Exploring lesson design through algebraic misconceptions

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Abstract/Summary
The project explored the theme of algebraic proficiency through one of the “big ideas” in mathematics teaching namely, misconceptions and errors for learning. Specifically, we looked at the idea of designing lessons around mathematical misconceptions rather than waiting for misconceptions to “tumble out” of a lesson. Misconceptions often lead to key learning points within a lesson and by anticipating misconceptions, and planning in such a way that they are exposed to students more regularly, deeper learning will take place.

The experience of many of our KS4/KS5 teachers, as with many schools, is that algebraic misconceptions are what (by and large) hold many students back from accessing questions of a more challenging nature including multi-step questions on the GCSE higher and A-level papers. This has particularly manifested itself with the inclusion of more problem solving activities in the new specifications for GCSE. We are aware of a wealth of strategies to support misconceptions when they arise but the work of this group challenged teachers to think critically about lesson design to ensure misconceptions do arise within the classroom and students develop their conceptual understanding of algebra through a variety of alternative approaches that were planned for by the teacher.

There were many key outcomes from the project inspired by those directly involved and also the wider mathematical community. It proved extremely difficult to confidently define the concept of misconception and to further differentiate between the concepts of misconception, misunderstanding and mathematical errors. Lots of strategies and resources to support the teaching of misconceptions were developed as a result of this project including effective use of ICT software. Lesson observations evidenced deep learning in the mathematics classroom with students unafraid to make mistakes and showing increased levels of mathematical communication as a result of the effective lesson design. Effective questioning was also a key feature of the work of the project.

Background
There were three schools involved in the collaborative project:

St Peter’s School Solihull
John Henry Newman Catholic College Solihull
Bishop Challoner Catholic College

All three schools form part of the Bishop Challoner Teaching School Alliance. Strong links between the three schools existed prior to the project with many of the participants having worked/trained in one of the other schools.

The idea of further exploring algebraic misconceptions was inspired by attendance at the NCETM Lead Professional development programme where research on the “big ideas” in mathematics was presented one of which was the concept of “using misconceptions and errors for learning”.

There is a wealth of research on mathematical misconceptions and errors for learning. Much of the material relates to resources that can be used to support misconceptions within the classroom such as “always, sometimes, never true” and other probing questions. There is a three way battle between warning of potential misconceptions, waiting to see if misconceptions arise and avoiding students developing new misconceptions. We have to tread carefully as conceptual understanding is crucial in developing knowledge of a subject. In this sense, lesson design needs to be carefully thought about to ensure deep learning takes place.

Research has shown that the same mathematical mistakes happen with students all over the world. This finding suggests that good (or bad) teaching has little to do with the formulation of misconceptions. It does however lead on to the idea that students construct concepts organically, meaning students often attempt to make sense of the world naturally through their own interpretations (Swan in Gates, 2001). It is also clear from the available literature that it is important to distinguish between misconceptions and “mathematical mistakes” or errors. Misconceptions are far more than just having a mistake pointed out.

**Aims of the Collaborative Teacher Project**
- To develop algebraic proficiency across a Teaching School Alliance.
- To explore lesson design through one of the “big ideas” in mathematics namely, mathematical misconceptions.
- To support the development of “deeper learning” in the mathematical classroom through multiple representations and effective questioning.

**Details of those involved in the Collaborative Teacher Project**
S Jethwa, Bishop Challoner (KS4 Coordinator)
L Clark, Bishop Challoner (NQT)
J Bloxidge, Bishop Challoner (NQT)
Ezran Little, John Henry Newman (KS4 Coordinator)
Holly Watson-Bryant, John Henry Newman (NQT)
Faruk Patel, John Henry Newman (NQT)
Simon Palin, St Peter’s (KS5 Coordinator)
Samantha Smith, St Peter’s (Teacher of Mathematics)
Luke Tudor, St Peter’s (NQT)

**A description of the Collaborative Teacher Project**
The project involved learning triads from each of the three schools. Each triad consisted of at least one experienced teacher and one newly qualified teacher. The triad worked collaboratively during the project and shared experiences from the classroom as the project evolved. A particularly pleasing aspect of the work of the group was that there was lots of cross-school collaboration both at and between the meetings.

The project evolved in distinct phases as detailed below with “gap tasks” for the group to complete between each phase:

**Phase 1:** Developing knowledge and understanding of misconceptions
Initially the group met to discuss the concept of misconceptions and to think about what constituted an algebraic misconception, when and where they regularly encountered them in their own teaching and how they might deal with them.

**Gap task:** The group were asked to reflect on the concept of algebraic misconceptions and keep records of any that arose in their own teaching.

**Phase 2:** Developing strategies to deal with algebraic misconceptions
In the next session the group looked at ways in which misconceptions could be approached through effective questioning, multiple representations and innovative resource choice/design. The group were given specific training and ideas to help support them with the challenge of developing conceptual understanding.
**Gap task:** Members of the group were given the opportunity to reflect on the session and respond to several of the misconceptions they had seen in the classroom by producing resources and developing effective questioning techniques.

**Phase 3: Lesson design from algebraic misconceptions**

In the third session, after a revision of how to respond to misconceptions, the group were given time to design either a sequence of lessons or a single lesson to be delivered back at their school. The lesson design focused on an inside out approach starting from the misconceptions and subsequently building the lesson around them.

**Gap task:** The lesson/lesson sequence was delivered and observed within each triad. Informal feedback from teachers and pupils was collated in the form of interviews and copies of pupil work.

**Phase 4: Feedback, reflection and cascade dissemination**

In the final session there were opportunities to reflect on the lesson, share common experiences and collate feedback. We then discussed ways in which each of the triads could disseminate their work to their own departments and challenged them to do this at some point in the future.

**Gap task:** Cascade dissemination from triads to their own departments.

For a detailed description of what happened as the project evolved see below.

**What has been learned from the project**

The first session took place in October.

**Session 1 Aims**

- To define what is meant by a mathematical misconception.
- To distinguish between mathematical errors and mathematical misconceptions
- To look at typical lesson design and how misconceptions fit in with this.

We began by looking at the groups understanding of the meaning of misconception. Each teacher was asked to define the word misconception (in 30 seconds) and then explain what they thought it looked like in a maths lesson. This then led to a fruitful discussion on misconceptions teachers in the group had previously encountered. Here are some of the definitions given by the teachers.

- “A misunderstanding caused by a preconceived idea of what something is.”
- “A common error displayed by a pupil.”
- “Where there is a preconceived idea that is the basis for another idea.”
- “An alternative, but incorrect, view of how a problem should be solved.”
- “An idea or process which is not well understood and therefore leads to incorrect solutions.”
- “A mistake which arises from a lack of understanding – or they feel they understood something but do not fully.”
- “A misconception or common error. An assumption made in error.”

We have collated many varied definitions of the concept of “misconception” from a wide variety of teachers and other mathematics educators.

The next activity in the session was to look at effective maths teaching. The group were split into subgroups and given nine statements relating to maths teaching and asked to arrange them into a “diamond nine” with the statement they agreed with most at the top of the diamond. One of the statements read as follows:

- “Good maths teaching happens when … students make mistakes and are given the opportunity to reflect on, and learn from, them.”

In each subgroup this statement was positioned at the top of the diamond nine or in the second row of the diamond. This suggested the teachers in the group valued the importance of students learning from their mistakes and suggested misconceptions are key to their interpretation of good teaching and learning.
The next activity the groups were given attempted to further unpick the concept of a misconception as opposed to a misunderstanding. Teachers were given a series of cards with different, but incorrect, mathematical statements. They were then asked to categorise them as misconceptions or misunderstandings. This led to lots of discussion. One group decided to rank the cards on a five point scale from certain misconception to certain mistake. An example of such a card is seen in figure 1.

![Figure 1](image1.png)

This activity led to lots of fruitful discussion/professional disagreements! Teachers who tried this activity tended to fall into two categories:

1) Those who were confident they could distinguish between statements that were misconceptions and statements that were misunderstandings.
2) Those that felt unless the problem was seen in context (i.e. with the student in front of them) there was no sure way of telling which statements were misconceptions and which were misunderstandings.

Finally, we looked at “typical” lesson design and perhaps how this could be flipped around to build a lesson based on misconceptions (see figures 2 and 3).

![Figure 2](image2.png)

**Typical Lesson design**

![Figure 3](image3.png)

**Alternative Lesson design**
Whilst the above figures are quite a crude representation of how planning may happen, the idea was to challenge the group to think about how often they had taught a lesson and the students just “got it” even though the work may have been challenging, and relevant, and interesting. The idea was to get the group thinking about what the key things were that they really wanted to come out of a lesson i.e. the “light bulb” moments and then plan out from here to ensure these moments actually happened within a lesson.

The teachers were then given the opportunity to design their own schematic view of a typical maths lesson using a series of card prompts.

**Gap Task 1**
At the end of the first session the group were given a gap task to complete. The task was simply to record misconceptions they saw in their lessons or in pupil work and share these with the group. Many of the misconceptions seen were recorded on the NCETM portal.

**The second session took place in November.**

**Session 2 Aims**
- To understand what is meant by multiple representations and effective questioning.
- To look at, and evaluate, a range of strategies for teaching algebraic misconceptions.
- To design some new resources to support the teaching of algebraic misconceptions.

The group was again split into small subgroups and two main tasks took place in this session. The first was a carousel activity looking at five different types of resources that could support the teaching of algebraic misconceptions. The second was an opportunity to design a new resource to support the teaching of misconceptions either by using one of the ideas shown or by developing a new idea.

The five resources showcased were:

(i) Effective questioning
(ii) Multiple representations
(iii) Multiple right
(iv) Sometimes, always, never true
(v) Dynamic maths

The first two resources were inspired by ideas taken from the NCETM Lead Professional Development Programme. Multiple right was about designing a multiple choice question where answers had been carefully chosen to draw out misconceptions e.g. see figure 4.

**Question 4**

\[
(2a^2 b^3 c^4)^4
\]

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<td>D</td>
<td>2a^4 b^2 c^12</td>
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<tr>
<td>E</td>
<td>16a^4 b^2 c^12</td>
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**Figure 4**

Sometimes, always, never true is a fairly well-known mathematical teaching concept although surprisingly not well-known by this group (in fact it was the resource they least liked)! Dynamic maths consisted of files created on GeoGebra to explain and draw out misconceptions e.g. see figure 5.
The group were given time to develop their own resources and this was left as the gap task before the next session in January.

**Session 3 Aims**
- To collaboratively plan and share best practice with other professionals.
- To design a lesson centred on algebraic misconceptions.
- To reflect on ways of feeding back lesson observation and collating evidence of learning in the mathematics classroom.

Session 3 was largely devoted to offering time to each group to plan a lesson or sequence of lessons based on misconceptions. We discussed as a group the importance of choosing the right group to trial this new approach with. The importance of “taking a risk” with a new style or approach was emphasised as well as choosing a group you could trust and would easily engage. In planning the session it was felt that to change peoples’ attitude and approaches to different pedagogy requires teachers to be risk takers and see the value in what they are doing.

It was very pleasing to see that within the collaborative group teachers mixed cross-school to jointly plan lessons. Triads also worked together to plan lessons that could be differentiated to meet the needs of several different teaching groups back in each school context. All teachers were given the following planning template to help shape their ideas (see Figure 6). The template kept the core central theme of misconceptions whilst highlighting the importance of the consideration of both effective questioning and also multiple representations and activities to draw out misconceptions. Teachers used this template together with the ideas gleaned from sessions 1 and 2 to plan their lesson.
After some time spent on planning we turned the focus to thinking about giving effective feedback. The following points were identified as key things for the group to look for and make statements about when observing.

- Focusing on the maths and the way it is being taught rather than specifics e.g. behaviour (unless learning related) and structure of the lesson per se.
- Pay particular attention to the misconceptions being addressed in the lesson.
- Make judgements about the progress of students as a result of the design of the lesson.
- Evaluate the usefulness of the resources used.
- Look out for use of effective questions and multiple representations.

We felt that it was important not to grade the lessons in any way to keep the experience positive and focussed on the mathematics and the way it was being taught. This also gave teachers confidence to take more risks without fear of failure.

Collaborative Lesson Observations
Lesson observations took place mostly in early March 2013. The groups liaised via email so all participants were aware when and where lessons were happening. It was pleasing to see several colleagues visiting other schools to observe lessons with different groups of students. A wealth of evidence of pupil work, photographs, videos, teacher notes etc was collated. Details on key points that arose from the lesson observations and paperwork is recorded below.

The group met for a final time in late March.
Session 4 aims
- To evaluate the work of the project so far.
- To share key findings with the rest of the group.
- To reflect on ways of best disseminating the work of the project.

In this session each group recorded their experience of their respective lessons in poster form (see Figures 7 and 8).
Each group was given the opportunity to talk through their approach with the rest of the project participants and opportunities to ask questions were given.
Key outcomes

The approach to misconceptions
The group valued the importance of making mistakes as a powerful learning opportunity and saw this as a key ingredient in good maths teaching. The discussion on the NCETM portal led to an interesting conversation over whether a potential misconception remains a misconception if it can be solved in a new context. The question posed was to estimate,

\[
\frac{7.19 \times 19.7}{0.46}
\]

Here the student got the answer 140 by rounding to 7, 20 and the denominator to 1. Had this problem been posed in terms of money would the student still arrive at the same answer? Discussions on the portal also highlighted the importance of discussing solutions with students and also students discussing with students. It was felt that, particularly with misconceptions, the ability to articulate mathematically the processing that is happening is key to determining the underlying problem.

“Multiple right” was seen to be a useful resource and one group felt this could be developed further by giving multiple solutions that included the method used. Also one group felt effective questioning could be used to draw out why certain multiple choice answers were wrong. It was a hard task to clearly separate misconceptions from misunderstandings. A large pool of algebraic (and other) potential misconceptions was created.

Lesson observations/evidence

St Peter’s School
Teachers at St Peter’s School delivered a lesson focussing on approaches to solving linear equations. A similar lesson was taught to groups of varying ability and the resources used and approach were differentiated as necessary.

There were four different approaches to solving equations presented:

- Balancing
- Use of function machines
- Stream of thought
- Division method

Prior to the lesson teachers spoke about their concern over methods previously acquired:

“Many of them seem to have picked up bad habits e.g. float and ping, change sign when you go across an equals sign etc. I was also concerned about teaching a lesson where I knew pupils would make a lot of mistakes.”

In this lesson students were being challenged to change approaches to solving equations, where necessary, to adopt mathematically sound reasoning.

As part of this lesson the ‘multiple right’ activity was used. Students were given a set of differentiated problems to solve with carefully chosen options for the final solution.

“It was good including common incorrect answers as pupils instantly thought they were right as it was in the list of choices.”

Teachers spoke about the importance of mathematical communication:

“I was looking forward to the ‘mathematical discussion’ that we would be having as a class. I like the fact that pupils have different ways of solving problems and always try to actively encourage them to discuss these various methods and why some work better than others. Through doing the ‘multiple right’ activities it eases pressure on pupils who are not always that confident to provide answers themselves.”
In this lesson the pupils showed a really good approach to misconceptions when they arose,

“As well as discussing correct solutions we also discussed common incorrect answers and definitely appreciated the value in doing this. The majority of the class were happy to make mistakes on the main exercise which demonstrated a really mature approach.”

The impact of developing conceptual understanding was clearly evident:

“From the previous lesson it was clear that many pupils had been drilled into memorising certain methods, without really understanding why they were doing what they were doing. During this lesson, the students appreciated the value of balancing an equation and understood the flaws that arise when using shortcut methods.”

The concept of trail of thought was drawn out within this lesson i.e.

\[ 3x + 2 = 11 - 2 = 9 \div 3 = 3. \]

The teacher emphasised that,

“The main thing I wanted pupils to get out of the lesson was the importance of demonstrating clear and concise working that was mathematically sound. I was not concerned about getting the right answer at the first time of trying.”

A version of the popular game show “Pointless” was used to engage students encouraging them to opt for methods that were mathematically sound and also yielded the correct solution to equations posed – these scored the lowest points. Methods that were quicker but less mathematically sound scored higher points.

There was evidence of deep learning taking place from the students:

“They were constantly questioning themselves which meant that they got a firm understanding of errors they could make.”

Many misconceptions/errors were drawn out in the St Peter’s lessons such as,

\[ 2x = 6 = 3, \]

and

\[ 6x + 2 = 44 \]
\[ 6x = 46, \]

to name but a few. One student was attempting to solve \(2 + 4p = 10\) and opted to take \(4p\) from both sides. Another student was solving \(7k + 3 = 5k + 9\) and tried adding \(5k\) to both sides. In both cases it was interesting to see students exploring these ideas and fascinating to see the “light-bulb” moment when they realised that perhaps they were not simplifying the situation with such approaches.

Another interesting example drawn out was when solving \(3x - 7 = 21\). One student said, “you start off by doing 21-7” they were quickly corrected by another student who said “you’ll end up with \(3x = 14\) which can’t work!” Two misconceptions/errors with one stone! In the ‘multiple right’ activity the teachers who planned this lesson were careful to include both fractional and decimal answers as possible choices.

Evidence from lesson observations at St Peter’s School highlighted the following key features:

- No fear of mistakes.
- Good discussion of alternate methods.
- Emphasis on method and conceptual understanding over answer getting.
High quality questioning to draw out understanding.
Lots of independent learning taking place and excellent levels of mathematical communication evident between pupils.
Students were given the freedom to use methods that worked for them as long as they made sense mathematically.

**John Henry Newman Catholic College**
Teachers at John Henry Newman choose to look at forming and solving equations. The approach was one of “tell me the story of \( x \)”. This encouraged students to articulate the operations that were happening in a stated equation and helped to deconstruct the algebra. The use of written instructions drew out the misconceptions e.g. the difference between \( 3g + 5 \) and \( 3(g + 5) \).
Students were encouraged to think about operations and their inverses to support learning. By the end of the lesson students were solving complex equations such as,

\[
\frac{(2b - 5)^2 - 2}{2} = 7
\]

The “Grid Algebra” software (ATM) also supports this sort of algebraic reasoning and is an extremely effective tool in making the link between algebra and number.

There was some great feedback from students in the lesson such as,

“I like to make mistakes in order to learn.”

They also valued the importance of teacher explanations,

“We are allowed to make mistakes and work things out for ourselves.”

Evidence from the lesson observation at John Henry Newman highlighted the following key features:

- Effective questioning drew out explanations from pupils and focussed on the process not the answer.
- There was good use of “visualisation” to encourage students to explain the mathematics in terms of “tell me the story”.
- Mistakes were deliberately drawn out to allow opportunities to address them.

**John Henry Newman Catholic College/Bishop Challoner Catholic College**
One teacher from John Henry Newman decided to pair up with another teacher at Bishop Challoner to co-plan a lesson on approaches to expanding single brackets. The sorts of mistakes/misconceptions being addressed were:

\[
3(x + 3) = 3x + 3,
\]

and

\[
3(x + 3) = 3x + 6.
\]

Students were presented with several multiple representations of how to expand single brackets:

- Pictorial representations
- Arrow methods
- Grid method
- Making connections with areas of rectangles
- “Lots of” method

The students were placed into groups of five and each given one representation of expanding brackets to explore. They were all given the same set of problems to solve using their method. At
the end of the lesson feedback was given by each group. Ipad were used effectively to support learning and videos were created to fully explain the process students had gone through.

Evidence from lesson observations highlighted the following key features:

- Mathematical communication was again excellent and students worked well together.
- Advantages and disadvantages of alternative representations were discussed by students.
- Misconceptions were naturally tumbling out of the lesson as a result of the planned activities.

**Bishop Challoner Catholic College**

Two teachers at Bishop Challoner paired up to jointly plan a lesson that was delivered to two different teaching groups. The focus of the lesson was on sequences and the distinction between term to term rules and position to term rules. The misconception being addressed was the confusion between the two forms e.g. writing n+2 for the sequence 2,4,6,… .

The lesson began with students constructing sequences on mini-whiteboards. Later in the lesson students were told the rule was add 5 and then a variety of answers for the first five terms of this sequence were given. The teacher skilfully questioned the students asking how could all the sequences be made the same? This activity was designed to draw out the conceptual understanding that it was not just the term-to-term rule that was important, the start point was also of key value. Students were then asked to debate which was the best approach to describe a sequence. One student responded,

"If you are dealing with larger numbers the nth term helps you more."

This was a key "light-bulb" moment in the lesson and helped students understand the importance of having different ways of describing sequences.

There was lots of evidence of effective question stems being used within the lesson such as,

"What happens if..? Describe… Which is best…? Why is this wrong…?"

Evidence from observation notes showed that students had started to think beyond the initial problem,

"What if we had a sequence with the differences +1, +2, +1, +2?"

Multiple representations were used to explore sequences further such as an image of the Heron Building. Each floor of this building is made up of steel girders and students were challenged to make connections between the floor number and the number of steel girders.

Evidence from lesson observations at Bishop Challoner highlighted the following key features:

- Encouraging students to communicate their mathematical thinking both within their paired groups, as well as whole class.
- Questioning probed for deeper understanding throughout the lesson.
- Nice use of debating to compare two alternative approaches.
- Multiple representations evident in planning emphasised the link between maths and the ‘real world’ e.g. Heron building.
- The interactivity in the ‘starter’ activity immediately engaged students and immersed them in the concept of sequences.
- Good explanations of the key information.
- Good use of mini whiteboards to check whole class response throughout the lesson.
Impact on teachers’ practice
The following quotes are taken from the teachers involved in the project and give a feel for the key impacts of the project.

“It has opened my eyes to how misconceptions can be used as a basis for lesson design and how successful this can actually be in teaching and learning.”

“I was exposed to a wonderful array of innovative approaches to teaching algebra topics e.g. multiple representations and multiple right”

“Having taught this topic several times before I thought I would be able to know what was going to happen before the lesson; but it was not what I expected” (experienced teacher).

“Collaborative planning took a lot longer but I gained far more from the experience.”

“I spent a lot more time in lessons thinking about outcomes of activities.”

“Pupils are used to me considering misconceptions. True/false exercises. Why is this wrong? What would be a common incorrect answer to this question?”

“Effective questioning is something I have always done well, however I have developed this further and think about it more thoroughly.”

“The project has really developed me as I have ideas of how to plan a lesson thinking about the misconceptions firstly. I have also developed ideas on how to create resources to draw out misconceptions.”

“I now think about misconceptions firstly when I think about my lessons. I incorporate wrong answers into my worksheets and teaching aids.”

“The impact has been I have explored a broader range of approaches when teaching. Before I would just rely on one or two (personal) methods to teach. However, now I am seeing more.”

“I was surprised by the outcome of the taught lessons. I perhaps underestimated the low ability group in particular and would be happy to attempt more challenging activities in the future.”

“Planning to address the misconceptions gives me more confidence when they arise rather than being surprised and unprepared. Using the various teaching techniques gives the pupils a wider variety of activities and greater engagement.”

“It has reinforced that questioning is at the heart of every successful lesson.”

“The initial sessions on the exploration of misconceptions were excellent and really questioned my understanding of what misconceptions truly are. The GAP tasks were great as they ensured ‘continuity’ of the project between sessions.”

“I have learnt the difference between an error and something that is deeper i.e. a misconception.”

“I now see even more value in addressing misconceptions as they arise rather than brushing over them or waiting for a more appropriate time to deal with them.”

Impact on the pupils
In relation to a lesson on solving equations
“The lower ability cared less about getting the answer right and more on the method of solving.”

“Multiple right was good having the wrong answers on there.”
“My year 8 set have really enjoyed the resources used and now think about possible errors they could make and how to avoid them.”

“More emphasis on the potential wrong answers is clearly helping students’ understanding and gives them more confidence in the future.”

“They are all [pupils] mentioning the word ‘misconception’ and a large majority of my groups are determined to overcome common misconceptions.”

**Impact on the schools involved**

“It has been inspiring to discuss with other mathematics departments their findings relating to student misconceptions.”

All the schools involved would like to pursue a further collaborative project possibly looking at misconceptions in A-level mathematics and supporting the transition from GCSE to A-level study for grade B/C candidates.

**Advice to teachers who may want to try something similar**

There are several pieces of advice we would give to teachers trying something similar:

- When developing new approaches to effective teaching be aware that the teachers involved might be reluctant to change. Encourage them to try out new resources and activities with a group they trust first rather than trying to completely overhaul their teaching approach. We found that having triads of teachers including experienced and less experienced really helped.
- Provide opportunities and time to carry out lesson study and observations as part of the project.
- Collaborative planning is time-consuming but can be extremely rewarding so build in time for this to happen.
- When planning a collaborative project the “gap tasks” between sessions are crucial in moving the project forward and developing understanding of effective pedagogy. The tasks need to have value but also be manageable given the busy teaching load of many professionals.
- When feeding back or observing lessons focus on the mathematics and how it is being taught rather than other logistics such as lesson structures and plans.
- When preparing ideas for suitable resources give stems rather than a complete resource and let the group develop these stems further i.e. resources do not have to be fully prepared, let the group develop them.
- Start with a small group of teachers and ensure that several teachers from each school are involved to build confidence within the group and a sense of shared experience.
- Look to use ICT effectively as good pedagogical practice where possible, but, at the same time, be aware of the limitations that might exist in some schools in terms of ICT infrastructure.
- Be prepared to support colleagues in other schools and be “hands-on” with any activities the group are asked to do.
- Begin by challenging teachers to look for misconceptions and record these as and when they arise in their own lessons. Make the concepts relevant to the daily teaching experience.
- Sometimes it might be easier to focus on arithmetic misconceptions before exploring algebraic ones.
- Be prepared for people to differ/disagree and have alternative views on what constitutes a misconception and what constitutes a misunderstanding. Allow people to disagree!
- Look for opportunities to share your work with the wider mathematical community this can help shape your own views on the work and allow you to reflect on your own practice.
- Encourage cross-collaboration between schools where possible. Promote the sharing of good resources within the group.

References and resources produced or used