Learning from misconceptions - teachers and students

By Sharon Bolger of Harris Bromley School

Abstract/Summary
The project involved collaborative learning on the theme of discovering and addressing misconceptions in mathematics. We researched and trialled different learning strategies and produced resources and shared learning with other teachers within our schools.

We met regularly as a group of teachers, both to research and learn more about this methodology but also to develop some training materials which we could use with the teachers within our schools.

We decided that we would focus on the teaching of fractions as this was an area we identified in previous years as being a weakness when students started our secondary schools.

“The headlong rush into computation with fractions, using such mumbo-jumbo as ‘add the tops but not the bottoms’ or ‘turn it upside down and multiply’, has often been attempted before the idea of a fraction or fractional notation has been fully understood.”


We researched different pedagogical approaches to learning fractions and decided that we would explore further the methods taught in Singapore schools where there is a heavy focus on teaching for understanding.

Background
Three secondary schools were involved in this project. The schools have a fully comprehensive intake and have a high percentage of FSM and a lower than average KS2 point score. Much work has been done in KS4 to close the gaps in mathematics attainment and teachers now want to focus on KS3. The schools involved in this project have identified a significant number of students entering secondary school with misconceptions, which if not challenged, hinder students learning, progress and enjoyment in mathematics. We want to move students to a focus on underlying understanding to develop sound mathematical skills proficiency.

Aims of the Collaborative Teacher Project
The aims of this project are to identify the common misconceptions relating to number and calculation and identify ways to address these misconceptions through research into different learning theories, teachers’ trialling different strategies in the classroom and evaluating their effectiveness.
Details of those involved in the Collaborative Teacher Project
The mathematics coordinators of the three schools were involved in this project. The project started with a small group of teachers but then extended as more maths teachers across the schools became involved.

A description of the Collaborative Teacher Project
A group of teachers met in the Summer Term of 2012 with the idea of working together to investigate and address students’ barriers to learning.

The first meeting involved the three teachers initially involved, meeting at one of the schools in order to decide the direction of the project. We decided that a useful starting point would be to assess the mathematical skills that our students came to school with and to identify specific areas of weakness and common misconceptions.

We had another meeting the following week where we agreed the content of a skills test that would uncover misconceptions. Our ideas were pulled together by the Federation consultant and a first draft was produced with a curricular analysis spreadsheet that schools could use.

The Federation organises regular KS3 meetings and we had an opportunity to discuss the aims of the project and share the skills test with other KS3 coordinators at the next KS3 meeting.

The three schools involved, used the skills test with their Year 7 classes and completed the curricular analysis. The teachers involved met again and one of the topics identified as being an area where there were common errors was calculating with fractions. In the skills test, students were asked to calculate with fractions and after further probing, it was found that many didn’t have a sound understanding of fraction notation and what it represented. Some were not secure with the concept of the numerator or denominator and had learnt rules for calculating with fractions that they didn’t understand and could not use. One of the conclusions we came to as a team is that we needed to rethink our approach to teaching fractions ensuring we teased out misconceptions and addressed them head on. We looked at where the topic of fractions was placed in the scheme of work ensured that we had planned adequate time to go into this topic at some depth.

The group met three more times before the half term. During these meetings we researched different pedagogical approaches to learning fractions and decided that we would explore further the methods taught in Singapore schools where there is a heavy focus on teaching for understanding. Much of that time involved exploration and discussion of learning through the development of concrete, pictorial and abstract understanding and how students can develop misconceptions if students are moved on to abstract before they fully understand the concrete and pictorial.

We developed resources on teaching fractions through pictorial representations, based on the bar modelling heuristics used in the Singapore curriculum, and these were used in school as a stimulus for working with other teachers on planning collaboratively and team-teaching lessons on fractions.

Before the end of term, we organised for a teacher who was an expert on teaching using this methodology to teach a Year 7 class in two of our schools with maths teachers from the schools observing. We met after the lessons to discuss what we had observed and identified the good practise that we could incorporate into our lessons and share more widely.
We adapted and developed further our resources for teaching fractions and we shared them more widely across other schools in the Federation. Resources such as lessons and teacher guidance packs have been developed at our meetings to support learning through concrete, visual then abstract representations of mathematical concepts and learning through misconceptions.

In each of our schools, we have had regular department meetings during the Autumn and Spring terms where we discussed the approach we would use to teaching fractions, shared resources and ideas and supported other members of the department through collaborative planning, team-teaching and peer observations.

The group involved in this project have shared their learning across a number of different academies and we plan to develop this further in the Summer term where we will invite other teachers from other schools to be involved in the next stage of the project.

**What has been learned from the project?**

By using a test where each concept is at the most basic form, it is clear there are several basic skills where students hold misconceptions. The test, while good for identifying areas where work is needed, is not enough explore the misconceptions themselves, this needs to be done through working with the students on the areas identified. Fractions were identified as a topic in which students across a wide range of abilities find difficult. Through discussions and working with students we learnt that many did not use any intuitive understanding of fractions or division, but applied learnt methods and hence often made errors.

Bar modelling was an effective approach as it allowed students to develop ideas from their own understanding. There are many examples of how bar modelling can be used, it is tempting to fix what you want students to do, however we learnt that problems can be set up in different ways and although you will give examples, it is important that students develop their own approaches otherwise it is just another method.

Bar modelling is a transferable skill. Students first need to be comfortable using the bar modelling methods with whole numbers before adapting for fraction work. Using the modelling students can progress building on prior knowledge and the methods then come from their understanding rather than being given to them by teachers.

**Impact on teachers’ practice**

Before the project, the focus of collaborative planning had mainly involved discussion of teaching the learning objectives and choosing an appropriate task rather than how students learnt. Over the last year, teachers are now involved in deeper learning conversations and are looking more at research and pedagogy.

Investigating why many students find fractions difficult during this collaboration, has allowed us to discuss our own reasoning in terms of fractions and how we teach key concepts. The feedback from teachers has been that they now think about what misconceptions students may have, how the misconceptions may have developed and how to unpick them when they are planning lessons. They understand the importance of allowing students to build on prior understanding and making links and connections, even in older year groups, rather than a quick fix or reminder of a standalone method.

Teachers plan for challenging misconceptions through the concrete and pictorial representations. They have also learnt about the different ways to scaffold tasks using pictorial representations to
support student understanding including using area and bar models. One school highlighted that this was particularly effective in converting between mixed numbers and improper fractions.

Impact on others
Teachers who had used the techniques developed by the team reported that in a test students were able to represent and hence solve more complex worded questions. When presenting their answers the majority of students were able to explain their understanding thoroughly and present logical reasoning using their models. Moving onto other topics students were able to transfer the bar modelling skill where appropriate, so have acquired a new strategy for dealing with problems in mathematics.

Feedback from students has been positive. For example, students were introduced to finding a fraction multiplied by another fraction using the concrete representation involving paper folding. Students were asked to fold their piece of paper into a quarter and then to fold the quarter into a half. This provided a concrete representation of a ¼ multiplied by a ½. Some students commented that this concrete representation of folding paper helped them understand this mathematical concept of multiplying fractions. This was done before students met the general rule and students found this helpful in remembering the general rule.

Other feedback received from students was that the bar model was useful in understanding a problem but some didn’t like drawing the bars as it sometimes took a long time.

Following this we will track these students to see if the methods support better retention as they approach the topics again next year.

An inset was delivered on the techniques we trialled and developed at the KS3 meeting for all our academies. There was a lot of interest and we have now set up an inset over several days at which we will have 20 of our teachers across primary and secondary academies.

Advice to teachers who may want to try something similar
Visual representation of fractions helps students understand equivalent fractions very well. If you keep splitting the bars up then you can find equivalent fractions yet the actual size stays the same which is something very key that can be missed.

One problem is if the students have already been taught certain parts of calculating with fractions using a different method, they see the visual representation as unnecessary as they already know how to do some calculations. However, when they get on to hard calculations they get stuck, so it may be worth getting them to do the easier parts with the visual representation even though they may be able to work the answer out in their heads. Ideally they would be taught fractions in this way right from day 1 so this would not be a problem.

The only other thing is to make sure the students know that using bar models the models have to be stacked on top of each other starting at the exact same position on the left as otherwise subtraction and addition, division does not really work.

References and resources produced or used
Skills test
Skills test analysis spreadsheet
Teacher guidance powerpoint on concepts of fractions and calculating with fractions
INSET on Singapore maths
References:
Singapore math: Simple or Complex?
Source: Educational Leadership 65 No 3 2007

Teaching of fractions from research to practise
Authors: Douglas Edge, Yeap Ban Har

Handbook for Maths teachers in primary schools
Publisher: Marshall Cavendish International