example: • Symmetric pairs such as 2,7 and 7,2 produce symmetric points (as they are balanced around the line $y = x$). • If we consider two pairs of values such as 2,7 and 5,4, we can see that 5 is 3 more than 2 and 4 is 3 less than 7; similarly, the distance between their points is 3 units to the right and 3 units down. Now plot some more points that fit. Draw a straight line through them. Discuss how to describe the line visually, e.g.: • It is at 45° to the axes. • It cuts both axes at the 9 marks. Discuss the line numerically, e.g.: • It represents pairs of values that add up to 9. It represents the relation 9 =? +?.
Slide 12 This slide shows another image of a balance beam, along with a table and graph that are initially hidden.  	This time the weights in the two pans don't balance, so that when we graph pairs of values for the weights of the square and triangular shape, the points don't lie on a straight line. Instead, they lie on a region bounded by a straight line. You can approach the task of finding values, recording them in a table and representing them as points, in a similar way to the previous task, but perhaps in not quite the same depth. *'Describe how the plots would look if the square and triangle together weighed more than the 9.'*
Slide 13 Here we have a balance beam with a known weight in one pan, and a square and triangular weight in the other pan.  	When we plot pairs of values for the weights of the square and triangular shapes, the points lie on the blue line shown on the graph. *'What does this tell us about the angle of the beam of the balance?'* *'What does it tell us about the known weight in one of the pans?'* Sketch the balance. Write some values in the table. Describe the relation between the weights in the pans (in words and/or symbolically). Compare the line on the graph here to the line we found for 9 = □ + △

© Crown copyright 2023

Task 4: Hidden balance beam

In this task we have a hidden balance beam with a weight of 10 units in one pan, and which is heavier than the weights of a square and triangular shape in the other pan. In other words, the balance beam shows the relation 10 > 10 > ☐ + △

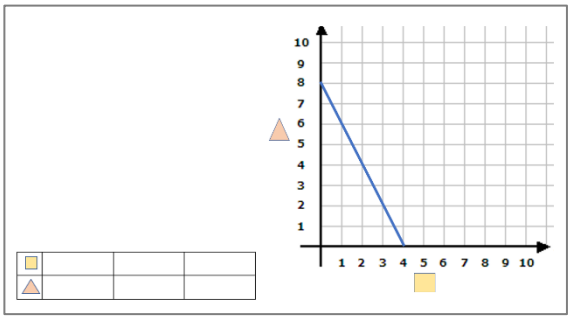
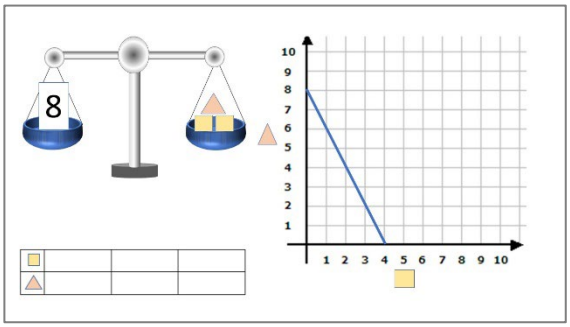
The pupils' task is to guess the relation based on the points shown on the graph that either fit (blue) or don't fit (red) the relation.

Slide 15	Commentary
	Tell pupils that you've got a hidden balance with a specific weight on the left pan, and a square and triangular shape on the right pan. The pupils' task is to find what the specific weight is and the orientation of the beam. Start with just the point (6,3) marked on the graph. *'This point shows a pair of values that fits my balance. What could the relation be for the weights in the two pans?'* Write down some possible relations, e.g.: 9 = ☐ + △ 10 > ☐ + △ 8 < ☐ + △ 15 > ☐ + △ 2 < ☐ + △ Ask the class for another point that they think might fit the relation. Plot it on the graph. Colour it blue if it fits our actual relation, or red if it doesn't fit. Eliminate any of the written down relations that no longer fit. Ask for one or two new relations that do fit. Ask for a third point. Plot it in the appropriate colour. Continue until it is possible to deduce our relationship with certainty. If you wish, repeat the task for a new relation, e.g. 7 < ☐ + △.

© Crown copyright 2023

Task 5a: A challenge!

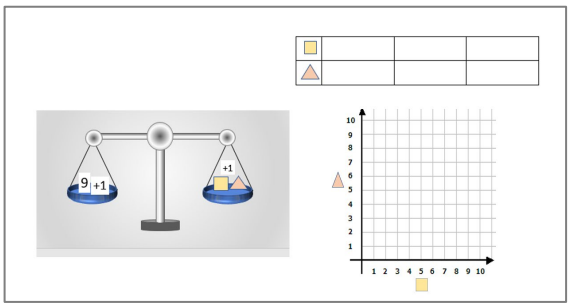
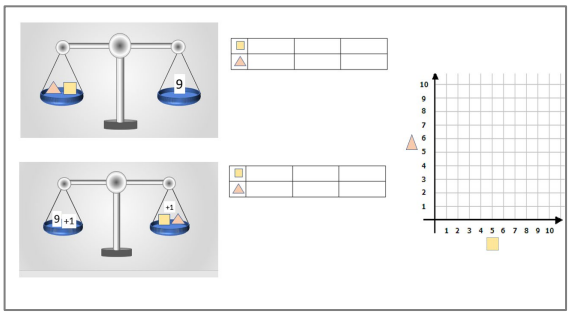
This is quite a challenging task.

Slides 17 and 18	Commentary
Slide 17  	Tell pupils we again have a balance involving a known weight and some square and triangular weights, but the situation is a bit more complicated than before. Points on the blue line give us values that fit the balance. *'Can you determine what the weights might be and what the balance might look like?'* It turns out that the relation is 8 = □ + □ + △. A balance that fits this is shown on the next slide.
Slide 18  	This shows that the beam balance for the relation is 8 = □ + □ + △, which fits the points on the blue line. Verify this by assigning values to the weights shown by points on the blue line, such as □ = 1, △ = 6. *'Does 8 = 1 + 1 + 6?'* You could extend this task by considering the straight line through the points (2,0) and (0,8). If you wanted to make the task even more challenging, choose the line through, say, (6,0) and (0,8). LATER you could explore what happens when we take the familiar balance on slide 9, which represents the relation 9 = x + y, and see what values would fit a balance where the square shape was in the other pan, alongside the weight of 9 units. This balance would model the relation 9 + x = y. What would the graph look like? What happens to the graph if we had a similar, balanced balance but with a fixed weight of 8 units, say, rather than 9?

© Crown copyright 2023

Task 5b: Exploring the balance beam in other ways

These final slides give the opportunity for pupils to explore what else they could do mathematically with a balance beam.

Slides 20 and 21	Commentary
Slide 20  **Slide 21**  	Ask: *'What other things can I do to still keep it balanced?'* Explore adding additional squares and/or triangles to each side. *'What would happen if you doubled (or other multiples) the amounts on both sides?'*