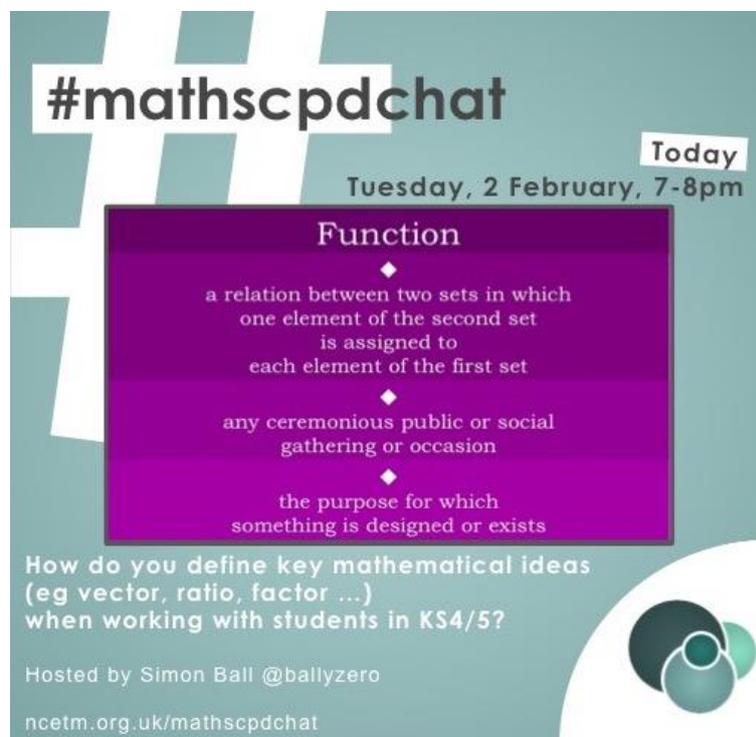


#mathscpdchat 2 February 2021

How do you define key mathematical ideas (e.g. vector, ratio, factor ...) when working with students in KS4/5?

Hosted by [Simon Ball](#)

This is a brief summary of the discussion – to see all the tweets, follow the hashtag **#mathscpdchat** in Twitter



#mathscpdchat

Today
Tuesday, 2 February, 7-8pm

Function

- ◆ a relation between two sets in which one element of the second set is assigned to each element of the first set
- ◆ any ceremonious public or social gathering or occasion
- ◆ the purpose for which something is designed or exists

How do you define key mathematical ideas (eg vector, ratio, factor ...) when working with students in KS4/5?

Hosted by Simon Ball @ballyzero
ncetm.org.uk/mathscpdchat

Among the links shared during the discussion were:

[Vocabulary](#) which are resources from *Boss Maths* ([Sudeep](#)) that aim to relate mathematical terms from other subjects (or words in common use) that share the same etymology. It was shared by [Sudeep](#)

[Examples for Mathematical Definitions](#) which is where you can see a brief definition of any mathematical term that you enter, and find properties of the mathematical idea named by that term. It was shared by [Mary Pardoe](#)

[Primary Mastery Professional Development](#) which are materials from the NCETM that will assist primary teachers in their professional development. It was shared by [Alison Hopper](#)

[Secondary Mastery Professional Development](#) which are materials from the NCETM that will assist secondary teachers in their professional development. It was shared by [Steve Lomax](#)

[The Problem with Math is English](#) which is a book by Concepcion Molina in which the author illustrates how teaching maths becomes an easier task when everyone understands the language, symbolism, and representations of mathematical concepts. It was shared by [Sharon Malley](#)

[Arithmetic](#) which is a book by Paul Lockhart in which the author reveals arithmetic not as the rote manipulation of numbers, but as a set of ideas that exhibit the fascinating and sometimes surprising behaviours usually reserved for higher branches of mathematics. It was shared by [Atul Rana](#)

[Differentiation in maths - scaffolding or metaphorical escalators!](#) which is a blog post by Charlie Harber of Herts for Learning in which the author outlines a few of the differentiation strategies that a teacher can deploy. It was shared by [Sharon Malley](#)

[Frayer-Model.co.uk](#) which is the teacher's one stop site for maths Frayer models. It was shared by [Peter Mattock](#)

[MRC Cognition and Brain Sciences Unit, University of Cambridge](#) which is some writing related to research on how the brain processes language. It was shared by [Sudeep](#)

[Mathematics glossary for teachers in Key Stages 1 to 3](#) which is a PDF resource from the NCETM. It was shared by [Guy Carpenter](#)

[Illustrated Mathematics Dictionary](#) which is a *Maths Is Fun* webpage providing easy-to-understand definitions with illustrations and links to further reading. It was shared by [Glo](#)

The example below (of screenshots of chains of tweets posted during the chat) shows part of a conversation about trying to define the word 'fraction'. It is a reminder that thinking about definitions can prompt useful and revealing discussions between pupils (and teachers). **Click on any of these screenshots of a tweet to go to that actual tweet on Twitter.**

The conversation was generated by this tweet from [Gemma Scott](#):

 **Director of Maths** @DirectorMaths · 16h ...
Replying to @ballyzero
It's very interesting to ask students to define something simple like a fraction or a decimal. They can draw them, they can write them but can they actually define them? Also an interesting thing to do with colleagues #mathscpdchat

and included these from [Tom Bowler](#), [Gemma Scott](#) and [Simon Ball](#):

 **Tom Bowler** @Ridermeister · 16h ...
I'm intrigued now... What definition do you give students for fraction? Can we have fractions within fractions? Is 1 a fraction? This could be fun!

 **Director of Maths** @DirectorMaths · 16h ...
I talk to them about rational and irrational numbers! #mathscpdchat

 **Tom Bowler** @Ridermeister · 16h ...
I suspected that may be the case. So is $\sqrt{2}/\sqrt{5}$ a fraction?

 **Director of Maths** @DirectorMaths · 16h ...
Good question and I'm going with no, the same way I wouldn't class $\pi/2$ as a fraction. What do you think? #mathscpdchat

 **Simon Ball** @ballyzero · 21h ...
I agree with you: a fraction should have whole numbers in its numerator and denominator. #mathscpdchat

these from [Tom Bowler](#), [Simon Ball](#) and [Gerry McNally](#):

 **Tom Bowler** @Ridermeister · 20h ...
Replying to @ballyzero and @DirectorMaths
This is interesting, I've never thought that numerator and denominators need to be whole. The everyday use for the word is part of a whole so I'm happy $\sqrt{2}/\sqrt{5}$ is just that! I guess that also means something like $7/2$ is part of several wholes!

 **Simon Ball** @ballyzero · 19h ...
Replying to @Ridermeister and @DirectorMaths
Aren't all whole numbers fractions? #mathscpdchat

 **Gerry McNally** @mcnally_gerry · Feb 2 ...
Replying to @ballyzero @Ridermeister and @DirectorMaths
.. and aren't some decimals fractions, too? #mathscpdchat

 **Gerry McNally** @mcnally_gerry · Feb 2 ...
... come to think of it, isn't every number in base 10 a decimal? #mathscpdchat

these from [Laura](#), [Sam Blatherwick](#), [Anne Watson](#) and [Tom Bowler](#):

 **Laura** @mathsteacher09 · 15h ...
I think there is a danger of being too pedantic at the expense of the big picture. I agree $\pi/2$ is not strictly a fraction but I don't see anything wrong with saying something like 'simplify that fraction' if they had $2\pi/4$ for example.

 **Sam Blatherwick** @blatherwick_sam · 15h ...
Is cancel down wrong? 🤔

You could argue that π has a coefficient of $1/2$ here? Could you?

[#mathscpdchat](#)

 **Anne Watson** @annemathswatson · 4h ...
The fraction $2/4$ is operating on π . So $2/4$ is the fraction to be simplified.

 **Tom Bowler** @Ridermeister · 3h ...
This is an interesting point of view, I like this description. How would you describe $(x+1)/(x+2)$, something often referred to as an algebraic fraction?

and these from [Gerry McNally](#) and [Simon Ball](#):

 **Gerry McNally** @mcnally_gerry · 17h ...
Words can also close down thinking. Is a/b always a fraction, or is it also a quotient? Is a always numerator and b always denominator, or are they also dividend and divisor? [#mathscpdchat](#)

 **Gerry McNally** @mcnally_gerry · 17h ...
What about $(2/3)/(3/4)$ - what kind of thing is that? What could we say about numerators, top numbers, denominators and bottom numbers here?
[#mathscpdchat](#)

 **Simon Ball** @ballyzero · 17h ...
An expression that needs simplifying!

(to read the discussion sequence generated by any tweet look at the replies to that tweet)

The areas where discussion focused were:

how important teachers believe that it is for definitions to feature in their maths lessons:

- 'very important and easily overlooked' ... teachers may assume wrongly that their pupils, if asked, would be able to articulate definitions of mathematical words that they are being expected to use appropriately ... that definitions of maths words (such as 'regular' and 'similar') that have different meanings in ordinary usage are particularly important ... there was a brief discussion about the extent to which pupils may or may not know what are the kinds of mathematical object that the word 'factor' can denote;

- that it is as **important for pupils to know what a word does NOT denote** as it is for them to know what it does ... ways of working with examples and non-examples ... **'examples and non-examples** more often than not come before definitions for me';
- that the journey in a lesson episode that is **'towards creating a definition through carefully sequenced questions that include boundary examples and non-examples** can be really powerful for deeper understanding' ... 'but it's something that I wouldn't necessarily have thought about explicitly without this #mathscpdchat';
- some teachers believe that the way to help students arrive at a definition of a mathematical 'object' is through **'Englemann Direct Instruction**, for example, this is a polygon, this is a polygon, this is not a polygon ... now give me some more examples of a polygon';
- some teachers sometimes **struggle themselves to think of 'student-friendly and succinct' definitions** ... that a **glossary of mathematical words** (such as the NCETM's glossary ... link provided above) can help teachers 'have a consistent approach to definitions' ... some teachers like to bring the etymology of words into discussions about definitions;
- other teachers do **online searches for definitions of mathematical words**, and then write their own definitions ('being as clear as possible for the KS3/4 audience they are aimed at') into their **pupils' knowledge organisers** ... teachers within their departments discuss and refine their definitions in order to arrive at **shared and agreed understandings**;
- some teachers, while believing that it is crucial to teach students to use language accurately, are less keen on students 'copying definitions into books' ... definitions may 'become too long and potentially confusing ... often the more I understand about the concept, the more complex it becomes to define';
- that **contexts can illuminate the value of using mathematical words correctly** ... for example using the words 'multiplicand', 'multiplier' and 'product' correctly when talking or thinking about or comparing statements such as ' $15\text{ cm} \times 2 = 30\text{ cm}$ ' and ' $2\text{ cm} \times 15 = 30\text{ cm}$ ';
- some teachers deliberately **use 'evolving' language to try to facilitate understanding** ... 'so for multiples I will use the word 'copy', then replace it with 'multiple' later on' ... eventually moving to symbolic representation and using brackets ... 'the definition gets more refined with time' ... moving from 'street lingo to proper maths language' ... 'moving from 'to multiply out' to 'to distribute' ... there was a brief discussion about distributing multiplication and division by ~ 1 over addition and subtraction;
- there was a discussion about **whether teachers use the same definitions/language with 'top sets' as they do with 'bottom sets'** ... some teachers believe that it is

important for definitions to be ‘age appropriate’ ... ‘we need to expose them to subject specific vocabulary, whilst keeping it accessible’;

the host asked teachers how they would define ‘vector’ with their students:

- a popular response was **‘a vector is a quantity with both magnitude and direction’**;
- teachers agreed that **they would not give a definition without also giving examples** ... some would give their examples ‘in multiple representations’ (by writing some as column vectors, and showing others by drawing directed line segments?) ... at least one teacher would ‘physically walk a vector, e.g. three steps towards the whiteboard, but from different positions in the room’ (how to model that during remote teaching?);
- a teacher commented that **‘you don’t give definitions to get hold of a concept, you use the concept to get hold of the definitions’**;
- there was some discussion about **whether ‘directed line segment’ is an adequate definition of ‘vector’** ... ‘what if you are describing something in 6-D space?’;
- that **many different definitions of ‘vector’ can be found online**;
- that when defining ‘vector’ it is important to give examples of ‘forms’ of vectors (e.g. velocities and forces) ‘to help students appreciate why they’re used’ ... that ‘if you are studying Mechanics then you will treat \vec{v} as both a scalar quantity and a vector quantity depending on context’ ... at least one teacher’s view is that **a vector quantity is something that can be represented by a vector, rather than a vector itself**;
- there was a **brief discussion about working with vectors written in column vector form** ... that the magnitude of a vector is always positive and that ‘Pythagoras’ Theorem can be used to calculate the magnitude when the vector is drawn on a Cartesian plane’;

the host asked teachers how they share definitions with students:

- much discussion was prompted by a teacher who responded by remarking that it is **‘interesting to ask students (and colleagues) to define something simple like a fraction or a decimal’** (tweets that formed part of this discussion are shown above);
- many teachers share definitions ‘topic by topic’ ... ‘if a student does not explain using the correct mathematical language, I re-explain, then ask the student to try again’ ... other teachers responded by pointing out that **learning is not facilitated by the teacher always re-wording a student’s definition using ‘correct’ mathematical language** ... by the teacher ‘overhelping’;
- some teachers have definitions of ‘key words’ that are likely to be used during current learning on display in the classroom, and written in the knowledge organiser for that ‘topic’ ... students highlight definitions of words in their own notes ... KS5 students do tests (‘they get given some definitions to learn, then we have a ‘spelling-test’ kind of test like in a primary school’) ... **Fraye Models** (used as a resource for organising information for learners) were mentioned (link provided above) ... although some

teachers' students sometimes **use Frayer Models just for reference**, they may also sometimes be presented with one with **'empty boxes' in which they are challenged to write their own examples and non-examples**;

- some teachers share definitions topic by topic in a **three-column table with column headings 'word', 'definition' and 'example'** (and 'the use of correct terminology is reinforced everywhere in all lessons') ... this document (the three-column table) forms part of students' **'half-term knowledge booklet'** which they have in lessons and take home at the end of the half term ... it keeps the process of note-taking by students away from their main learning activity during lessons;
- teachers commented that 'reinforcement (of knowledge of definitions) is key' ... **the teacher frequently asks 'remind me what ____ means again'** ... 'getting students revoicing' ... how easy is it to do this during remote teaching?
- some teachers like to **give a 'slack' definition followed up by a more formal definition**, so that a definition is 'tightened' over time ... or the 'tightening' may be immediate (e.g. 'we need to use formal language or we won't be sure about what we're trying to do');
- some teachers, noticing that their classroom displays of definitions 'weren't actually being read', moved to having **students write definitions in their notebooks 'as we go'** ... students keep the same 'notebook' (to which they can refer) as they progress right through their secondary education;
- **some teachers have never experimented with the use of a glossary** ... because 'often the worded definitions on a static page are harder to take in than a diagram accompanied by a human saying a few words while pointing at parts of the diagram';
- some teachers 'have a word of the day each morning to explore'; ... **students record their own definitions after exploring the word**;

whether teachers' definitions are different in Higher Tier lessons compared with those that they use in Foundation Tier lessons:

- some teachers reported that **definitions are the same in all KS4 lessons ... and they are the same in KS3 and KS4 lessons** ... that examples and non-examples may differ, but definitions do not;
- some teachers 'confessed' that they are **'guilty of occasional 'dumbing down' of language' when they are teaching Foundation Tier classes** ... e.g. saying 'top number' rather than 'numerator' ... that if a teacher say 'top number' after saying 'numerator' they are facilitating learning rather than 'dumbing down';
- some teachers mentioned that they sometimes use **'sentence stems' and 'talk structures'** as learning 'scaffolds';
- some teachers reported that their **definitions are consistent across KS4 to KS5 ... later definitions expand on earlier ones, but don't contradict them.**