Improving children’s understanding of written algorithms through the use of concrete models

By Gemma Godwin of Saltford C of E Primary School

Abstract/Summary
Following a drop in KS2 maths results, two village primary schools collaborated in investigating KS2 children’s approach and attitudes to questions about the four operations of number. They found that teaching staff lacked confidence in teaching written algorithms, especially division. Lead teachers completed their research with individual children across KS2 year groups in each school, using concrete models to support teaching. The reports back to staff were the catalyst for staff professional development and confidence to develop new school Progression in Calculation policies and guidelines, which develop children’s conceptual models rather than simply teaching written methods. The findings are being shared with a wider Teaching School Alliance community of 30 primary schools.

Background
Bathford is a small primary school (approx. 120 children) serving a village on the outskirts of Bath. The Head has been in post for one year. Saltford is a medium sized primary (approx 350 children) serving a larger village between Bath and Bristol. The Head has been in post for one year. Both schools have a well below the national average percentage of children entitled to free school meals.

Both schools have recently experienced a dip in KS2 maths results. Through observation of children and analysis of SAT results, it was identified that children struggle to retain written methods of calculation and to apply them in different situations. Analysis of common mistakes indicates that the children’s understanding of the concepts behind written algorithms is weak. Teaching staff have reported that they lack confidence in teaching written algorithms, and would like further training on how to develop children’s conceptual models rather than simply teaching written methods.

Maths was a School Development Plan focus for each school and both schools identified progression in written algorithms as a key area for improvement. Using effective concrete models to support this progression was identified as a vital component in improving children’s conceptual understanding. The project has allowed both schools to explore the attitudes and understanding of children and staff, and to explore the conceptual models that underpin the written algorithms, informing school policy, CPD and classroom practice.

Aims of the Collaborative Teacher Project
The aim of the project was to examine how concrete models can be used to improve children’s understanding and retention of written algorithms

Details of those involved in the Collaborative Teacher Project
Gemma Godwin is the maths co-ordinator at Saltford C of E Primary School. Her current School Development focus is to improve children’s understanding of and proficiency with written algorithms with the aim of improving attainment in maths. She is actively involved in working with
parents to support their children with mathematics, and regularly delivers CPD within school. She has recently completed the NCETM lead support programme.

Karen Sykes is Deputy Head and Numeracy Leader at Bathford Primary School. Evaluation of recent Numeracy results has led to the school looking at innovative ways to improve attainment and progress in Numeracy.

Fiona Bell is an independent consultant in mathematics, and is working with Saltford Teaching School on the research, design and management of a CPD programme. She has been delivering CPD to teaching assistants & SENCOs since 1998, holds an RSA Diploma in Specific Learning Difficulties and is completing an MA in Early Mathematics Interventions. To complete the NCETM Lead Support accreditation, she undertook action research into the use of concrete materials to support the teaching of the division algorithm.

A description of the Collaborative Teacher Project

During the project, the maths co-ordinators from each school met on five occasions to share findings and to plan next steps.

Before selecting concrete models to use in the schools, the attitudes, confidence and understanding of both staff and children were assessed. Questionnaires were formulated for the staff based on the NCETM self evaluation tool. These were completed by all staff in Bathford and by KS2 staff in Saltford. At each school a sample group of 24 KS2 children were selected: 6 children with a range of ability from the four KS2 year groups. The teachers in each school interviewed each child about their preferences and confidence in the four rules of number. The children were asked to complete an age-appropriate calculation in each of the four rules. Concrete materials, such as Dienes and counters, were available for the children to use, in addition to pencil and paper. The notes and observations from this activity informed the planning for the next phase of the project, together with the results of the staff questionnaires.

During the project, alongside the CTP, Fiona Bell and Gemma Godwin took part in the NCETM’s Lead Support Programme. The concrete methods explored as part of this training, including place value counters and Dienes blocks, formed an invaluable source of concrete methods to trial. Fiona was also able to share relevant reference articles from her Masters studies, particularly Moyer (2001) which addressed teacher attitudes to the use of concrete materials in the classroom.

A large range of possible concrete materials were collected. These resources included concrete models that were currently in use in each school, those introduced by the NCETM Lead Support Programme and those collected from research. The project teachers worked with the same group of 24 children, introducing them to the concrete models, and asking them to complete the same age-appropriate questions. The same targeted methods were introduced to staff at whole school INSET, staff meetings and planning groups. When staff were familiar with the methods they were asked to trial them with their classes.

The project teachers shared and compared their existing Progression in Calculation (PIC) guidelines for parents. In response to what has been learned from the CTP, both schools have reviewed their PIC guidelines for parents. Meetings have been held with the link curriculum governors in both schools so that the governing body is aware of the work that has taken place and so that they can support in resourcing and training needs.

What has been learned from the project?

Initial work with staff:

**Bathford:** The staff questionnaire results were reassuring in that there were no real gaps in teachers’ knowledge and understanding and all staff had positive attitudes towards maths. Most
teachers were quite confident about their own understanding of the maths that they taught with their own age group. The language of maths, such as the commutative and associative laws, caused confusion and needed clarification. No teacher was aware of using scaling to model multiplication. Most teachers found the language of division – quotient, dividend and divisor – tricky to remember.

Foundation and KS1 teachers were less confident about areas 'they did not use personally on a regular basis', such as multiplying and dividing decimals and using the order of operations. 'I just use the calculator on my phone,' was a common comment. Foundation and KS1 teachers were also less confident with areas they had not themselves learned at school, such as tests of divisibility and digital roots. When the staff conversation turned to multiplication and division methods, the Foundation and year 1 teacher asked if they could leave 'as this had nothing to do with what they taught in the classroom and they had other things they could be getting on with.'

There ensued quite a lively conversation about the role that all year groups have in the development of children's mathematical understanding. When discussing maths methods used, most KS2 staff were unhappy with chunking and using the expanded method of multiplication, as the consensus was that these created confusion for some children and were fairly time-consuming methods, i.e. inefficient. These have now been taken out of the school's maths progression policy.

There was some surprise amongst staff that children were not as confident with division and were struggling to remember a method. We spent considerable time discussing the use of concrete methods and different types of resources that could be used to support the teaching of multiplication and division. Lower KS2 teachers agreed that they were withdrawing practical methods too quickly and only tended to use equipment for SEN children.

**Saltford**: The staff questionnaires showed that staff feel confident and have relatively good subject knowledge in most areas, except for the teaching of division. In this area teachers are less confident, particularly with respect to the vocabulary of division and the teaching of written methods of division. “Chunking” in particular was considered too complicated and confusing, both to teach and for the children to retain. The results also showed a lack of confidence in teaching long multiplication. In discussion, it emerged that many KS2 staff only use concrete materials with SEN children, and do not know how to model more complex algorithms using concrete materials. One comment was that “I can't stand getting the Dienes out – it's too noisy." The work with teaching staff showed that we need to equip the teachers in the school with models and methods that support the four rules, with a particular emphasis on multiplication and division. The issue of how to use concrete materials to support short division and written multiplication will be a particular focus.

**Initial work with children:**

**Bathford**: Most children were secure with written methods of addition and subtraction. Year 3 children were less confident with multiplication. Children in years 3, 4, 5 and less able year 6 lacked in confidence doing division. A small minority of children from lower KS2 used fingers or apparatus to support them. Others who struggled to complete the maths calculations did not use any of the equipment available. A child who has only recently joined the school was the only one to automatically use the Dienes apparatus. The children appeared to be very reliant on 'doing it in their head' and made limited use of jotting. There was confusion about division with at least a third of children in each year group unsure what the division symbol was asking them to do or how to tackle the question.

**Saltford**: The majority of children, regardless of year group or ability, reported that division was their least preferred calculation; many of the older children attempted a half remembered written method, and several of the lower KS2 children had no strategy at all. Some commented that drawing it out might help, but they didn't know what visual representation to use. One year 3
commented “You can do all the others with your fingers, but division is hard because you have to count back”. Several year 4 children attempted to use the inverse operation by counting up in 6s to solve $56 \div 6$, but did not know how to handle the remainder. Children in years 3-5 commented that they could probably do the division practically “I could share out sweets with my friends” but discounted this approach, as “this would take ages for a bigger number”. Written multiplication methods also proved to be an area where upper KS2 children struggled to use an efficient method. They reported that “sometimes I get x and ÷ muddled up” and two children attempted to use the “bus stop” method of division to solve a multiplication problem. Many children’s first thought was to attempt mental methods irrespective of the magnitude of the numbers, and no child used any concrete materials, although they were on the table. This demonstrated the need to strengthen the children’s conceptual understanding of these operations, and to develop concrete models, which the children could use.

Researching and resourcing possible concrete models:
Both schools used current calculation policies as a starting point for collecting concrete models to support multiplication and division. Many of the models support the early stages of multiplication and division (arrays, area models, grouping and sharing) but were not suitable for use with bigger numbers. Online research also reflected this, with most models for division based on the sharing model using paper plates and counters, and on arrays of concrete materials to represent multiplication. Again, there were few concrete representations that could be applied to the larger numbers used in KS2. The NCETM Lead Support materials addressed this through the use of Dienes, straws and place value counters to model the grid method for multiplication, and for short division. It was these models that we eventually chose to trial with teachers in our school. Having researched and decided on the concrete models to be trialled, resources were purchased to enable staff to use the models with their classes. Plain counters were purchased and labelled with 1, 10, 100, 1000, straws to be bundled as HTU and additional Dienes were also purchased. It is possible to buy ready-made place value counters but these are costly to purchase in the quantity required to equip a key stage.

Using the concrete models with the children:

Bathford:
After the initial meeting with the children from each year group, those who had experienced difficulties returned to work on some division problems. This time we worked with concrete materials - counters and bundles of straws - and I modelled how the equipment could be used to help work out the problems. After two sessions with each year group, there was a definite improvement in most children’s understanding. The counters were the favourite piece of equipment. “It’s much easier to see what you’re doing, which helps you understand,” commented a year 4 child. I was particularly impressed with a year 4 girl who, independently, tried out dividing a 4-digit number by a 1-digit number, “Just to see if it still worked.”

Saltford:
Working individually with the children from the initial group, they were introduced to the use of place value materials for short division and multiplication. All children responded positively to the way that the concrete materials showed why the written algorithm worked, with one year 4 children commenting, “Oh, that’s why it works like that!” Even though higher ability upper KS2 children already understood the written algorithm without the concrete materials, one year 6 child commented, “If we’d had this while we were learning the grid method, it would’ve been a lot quicker.” When asked to try the questions from the initial assessment again, all but the higher ability year 6 children used the place value counters to solve the division question, and year 3 and 4 middle and lower ability children chose to model the multiplication question using Dienes or place value counters. They discounted straws as a resource as it was “too fiddly to split the tens and hundreds.”

Sharing findings with staff:
**Bathford:**
The work that was started with this project will continue this academic year. The maths subject leader will be monitoring work in classrooms and working with KS2 teachers to use as wide a range of concrete materials as possible for as long as necessary to support children’s learning and depth of understanding. The Headteacher found the results from the project were useful in that they supported his views on needing to slim down the school’s maths progression policy. It also provided evidence that the lower KS2 children should still be working more practically with concrete materials, a case he has supported for years.

**Saltford:**
The results of our research, and of our work with children, were shared with staff at an INSET day in September 2013. The children’s responses and approaches proved a powerful catalyst for change. Teachers were fully committed to improving progression and to the value of using concrete materials. They worked enthusiastically on developing a simplified and clarified progression in calculation policy. One teacher commented, “It made me think more than I have for years.” The place value counters were particularly well received, and provided a context for a professional dialogue about the progression of counting materials in general, and how counting materials should progress through one-to-one correspondence, then grouped materials such as Dienes and straws, to tokens such as coins and place value counters. The Headteacher has found numerous benefits from been part of this project, maths in school, including the strategic planning for maths, has been reenergised and has a greater clarity of focus and direction. This has shown greater impact on progress across all year groups and led to increased consistency of approach, including a calculation policy for all age groups, integrating effective concrete models.

**Impact on teachers’ practice**

**Bathford:**
At the beginning of the project, it was hoped that it would give some insight into why KS2 SAT results were flagging. Whilst it will be some time before any impact of changes can be seen, there has been positive impact for all teaching staff.

In my own teaching I have started using counters to model division and have encouraged children to use them if they wish. I have become more aware of the need to have more concrete materials available and will be far more enthusiastic about children using the equipment to help consolidate their learning. Working with selected children from different year groups and discussing the outcomes with class teachers has stimulated an educational dialogue that has been waning amongst staff over the last few years. It has stimulated teachers to ‘dust off’ the equipment that has been at the back of the classroom and ‘have a go’ at trying something different. It has also encouraged them to see that concrete materials are not just for SEN children. The impact on day-to-day teaching is that all teachers are now far more aware of the need to provide concrete materials throughout KS2 and to encourage children to be the ones who decide that they are ready to move on to the abstract.

**Saltford:**
As maths co-ordinator, I needed to identify why our maths results had dipped, while other subjects remained more stable. From my role as a year 6 teacher I knew that many children were reaching year 6 with little clear idea of reliable written methods, particularly for multiplication and division. The project has allowed me to explore exactly where children were encountering difficulties, and to identify ways to support their conceptual understanding. My teaching has already undergone changes as a result, with many more concrete resources in use in my classroom, and a much more straightforward route to teaching written algorithms. Although it will take some time for the impact of these approaches to bear fruit across the school, I am confident that all teachers are committed to using the approaches identified and that we will see a positive impact on our results.
Teaching staff reported that they needed a clearer understanding of the children’s learning journey through the four rules of number. Teachers in different phases reported little knowledge of the methods and objectives for phases above and below them. Many teachers also held the opinion that concrete materials had little relevance for KS2 children (other than SEN) and that these materials could not be used effectively to model more complex calculations. The project has enabled all staff to engage with our calculation policy, and feel that they have ownership and a vested interest in the progression in calculation. Using concrete models to support short division and multiplication were particularly well received, and demand for concrete materials has risen considerably.

At the beginning of the project, many staff in KS1 felt that they did not need to know about concrete models for KS2, as they felt that they already used concrete materials effectively within their own phase. After the INSET day, many KS1 teachers asked for a calculation workshop, so that they could brush up their calculation skills and try some of the new materials.

**Impact on others**

**Bathford:**
After the focussed work on division, only the less able year 3 children were still finding the process of using concrete materials to support work in division challenging. These children will be receiving additional support in maths as a result.

The collaborative work with teachers from another school has been quite powerful. I have been shown (and shared) different methods and approaches, which I have been able to bring back to my school to share and try out. It has been quite reassuring to find that some of the issues that I thought were specific to my school are, in actual fact, shared by others and it has been valuable to brainstorm ideas with people of varying experiences. Meeting and working with other professionals has also had a powerful impact in that we are now part of a group forming a larger support network, which will be invaluable with the demise of Local Authority support.

**Saltford:**
The initial work with children identified division as an area needing development. After introducing the concrete model for short division, all children were able to use this effectively to solve the division questions. We will need to revisit this to properly assess its long-term impact on their understanding. Our KS2 teachers are currently trying the new concrete models with their classes as multiplication and division units are taught, and teachers report that they feel confident that these models will improve the children’s conceptual understanding of why the algorithm works. The impact on the school is that the project has reinvigorated our maths teaching and unified our approach to calculation. Teachers are more aware of the need to use concrete materials as a whole, not just in the specific areas addressed by this project.

Taking part in the NCETM’s Lead Support programme at the same time as the project has also been extremely beneficial, enabling me to access the most up-to-date thinking about concrete models and providing a vehicle for sharing ideas and resources across a wider group of professionals.

**Advice to teachers who may want to try something similar**
Ensure that the whole staff is involved, from Headteacher and Governors to TAs. To maximise the impact, this should be at all stages of the project, so that there is a joint responsibility for its success. We shared findings at each stage of our information gathering and involved colleagues with research. We also asked colleagues to try the concrete models we had identified as being effective, to ensure that their wider appeal. Once we had agreed the models to be used, we updated our calculation policies jointly with colleagues, so that all parties had a sense of ownership for the policies and a shared responsibility for their implementation.
Use project funding to devote quality time to focus on the project. This includes time to work with children individually, to meet with project colleagues and reflect/write up outcomes. This dedicated time will ensure that the project remains a high priority and that the work undertaken will be of high quality.

Audit the maths equipment in your school to ensure that you have the equipment to properly implement the project work, both for work with children individually and in a whole class. Also be prepared that demand for concrete equipment will rise as colleagues begin to use concrete resources more widely.

Be prepared for surprises – the research stage of the project may reveal some surprising attitudes from colleagues (such as Early Years staff thinking that discussion about written maths methods are not relevant to their teaching or Year 6 teachers thinking that pupils don’t need equipment) and some shocking results from children (such as trying to use the bus stop method to solve multiplication problems). Be ready to respond constructively to attitudes, comments and misconceptions that people may have regarding their own mathematical abilities and approach to the teaching of mathematics.

Communicate your findings with Governors and parents, so that they can fully support the changes to the teaching of mathematics that will inevitably come about as a result of the project. We have found that parents’ meetings, information leaflets and calculation guidance documents have all needed review in the light of the project findings. You may also find that the findings have a wider impact on resources for the school, so Governor support is essential in budgeting for this.

References


Other NCETM Portal references:
Developing the use of models and images to support progression and proficiency in subtraction and division methods throughout the school (CTP0412)


Article - http://math.coe.uga.edu/tme/issues/v18n1/v18n1_Ojose.pdf
Examples of concrete models for division:
http://www.learner.org/courses/learningmath/number/session4/part_b/index.html

Resources Used

<table>
<thead>
<tr>
<th>Models for multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Practical equipment</td>
</tr>
<tr>
<td>Multilink/unifix towers GG/KS</td>
</tr>
<tr>
<td>Counters/objects GG/KS</td>
</tr>
<tr>
<td>Dienes for larger numbers GG</td>
</tr>
<tr>
<td>Cuisinaire GG</td>
</tr>
<tr>
<td>Place value counters GG/KS</td>
</tr>
<tr>
<td>Straws GG/KS</td>
</tr>
<tr>
<td>Coins GG</td>
</tr>
<tr>
<td>Visual/Picture</td>
</tr>
<tr>
<td>Arrays GG</td>
</tr>
<tr>
<td>Drawings of blocks GG</td>
</tr>
<tr>
<td>Dienes (pictures of) GG/KS</td>
</tr>
<tr>
<td>Number line KS</td>
</tr>
<tr>
<td>Explore whiteboard representations GG</td>
</tr>
<tr>
<td>Models for division</td>
</tr>
<tr>
<td>-----------------------------</td>
</tr>
<tr>
<td>Place value counters GG/KS</td>
</tr>
<tr>
<td>Dienes GG/KS</td>
</tr>
<tr>
<td>Straws GG/KS</td>
</tr>
<tr>
<td>Coins GG</td>
</tr>
</tbody>
</table>