Lesson Materials to Support Teaching of Algebra in KS2/3

Produced through Collaborative Teaching Project

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This project aimed to support transition across the KS2/3 boundary through the collaborative development of a consistent and systematic approach to algebraic teaching and learning. The following lessons were developed to support teachers in utilising resources across this transition period.

Three main aspects of algebra were identified for the lesson materials based on aspects within the New Curriculum.

• Symbol Representing a Number
• Geometry and Algebra
• Equivalence
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• Year Seven

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NEW NATIONAL CURRICULUM

Year 6 Algebra
Statutory requirements
Pupils should be taught to:
- use simple formulae
- generate and describe linear number sequences
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with two unknowns
- enumerate possibilities of combinations of two variables.

Notes and guidance (non-statutory)
Pupils should be introduced to the use of symbols and letters to represent variables and unknowns in mathematical situations that they already understand, such as:
- missing numbers, lengths, coordinates and angles
- formulae in mathematics and science
- equivalent expressions (for example, \( a + b = b + a \))
- generalisations of number patterns
- number puzzles (for example, what two numbers can add up to

Key Stage 3 Algebra
Pupils should be taught to:
- use and interpret algebraic notation, including:
  - \( ab \) in place of \( a \times b \)
  - \( 3y \) in place of \( y + y + y \) and \( 3 \times y \)
  - \( a^2 \) in place of \( a \times a \), \( a^3 \) in place of \( a \times a \times a \); \( a^2 b \) in place of \( a \times a \times b \)
  - \( a/b \) in place of \( a \div b \)
  - coefficients written as fractions rather than as decimals
- brackets
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by:
  - collecting like terms
  - multiplying a single term over a bracket
  - taking out common factors
  - expanding products of two or more binomials
- understand and use standard mathematical formulae; rearrange formulae to change the subject
- model situations or procedures by translating them into algebraic expressions or formulae and by using graphs
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- work with coordinates in all four quadrants
- recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in \( x \) and \( y \) and the Cartesian plane
- interpret mathematical relationships both algebraically and graphically
- reduce a given linear equation in two variables to the standard form \( y = mx + c \);
- calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically
- use linear and quadratic graphs to estimate values of \( y \) for given values of \( x \) and vice versa and to find approximate solutions of simultaneous linear equations
- find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs
- generate terms of a sequence from either a term-to-term or a position-to-term rule
- recognise arithmetic sequences and find the \( n \)th term
- recognise geometric sequences and appreciate other sequences that arise.
Symbol Representing a Number

Year Six

Prior Knowledge
• Four operations
• Inverse operations
• Multiplication as repeated addition (Potentially discuss as starter)

Activity
Swahili Letter Activity
• Explain how symbols can be used to represent missing numbers in calculations. Give example calculation for children to solve e.g. 89+? =100. How do we know what missing number is? How can we check this answer?
• Introduce activity to identify Swahili number worth from missing number calculation, explain will be applying knowledge of missing number calculations to find answer.
• Children to then work out the Swahili numbers in the given calculations. Use their results to substitute into expressions. Children to then create their own questions using Swahili numbers.
• Feedback- class discussion. How did they solve the missing letter through the calculations provided?

Number Grid
• Show number grid with every ball representing number value. How can values of each ball be calculated?
• Refer back to missing number calculations, children to find missing numbers recording calculations.
• Discuss how children found answers, how could values be checked?

Formulae
• Use shape formulae children are familiar with e.g. finding area of rectangle.
• Extend to use of more complex formulae, such as those used to find the volume of 3D shapes.
• Emphasise how letters are used to represent values, and how this is recorded in a calculation.

Future Learning
• Build on finding missing numbers recording algebraically when working.
Symbol Representing a Number

Year Seven

Prior Knowledge
- Four operations
- Inverse operations
- Multiplication as repeated addition (Potentially discuss as starter)

Activity
Fairground Grid
- Introduce the blank fairground grid (tab1) and inform students that the aim is to get the highest end number by arranging the numbers 1-5 in the first column in any order. Students do not know the process in which the final number is calculated. Columns 2, 3 and 4 are blank.
- Students at ‘random’ are selected to input the numbers into the grid and record their output with the aim to get the largest output.
- Students are now shown the grid with all columns visible and discussion is opened (perhaps students discuss in pairs before sharing as a group) as to how each number is calculated (each is the sum of the two previous).
- Students are given 6 copies of the grid to then try and make the largest output they can by rearranging the input numbers.
- Outputs are shared as a class “Who got the highest?” and “Can we prove that 61 (or whatever the highest found output was) is the highest?”
- The numbers 1-5 are replaced with the letters a - e in order to generalize and prove that “61” is the biggest. Students are asked to consider where the numbers in the second column have come from and apply the same idea to the letters. Ie a + b as opposed to 1+3
- Students are given the opportunity to independently complete the grid. As they go they should spot that it gets more difficult to fit the expressions into the boxes and look for a way to simplify (if necessary teacher directed). “How else could we write 3+3+3+3?” Opportunity for correct algebraic notation discussion. Ie $4 \times b \text{ as } 4b$

![Fairground Grid Diagram]

- Looking at the output of the grid above in the context of the original problem can we find where to place the number to maximize the output? Leading to discussion of the largest number needing to be in the middle as that occurred the most getting smaller towards the edges.
- Opportunity to discuss substitution to solve the problem of proving 61 as the largest output and discuss how algebra helps us solve problems.
Future Learning
- Substituting into expressions
- Simplifying expressions with four operations
- Extend to output given and solve for the input
Geometry and Algebra

Year Six

Key Concepts
- Able to collect numbers when calculating perimeter (understand what perimeter actually means – not a formula)
- Understand the concept of area when using compound shapes. When you arrange 2 shapes their area will not change, perimeter however may change.

Prior Knowledge
- Area and perimeter of rectangles
- Compound areas (rectangles)

Activities

Paper algebra (using numbers only)
Pupils should be able to complete the worksheet on paper algebra using prior knowledge of rectangle area and perimeter using numbers. (A4 shapes of paper will help this task as pupils will be able to visualize what they are actually doing – compound area is better understood when pupils can literally move the rectangles around and see why they give the same areas but different perimeters. Pupils need to build their own compound area shapes). Focus should be on area and perimeter being calculated correctly and the understanding of why it is calculated in a particular way.

Taktiles (using numbers only)
Pupils should be able to complete the worksheet on taktiles with guidance. Focus should be on the questions where shape A has an area of, say, 10, shape B has an area of, say, 5 and shape C has an area of A – B (only using the numerical values). This is fundamental for further building the concept of simplifying.

Future Learning
- To apply letters to represent numbers
- Simplifying expressions
- Substitution
- Formulae
- Constructing expressions and equations
Geometry and Algebra

Year Seven

Key Concepts

• To be able to simplify simple expressions
• Understand important algebraic notation (\(a \times b = ab\) etc)
• Understand a letter represents a number

Prior Knowledge

• Area and perimeter of rectangles
• Area of compound shapes (rectangles)

Activities

Paper algebra (using letters)
Similar to Year 6 task, pupils will work through the worksheet with some input from the teacher. A revision of using numbers would be useful to recap pupils’ knowledge and then move onto giving pupils letters instead of numbers. It is useful to use a solitary rectangle to demonstrate some of the important algebraic notation (\(x \times y\) is not written in this way – why not? Explanation). Pupils should then be encouraged to complete the task with as little teacher input as possible. (Again it is useful to use A4 paper to help the pupils visualize what they are doing.)

Taktiles (using letters)
Similar to Year 6 task but pupils will be using letters instead of numbers. Key understanding of describing how to work out areas (not actually finding the area).

Future Learning

• More complex substitution
• Simplifying expressions involving multiplication (e.g. \(3a \times 4b = 12ab\))
• Expanding brackets and factorising
**Equivalence**

**Year Six**

**Prior Knowledge**
- Calculations
- Inverse operations
- Number bonds - flexibility recognising and applying facts to a range of numbers

**Key concepts**
- Understand role of the equals sign in calculations
- Begin to understand and apply the concept of balancing

**Activities**

**Numicon Scale**
Use a balance scale with Numicon pieces to demonstrate balancing aspects of number. Show how 15 (10 and a 5 piece of Numicon) balances out a 9 and 6 piece on scale. Introduce calculation 5 + 6 + 4 using Numicon pieces, which other shape would be needed to balance with 8 + 3 pieces of equipment? Extend to introducing other operations such as subtraction and multiplication using Numicon pieces e.g. how to do you make 20 with only 5 and 4 pieces?

**Whiteboard Calculations**
Children to write calculation on whiteboard using given digits to set criteria e.g. use of odd numbers to 9- 3+5 They then need to locate partner in class with a calculation with the same answer in room. Discuss how different calculations can result in the same end result. Extend to using numbers with different operations, e.g. the numbers 2 and 1 and the symbol for multiplication. Write each aspect on an individual whiteboard and demonstrate how this can be equivalent to a division calculation of 10 and 5. Model how the multiplication can be changed around to still provide an equivalent result, whereas the division can not. Can children make their own equation with the same balancing answer as above?

**Balancing number questions**
Share with children incorrect model e.g. \(6 \div 2 = 8 \div 4\) and discuss with children whether the answers to both sides of the equation sign are equal, reiterating the meaning of the equals sign in calculation. Correct and discuss whether there are a range of possibilities- can children suggest different calculations to use in number sentence? Children to check different given calculations, correcting those that are incorrect as required. Extend to children completing their own made sentences in same manner as above.

**Branching diagrams**
Introduce branching diagram, demonstrate how a number on one branch on the diagram must be equivalent to the numbers related to two branches on an opposite arm, and move to more complex examples with various branches and offshoots.

**Extension**
- Use of number cards (+1 and -1) to support the balancing of more complex equations. (Appendix )
- Extend to introducing letters into equations when children are secure e.g. \(a+b=b+a\) is the same, \(a+a+b+b=b+a+b+a\)
• Extend to scale problem, how can this calculation be solved by utilising the principles behind equivalence? (Appendix)

Future Learning

KS3- To simplify and manipulative algebraic expressions to maintain equivalence by:
  • Collecting like terms,
  • multiplying a simple term over a bracket,
  • taking out common factors
  • expanding products of two or more binomials
Equivalence

Year Seven

Prior Knowledge

• Calculations
• Inverse operations
• Number bonds- flexibility recognising and applying facts to a range of numbers

Key concepts

• Understand role of the equals sign in calculations
• Begin to understand and apply the concept of balancing

Activities

Scales
Use a scale with weights to demonstrate balancing aspects of number. Show how 800g balances out a 500g and five 60g weights on scale. Introduce calculation 80g + 1kg + 50g = 20g + 10g + 100g + 500g + 500g. Extend to introducing other operations such as subtraction and multiplication to make equal weights. Three lots of 50g are the same as...?

Balancing number questions
Share with children incorrect model e.g. 6 ÷ 2 = 8 ÷ 4 and discuss with children whether the answers to both sides of the equation sign are equal, reiterating the meaning of the equals sign in calculation. Correct and discuss whether there are a range of possibilities- can children suggest different calculations to use in number sentence? Children to check different given calculations, correcting those that are incorrect as required. Extend to children completing their own made sentences in same manner as above.

Weight problems
Show children weight problem (appendix), how would they work out the value of the smaller weights shown on the left hand side? Model how to take out common factors to balance and find the answer. E.g.:

\[
800 = 500 + 6x \\
-500 \quad -500 \\
300 = 6x \\
\div 6 \quad \div 6 \\
50 = x
\]

Give alternative weight problems to practise taking out common factors.
Extension
  • Branching diagrams. Students to find the values of given letters within the
diagram from total amount provided. Ensuring branches are balanced at the
appropriate points. Record work using formula

Future Learning

KS3- To simplify and manipulative algebraic expressions to maintain equivalence by:
  • Collecting like terms,
  • multiplying a simple term over a bracket,
  • taking out common factors
  • expanding products of two or more binomials
Appendix
Symbol Representing a Number

Appendix One
Swahili is a derivative of the Bantu language and remains loyal to Bantu grammar, however its vocabulary has been influenced by Arabic (through culture and trade) and more recently by English (through technology). The word Swahili comes from the Arabic word for coast, since the language developed along the East African coast where several distinctive dialects still remain. Swahili has been described as "One of the twelve great languages of the world" and is spoken by millions of people in Central and Eastern Africa.

Work out the Swahili numbers from 1 to 10 using these clues:-
- saba + 6 = 13
- 3 x nne = 12
- sita – 3 = 3
- kumi ÷ 5 = 2
- 4 x mbili = 8
- (4 x tatu) + 3 = 15
- 3 x (12 – nane) = 12
- \( \frac{1}{2} \) tisa = 4\( \frac{1}{2} \)
- 2 ÷ moja = 2
- tano x tano = 25

Now you have worked out the numbers from 1 to 10, complete these calculations – in Swahili!

a) moja + mbili  
b) tatu + tano  
c) sita – tatu  
d) saba + mbili  
e) nne x mbili  
f) nane + mbili  
g) tisa x moja  
h) tatu x sita – tisa  
i) nne x mbili – tatu  
j) (nane – tatu) x mbili
# Number Grid

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Fairground Activity (electronic Excel Spreadsheet)

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Geometry and Algebra

Appendix Two
Paper Algebra

Without measuring, how could we describe the area of this piece of paper?

How could we describe its perimeter?

Cut piece 2 along the dashed line, and put one half aside.

What are the dimensions of this new piece? (1 and $\frac{1}{2}w$)

Cut piece 3 along the dashed line, and put one half aside.

Can you find the areas and perimeters of these shapes?

What are the dimensions of this new piece? ($\frac{1}{2}l$ and $w$)
Find the perimeters and areas of these compound shapes:

And more challenging:
Post-it note Algebra

Take a post-it note and draw the lines shown:

![Diagram showing a square with two lines drawn from opposite corners]

Label the lengths shown p and q:

![Diagram showing a square with a diagonal line and labels p and q]
Cut along the lines to make three pieces:

Fit your pieces edge to edge to make different shapes. What different perimeters can you make? eg
The full set of tak-tiles
Tak-tile Areas

This has an area of $a$ and this has an area of $b$

So this shape has area $a + b$

Think about this shape:

It could be made like this or like this

$2a + 2b = 2(a + b)$
How many different ways can you find of writing the areas of these shapes?

a)  

b)  

c)  

d)  

e)  

f)  

So now can you do these?

Remember to write them in as many different ways as you can find!!

a)  

b)  

c)  

d)  

e)  

f)  

Equivalence

Appendix Three
Positive and Negative Number Tiles

+1  +1  +1
+1  +1  +1
+1  +1  +1
+1  +1  +1
+1  +1  +1

National Centre
for Excellence in the
Teaching of Mathematics
\begin{array}{ccc}
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Scales and Branching Diagrams