Developing pupils’ understanding of arithmetic, fractions and equality through primary mathematics teaching to further improve rates of progress through Key Stage 2

Emma Penn

Abstract/Summary
In this project, we have explored and trialled different approaches to teaching fractions, multiplication, calculations and introducing pupils to the concept of equality. This includes exploring fractions visually, exploring and evaluating approaches to multiplication, creating different ways of explaining and verbalising mathematics, and posing mathematical problems differently in order to bring out links between operations and methods. We have also engaged several teachers in mathematics CPD that would otherwise have not had the opportunity. This is not just the teachers and numeracy coordinators directly involved in the project, but these teachers have gone back to their schools and delivered whole-school CPD and disseminated approaches to teachers, who have subsequently reported that their confidence in teaching these aspects of mathematics effectively has increased. Through use of a baseline assessment and final assessment, we have been able to evaluate the impact of our work on pupils. As a consequence, pupil progress with regards to these aspects has been noted.

Background
Great Alne is a small rural primary school with 90 pupils. They had an OFSTED inspection in May 2012 and received a grade 3. Their attainment in mathematics is good but progress from KS1 – KS2 needs improving. To quote directly from the report: “They should improve attainment in mathematics to match that in English by: devising ways of improving pupils’ problem-solving skills and providing more opportunities for pupils to use their numeracy skills across a range of subjects.”

Haselor is a small rural primary school with 90 pupils. They had an OFSTED inspection in December 2011 and received a grade 3. Their attainment in mathematics is in line with national figures but progress from KS1 –KS2 needs improving. To quote directly from the report: “Haselor needs to raise attainment in mathematics by devising strategies to improve pupils’ calculation and problem-solving skills.”

This project has allowed both schools to establish more robust assessments of attainment and then subsequent tracking of progress within a key area of mathematics. They have used the research focus to further develop arithmetic by embedding key elements from the development process into schemes and topic approaches.

Tudor Grange Primary Academy, St James is a small primary school in Solihull that converted to become part of the Tudor Grange teaching school alliance in January 2013. In the latest OFSTED inspection before conversion they received a grade 3 in every category. The Headteacher has created new subject leader posts, and developing mathematics provision is a key area of focus.

St Alphege CE Junior School is a high achieving school. However they are experiencing issues with the conversion from KS1 to KS2. This is particularly evident with children who have 2c or 2b
at the end of KS1, but do not attain level 4 at the end of KS2. At the other end of the scale they have children who have achieved level 3 at KS1 and are struggling to convert to level 5. They also have some very high achieving children who are working at level 6. With such a vast range of ability, this project enabled the school to develop a strategy that will improve attainment and progression for all pupils in mathematics.

Widney Junior School is a medium sized junior school in Solihull. It achieved a grade 3 in its last Ofsted inspection and set targets for year 5 attainment and progress between KS1 and KS2 in mathematics. Embedding this key aspect of provision within the curriculum addressed some Ofsted concerns about too many approaches being a barrier to pupil learning. Widney Junior School achieved a grade 2 in the recent Ofsted inspection.

Aims of the Collaborative Teacher Project
The aim of this project was to support progression in mathematics through KS2 by developing pupils’ understanding of key concepts and processes such as the meaning of equality and the equals sign, arithmetic fluency, and use of fractions. Through collaborative working, sessions aimed to develop teachers’ awareness of the key concepts, misconceptions and pedagogy. In addition, the project has developed teachers’ awareness of what is involved in planning and implementing a small-scale research project. A set of resources have been produced collaboratively, trialled, evaluated, improved and embedded into schemes of work (for pupils) and professional development programmes (for staff) so that they can be used long-term within schools.

Details of those involved in the Collaborative Teacher Project
Lee Gray is deputy head at Tudor Grange teaching school. He has oversight of research and development within the alliance, and has managed research projects in a variety of subject areas, establishing links with primary schools. Along with Donna Wright, he has acted as Improvement Agent for the project, ensuring effective communication across the participating schools.

Donna Wright is an experienced LA mathematics adviser with significant experience in acting as an Improvement Agent for NCETM-funded projects. Donna has previously worked for the NCETM as a regional co-ordinator supporting regional and national funded projects. She also holds the NCETM CPD Standard for Solihull LA. Donna has supported networks of schools, in Solihull and Warwickshire, researching teaching and learning in mathematics through providing training to teachers and supporting on-going monitoring and evaluation of the impact of activities undertaken. She has also taken a keen role in disseminating findings from the projects to other schools within and outside of Solihull LA.

Emma Penn and Jessica Devrell, Lead Teachers for Mathematics at Tudor Grange Academy Solihull, have lead on the project. They have a commitment to their own professional development and working collaboratively with others to improve the standard of mathematics teaching and wider teaching practises. They have ran the development sessions and developed, trialled and integrated resources and ideas for teaching approaches during the project, as well as disseminating findings to the rest of the mathematics department at Tudor Grange Academy Solihull.

Mark Williams is Numeracy Co-ordinator at Great Alne School in Warwickshire. Mark teaches the year 5 and 6 class.
Nick Harwood is Numeracy Co-ordinator at Haselor School in Warwickshire.
Jasvinder Bharj Subject Leader for Mathematics at Tudor Grange Primary Academy St James.
Joanne Jones is Specialist Mathematics Teacher at St Alphege CE Junior School.
Alan McClaren is Numeracy Co-ordinator at Widney Junior School.

All the above have attended the initial sessions and developed, trialled and integrated resources and ideas for teaching approaches during the project, as well as disseminating findings and delivering CPD to other staff and updated their numeracy policies accordingly.
A description of the Collaborative Teacher Project

Initially, all mathematics coordinators and headteachers met to discuss their current mathematics schemes of work and approaches to developing the concept of equality and arithmetic proficiency. Collaboratively, the pre-test material was developed in order to assess specific requirements. This was disseminated by mathematics coordinators and used with targeted pupils. Results were analysed by the project lead and lead teachers.

This was followed by a series of professional development meetings and interim reviews involving the mathematics coordinators, including: exploration into current literature and pedagogy surrounding use of the ‘equals’ sign, use of fractions and developing arithmetic fluency, and collaborative development and trial of a common approach to teaching these.

Information and resources were disseminated to teachers and used with pupils. Finally there was a post-assessment to assess the impact of the project, and an event where findings were shared. A newsletter was presented and sent to other primary schools.

As part of the above activities, the self-evaluation tools on the NCETM website were used to audit and develop teachers’ subject knowledge. They were used in professional development meetings and mathematics coordinators will ensure their use by teachers in the schools. During the collaborative meetings, use of the research available on the research gateway on the NCETM website was explored, along with use of Mathemapedia. The community area on the NCETM website was used to seek other teachers’ approaches in this area (by using a thread) and a community will be set up in order for involved schools to communicate and to store electronic resources.

All teachers taking part in the project subsequently engaged staff at their own schools in CPD relating to the project, and at some stage delivered whole-school CPD relating to the project.

Feedback from Great Alne:
After conducting the baseline assessment I identified three key areas for development:
Understand using fractions as operators on other numbers.
Confidence with extending written multiplication strategies
Application of written methods for multiplication and division involving real life problems.
We explored fractions visually, using the image of the divided square or circle, and linked to the language of division. We shared chocolate bars equally between different numbers of people, and represented the operations as improper fractions, extending to fractional quotients and remainders, and thus mixed numbers. We found fractions of complex shapes, by subdividing fractions into smaller parts. We practised counting in halves, thirds and quarters, leading to multiplications of fractions. We used the commutativity of multiplication to find fractions of numbers, introducing the reading of multiplication as ‘of’ (e.g. 2/3 x 6 = 2/3 of 6). We related this to the operations of division and multiplication in order to find a proper fraction of a number.”

Feedback from Haselor:
“As part of our aim to develop written methods of calculation children produced teaching videos for the written calculations. This would provide the class with a good opportunity to talk about, understand and become more confident in their written calculation strategy. The process of creating the videos also enabled me to see if there were any misconceptions and address them. Children began the process by producing a script of what they needed to say in order to help someone else understand how to carry out their method successfully. Once this had been completed, the children read their explanation to a partner and they had to carry out the calculation using only the instruction of the reader. Once children had evaluated the effectiveness of their scripts, they started the recording using a simple video camera and they were then shared with the class.”

Feedback from St Alphege:
“I looked in more detail at individual scripts to find the specific areas where errors were made. A majority of the children were using the multiplication grid for short multiplication (TU x U, HTU x U and U.th x U) and for the long multiplication questions (TU x TU and HTU x TU). The errors that occurred were place value related e.g. 60 x 70 = 4200 or errors made with addition.
From my analysis there were two areas I wanted to focus on:
The methods of multiplication taught.
Teaching approaches and materials used to teach multiplication.

Questions I wanted to answer were: Would focusing on place value and enabling pupil's to check their own calculations highlight the errors they made using the multiplication grid help them to ensure increased accuracy when using this method? Would teaching the written algorithm result in pupils being more accurate and eradicate the errors that I was seeing, especially for the long multiplication problems where place value and addition errors were being made using the multiplication grid?

I chose to split my class into 3 groups to teach the different methods for short and long multiplication:
Multiplication grid for short and long multiplication.
Compact written method for short multiplication (algorithm) and multiplication grid for long multiplication.
Compact written method for short and long multiplication (algorithms).

The groups were of mixed ability (ranging from L4b to L5a) and the children were given an opportunity to choose the group they would like to be in. Usually I teach the compact method for short multiplication in the last term of Year 5, to all children, and the long multiplication compact written algorithm for pupils working at 5b or above, therefore the project brought my teaching forward for some children. I looked at the expanded method for both the short multiplication and long multiplication methods, to ensure they had a good understanding of the place value involved in both methods.

After discussing and exploring ideas of teaching multiplication with the teachers on the collaborative project I decided to base two mental maths sessions (5 – 10 minutes) a week on multiplication and place value. Opportunities were given to children in lessons and the mental/oral sessions where they were given the answer to a problem, so they could check their own calculation to highlight any areas they were making errors (place value and addition errors). At least once a week problem solving lessons ensured that children were tackling problems in the context of measure involving short and long multiplication, as well as specific lessons focused on teaching the written algorithms and the multiplication grid.

Feedback from St James:
Focus was on written methods for multiplication and division. As a result, calculation policy was updated and whole-school inset delivered on methods for multiplication and division.

Feedback from TGAS:
We began our project by identifying areas of arithmetic weakness within our own school with particular focus on year 7. After identifying arithmetic fluency, fractions, equality sign, multiplication and division we devised a pre-assessment focusing on these topics. Pupils completed these assessments in class and they were marked by the class teacher. Using the assessment grid, we were able to identify the areas of weakness for our cohort. We devised starter activities which to improve these skills.
We used these starters for each year 7 lesson across the department for a period of 6 weeks and both pupil and staff feedback noted an improvement in strategy and where appropriate speed. The same assessment was used pre and post to evaluate how much pupils had improved, though the pupils marked their own assessment with a pupil friendly mark scheme and compared their pre- and post assessments. A reflection sheet was then completed to display areas of improvement. Most pupils improved and many stated that the reason for their improvement was a result of work carried out in class.

What has been learned from the project?
Great Alne: Teachers have learnt about different approaches to introducing fractions that increase progress at KS2: Exploring fractions visually and using the image of the divided square or circle, linked to the language of division, practically sharing between different numbers of
people, representing operations as improper fractions, finding fractions of complex shapes by subdividing fractions into smaller parts. Practising counting in halves, thirds and quarters leading to multiplications of fractions. Using the commutativity of multiplication to find fractions of numbers, introducing the reading of multiplication as ‘of’ (e.g. $2/3 \times 6 = 2/3$ of $6$), and relating this to the operations of division and multiplication in order to find a proper fraction of a number.

**Haselor:** Approaches to teaching arithmetic that increase progress at KS2: Creating videos can be a motivational and effective way for pupils to consolidate their understanding and share their experiences and methods in arithmetic.

**St Alphege:** Approaches to teaching multiplication that increase progress at KS2: Benefits of using the compact method: “The children who were most accurate, were those using the compact written methods, with those scoring full marks for an answer ranging from 83% and above, while those using the multiplication grid was as low as 14%. The mistakes seen in the pre-assessment still remained for the children using the multiplication grid, for the long multiplication and U.th x U calculations. A majority of children felt the compact written methods were more efficient, 'less time consuming', and they were less likely to make mistakes. A child who had used the idiosyncratic method for the long multiplication question, felt it took too long so had chosen not to use the multiplication grid, they wanted to learn the compact written algorithm for long multiplication. Many of the children wanted to learn the compact methods by at least Spring term, so they would have time to practise the new method and be confident to use it by the Summer term.”

**St James:** Approaches to teaching multiplication and division that increase progress at KS2: The importance of having formal written methods in a calculation policy shared with staff.

**TGAS:** Teachers have learnt about different approaches to exploring equality and improving arithmetic fluency with pupils: For multiplication we used ‘show that’ questions to encourage pupils to focus on method rather than getting to the answer as quickly as possible. We used a similar method for division. In addition, timetable grids were used to improve confidence in recall. All were effective. For the equality sign we devised starters where the sign was not always at the end of the equation, furthermore, these starters also helped to improve arithmetic fluency through development of number relationships and number bonds.

**Impact on teachers’ practice**
All involved teachers mentioned an increase in confidence regarding approaches to teaching the topics that we covered. In addition, they have all held CPD at their school and adapted their numeracy policies accordingly, and as such many teachers who would not have had the opportunity to engage in mathematics-related CPD have done. Many of the numeracy coordinators involved in the project and the vast majority of the teaching staff have disseminated their findings to those who were not previously aware of the existence of NCETM and did not know that there was mathematics CPD available on the NCETM website. They have used things, such as the self-evaluation tools, at their own schools and are now confident in searching for resources and CPD on the NCETM website, which they have said that they will continue to use.

In particular, at Haselor School for example: “Through taking part in the Collaborative Teacher Project teachers at Haselor School have had a good opportunity to think about how calculations are taught. Children were given questions which would get them to use their written methods in all four operations and staff did an analysis of their own class to see what methods were being used by the children and how effective they were. Through this analysis teachers discussed the implications of our calculation policy. We found that children were broadly in line with the policy but felt as though children in the lower year groups were given too many calculation methods to apply, which caused confusion. As a result, we have modified the policy so that children are given one method and understanding of the one method is focussed upon before introducing more choice in KS2. We hope to see that children will become more confident with these methods lower down the school before trying to apply their understanding to more complex algorithms. Staff also looked at the way calculation methods were displayed around the classroom. All classes displayed the school calculation policy on the wall, but through a staff discussion we concluded that no children used it and that it was just
turning into wallpaper. We came to the conclusion that these were not overly child-friendly and that our displays in the classroom should focus on the process of learning as opposed to the celebration of work. Teachers now display models of calculations as classroom displays."

**Impact on others**

*Great Alne:*

At the end of the project, seven of the target group had made 2 sub-levels progress, two had made a whole level, and one had made 4 sub-levels progress in the five months. The highest rate of progress was at the bottom end of the target group, where 50% of 3A/4C children made a level or more of progress. The follow-up assessment showed a significant increase in confidence with fractions among this group.

*Haselor:*

Children enjoyed taking part in the making of their videos and were motivated by the fact that their videos were going to be used in a real-life context. They felt that this was a much more interesting way to apply their knowledge of written calculations as opposed to doing page after page of sums. In a self evaluation completed by the children, the majority noted that they felt more confident in their written calculations. Children also developed a better understanding of the place value of the numbers within calculations and are less prone to making mistakes. Due to the success of the videos I have decided that the next step was to put the videos up on the school website. This would provide a great chance to celebrate the children’s learning and also could prove to be a valuable resource in the future. The main reasoning behind the idea is that parents will be able to access the videos and they will be able to gain an understanding of the written calculations we use within our policy. Parents and children could then use these as help towards their homework. We are now in the process of opening the strategy up to more children within the school and hope to develop a whole range of calculation videos which cover all methods within our calculation policy.

*St Alphege:*

“The Tudor Grange Collaborative Mathematics Project has given me the opportunity to reflect upon how and when I teach the written algorithms for short and long multiplication to my Year 5 group. As part of my action research the pupils in my Year 5 Mathematics group also had an opportunity to reflect upon the approaches they used, and when they felt it would be best to be taught the written algorithms. I will be sharing the results of my research with the teaching staff at St Alphege C.E. Junior School and will be encouraging them to reflect upon how they teach multiplication to their Mathematics groups. The new Mathematics curriculum is also being introduced in 2014, so this research will give some basis to how we amend the school's routes through calculation."

*St James:*

Involvement in this project with teachers from other schools has had a positive impact on the thinking and strategies used at TGPA in the teaching of Numeracy. It has been really beneficial to have worked with other teachers and to have had the opportunity to share ideas and good practice. The project has enabled the staff at TGPA to recognise the importance of formal written methods of computation of the four rules. This was particularly a focus point because it became evident after the initial assessment, children were using a variety of strategies to calculate, many of which they weren’t secure with. The project has enabled us to unify the variety used so the pupils are competent and secure with at least one method. The project has also helped the staff focus on the importance of children verbalising their thinking in order to prevent mistakes being made and to extend their thinking. A lot more work is focused on allowing children to explain to one another the process they went through to reach their final answer. As a result of this project, the school has adapted their calculation policy to allow for a more formal approach to be adopted at an earlier stage.

*TGAS:*

Most pupils improved and many stated that the reason for their improvement was a result of work carried out in class. We have developed more resources on arithmetic fluency that can continue to be used into the future.
References and resources produced or used
Calculation videos at www.haselorschool.co.uk in the learning zone.
e-Newsletter produced by participants in the project detail references and resources used. Link to this will be provided once it is available online.
Some resources looked at were:
Draft National Curriculum documents
Pre-assessment material
Post-assessment material
NCETM Primary Magazine Issue 49- development session idea
Research articles (such as from BSRLM)
https://www.ncetm.org.uk/resources/34989
https://www.ncetm.org.uk/resources/40529
Written division (Ian Thomson) article